Time Frame: Approximately five weeks

Unit Description

This unit provides an examination of properties of measurement in geometry. While students are familiar with the area, surface area, and volume formulas from previous work, this unit provides justifications and extensions of the students’ previous work. Significant emphasis is given to 3-dimensional figures and their decomposition for surface area and volume considerations.

Student Understandings

Students understand that measurement is a choice of unit, an application of that unit to the object to be measured, a counting of the units, and a reporting of the measurement. Students should have a solid understanding of polygons and polyhedra, the meaning of regular, the meaning of parallel and perpendicular in 3-dimensional space, and the reason pyramids and cones have a factor of $\frac{1}{3}$ in their formulas.

Guiding Questions

1. Can students define and provide justifications for polygonal and polyhedral relationships involving parallel bases and perpendicular altitudes and the overall general $V = Bh$ formula, where $B$ is the area of the base?
2. Can students use the surface area and volume formulas for rectangular solids, prisms, pyramids, and cones?
3. Can students find distances in 3-dimensional space for rectangular solids using generalizations of the Pythagorean theorem?
4. Can students use area models to substantiate the calculations for conditional/geometric probability arguments?
## Unit 7 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

### Grade-Level Expectations

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
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</thead>
<tbody>
<tr>
<td>7.</td>
<td>Find the volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)</td>
</tr>
<tr>
<td>9.</td>
<td>Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)</td>
</tr>
<tr>
<td>12.</td>
<td>Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)</td>
</tr>
<tr>
<td>19.</td>
<td>Develop formal and informal proofs (e.g., Pythagorean Theorem, flow charts, paragraphs) (G-6-H)</td>
</tr>
</tbody>
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### Data Analysis, Probability, and Discrete Math

21. Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)

### CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
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<tbody>
<tr>
<td>G.GMD.4</td>
<td>Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two dimensional objects.</td>
</tr>
<tr>
<td>G.MG.1</td>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
</tr>
<tr>
<td>G.MG.2</td>
<td>Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).</td>
</tr>
<tr>
<td>G.MG.3</td>
<td>Apply geometric methods to solve design problems (e.g., design an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</td>
</tr>
</tbody>
</table>

### ELA CCSS

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
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<tbody>
<tr>
<td>RST.9-10.1</td>
<td>Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.</td>
</tr>
<tr>
<td>WHST.9-10.2d</td>
<td>Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.</td>
</tr>
</tbody>
</table>
Sample Activities

Activity 1: Experiment with Volume and Surface Area (GLEs: 7, 9, 19; CCSS: WHST.9-10.2d, WHST.9-10.10)

Materials List: paper, pencil, stiff paper, cardstock, transparency sheets, shallow box, rice or other filler

Begin this activity by having students complete a SPAWN writing (view literacy strategy descriptions) from the Problem Solving category. In this case, students should be given a prompt which will ask them to form a conjecture about the relationship between two cylinders with different bases and heights.

To present the writing topic, take two sheets of paper and create two baseless cylinders like the ones shown below.

Present the cylinders to the class and have them write a few sentences to answer the following prompt from the Problem Solving category (post the topic on the board or the overhead):

*Problem Solving:* Based on your prior knowledge of volume and surface area, how might you solve the problem of determining whether the volumes and surface areas of the two cylinders are equal or whether the shorter or taller cylinder has the greater volume and greater surface area. Explain the reasoning you used when forming your ideas about how to solve this problem.

Do not have the students calculate the volume and surface area at this step—their predictions about how to do this should be based solely on their prior knowledge and their observations.

After giving the students time to write their responses, have a class discussion (view literacy strategy descriptions) where students will share their writings. For this portion of
the activity, use the Think Pair Square Share strategy. When students wrote their SPAWN writing, they completed the “Think” part of the discussion. Now, have students pair up and discuss their conjectures for about one minute. Then, have the pairs join another pair to form a group of four, or a square. In these squares, the students should discuss their conjectures and their reasoning. After a few minutes of discussion, have groups report their conjectures and reasoning to the class. Then, perform a demonstration for the students (or have the students perform the demonstration by giving them the necessary materials) by taping two sheets of paper to form the two cylinders, one short and one tall. (Stiff paper is helpful. Transparency sheets may be used). Hold the tall cylinder upright in a shallow box and fill with rice. Now, fill the shorter cylinder and compare the two amounts of rice. Have the students determine whether their conjectures were correct.

Now, have the students calculate the volume surface areas of both cylinders. Lead the students in a discussion as to why the volume and surface areas are not the same even though the sheet of paper used for each cylinder is the same size. Include in the discussion, a study of the areas of the bases and a determination of the effects of squaring the radius when calculating the areas of those bases.

At this point, have students analyze the formulas for the volume and surface area of a rectangular prism and a cylinder which they should have mastered in eighth grade in order to generalize them for other polyhedra. Ask students, “What part of each of these formulas have you seen before?” For the volume formulas, \( V = lw \) and \( V = \pi r^2 h \), students should recognize the area formulas for a rectangle and circle. For surface area, \( SA = 2lw + 2wh + 2lh \) and \( SA = 2\pi r^2 + 2\pi rh \), students should also recognize the area of a rectangle, the area of a circle, and the circumference of the circle. Lead students to develop an informal proof that the volume of any polyhedron (a 3-D figure with polygonal bases) will be found using the formula \( V = Bh \) where \( B \) is the area of the polygonal base. Also, discuss that the surface area of any polyhedron is the sum of the area of the bases and all lateral faces (the term lateral face may need to be defined here). Point out how the surface area formulas for the rectangular prism and cylinder given above fit these definitions. Students will need help understanding how \( 2\pi rh \) relates to the area of the curved surface of the cylinder. Note: A cylinder is not a polyhedron but the development of the formulas for volume and surface area of a cylinder is the same which is why they are used in this activity to help students develop the generic formulas.

To conclude the lesson, present another SPAWN writing (strategy link here) prompt which challenges the students to reflect on or think more critically about what they have just learned:

**What If?**

Given a cylinder which has a base with a diameter of 10 inches and a height of 12 inches, what would happen to the height of the cylinder if the volume were to remain the same, and the base was increased by 50%? What would happen to the base of the cylinder if the volume were to remain the same, and the height was increased by 75%?

Once students complete their writings, have students share their ideas with a partner, then a group, and then the entire class as they did after the Problem Solving prompt. These
discussions should include their calculations of volume and the new measures for the height and base.

Activity 2: Building a Pyramid (GLEs: 9, 12)

Materials List: two 8.5” x 11” sheets of paper, two pairs of scissors, two rulers, tape for each pair of students; for each student—pencil, calculator

Have students work in pairs and use the materials provided to construct a square pyramid which has a height of 7 cm and a base length of 4 cm. Each of the students is to make a pyramid, but the use of pairs allows them to think through the process together. Ask students to think about what other information they will need in order to construct the triangle faces of the pyramid. Record students’ ideas on the board or overhead. It may be a good idea to make paper models with different measurements in advance of the activity and then allow students to dismantle the pyramids to determine needed measures. If necessary, lead a discussion of the location of each measurement on the net in comparison to the 3-dimensional pyramid. Students should see that they will need to use the Pythagorean theorem to figure the slant height (height of the triangle for one of the sides) in order to construct the faces of the pyramid. They should also understand that the four triangles are congruent and isosceles.

The most efficient way to build the pyramid is to make a net consisting of a square with an isosceles triangle drawn on each side. The isosceles triangle should be constructed so that the height of the triangle is the calculated length of the pyramid’s slant height. Students who have trouble visualizing the net may use other methods, such as drawing each side individually and taping them all together.

Have students discuss ways they might find the volume of the pyramid. This discussion should include the understanding that the volume of a polyhedron, in general, is $V = Bh$. However, students should realize that this will not apply to a square pyramid because the pyramid is only a part of a cube or rectangular solid (which is the volume that would be found if the general formula is used). The students may not be able to find the volume at this point—it will be discussed more in Activity 4. Also lead the students in a discussion about the information that would be necessary to find the unknown height of a pyramid (or possibly an unknown base length of the pyramid if the height were given). Ask students to apply this knowledge by creating another pyramid with a different regular polygon as the base.
Activity 3: Surface Area (GLE: 7)

Materials List: models of pyramids from Activity 2, pencil, paper

Using their models from Activity 2, have students determine the total surface area of their constructed pyramids and describe the process of finding the surface area of the pyramids. Students should find the surface area of the pyramids with square bases and other regular polygon bases as well.

Activity 4: Volume and Surface Area of Pyramids and Cones (GLE: 7; CCSS: G.MG.1)

Materials List: volume model kit(s), rice or other filler, pencil, paper

Compare the volumes of a pyramid and prism with the same base and height as a demonstration using a volume model kit. If enough model kits are available, have students work in groups of 2 or 3 when completing the activity. Students could also make their own models using old manila file folders and then complete the activity.

Fill the pyramid from the kit with rice or unpopped popcorn. Ask students to estimate how many times the pyramid must be filled in order for the prism to be filled. Do the same thing with a cone and cylinder. Develop the concept that the volume of a pyramid and cone is one-third the volume of a prism and cylinder, respectively, if the two solids have the congruent heights and bases. As an extension, ask students to estimate the relationship between the cone and the sphere which are also a part of the kit. Since the cone must be filled twice before the sphere is filled, the sphere is twice as large as the cone’s volume or \( \frac{2}{3} \) the volume of the cylinder. Provide real-life applications in which students must find the volumes of cones, pyramids, and prisms. These problems can include objects just as ice cream cones, cylindrical water tanks, ice chests, etc. and should not explicitly ask students to find the surface area or volume, but rather should require students to use this information to solve a problem. To meet the standard of modeling here, students should be able to describe the situation with which they are presented through mathematics by using the properties of the shapes they have studied to represent the objects addressed in the problem.

Sample problem type: The Art club will run the concession stand for the basketball game this week to raise money to go on a field trip. They would like to sell soft drinks and have access to three 48-quart ice chests. How many 12-ounce canned soft drinks will they be able to put in the ice chests, with ice, for the concession sale? Be able to justify your solution with your mathematics and reasoning. Solution: Solutions will vary. For this type of problem, students may need access to different ice chests and some 12-ounce cans to be able to investigate space and how the cans fit in the ice chest. Students should be able to justify their work using the formulas for volume discussed in this unit as well as stating any assumptions they may have made to help them solve the problem.
Students should be asked to present their solutions to various problems to the class. The class should be given the opportunity to critique each other’s reasoning in order to provide more clarity to solutions.

Activity 5: Rotations of Plane Figures (GLE: 7; CCSS: G.GMD.4)

Materials List: graph paper, pencil, “Turning” a Plane Figure Into a Solid Figure BLM, the Internet, chart paper, markers

Begin with a demonstration to pique students’ interest and assist them in understanding how a solid figure can be created by rotating a polygon about an axis. This can either be done as a whole class demonstration, or students can work in pairs at computers. Go to the website http://www.shodor.org/interactivate/activities/3DTransmographer/. On this website, there is an interactive grapher that allows the user to transform a polygon to visualize what would then be created.

1. Create a pentagon: type 5 in the box on the right where it asks for the number of vertices and click Go!
2. Type the values for these ordered pairs in the boxes on the right: (2,7), (4,6), (5,4), (4,2), (2,1). Then click Graph (you may have to scroll down to find the button.
3. Under the coordinate plane, in the box that has Reflect or Revolve, select the radio button next to “across x = 0.” Do not change the value in the box. Then click Revolve.
4. Then click on the coordinate grid and drag the grid so students can see various views of the solid that was created.
5. Discuss with students why they think the solid has the resulting shape. If necessary, create other polygons and experiment with other lines and axes. Do not create rectangles or right triangles.

Once students have seen the demonstration, have them work in pairs to complete a process guide (view literacy strategy descriptions) for the next part of the activity. This process guide will guide students through creating a solid figure (or visualizing the creation of a solid figure) by rotating plane figures about the axes of the coordinate system. Give each pair of students a copy of the “Turning” a Plane Figure Into a Solid Figure BLM. Have students work through answering the questions posed on the BLM. The process guide begins by having students graph the coordinates of the vertices of a rectangle then finding the dimensions and area of the rectangle. Next, the guide asks students to imagine the rectangle being rotated about the axes and to identify the dimensions of the object created. After they have drawn models of the figures created, they are asked to make a conjecture using their knowledge of volume and surface area of solid figures. Finally, they are asked to find the volume and surface area of the solid figures and verify their conjectures. Depending on the level of the students, discussions may need to occur as each question is completed to help clarify any misconceptions. Once students have completed the process guide, lead a class discussion and have pairs
report their thinking to the class. Pairs of students should present the following information to the class by drawing/writing the following on chart paper:

a. Draw a representation of the graph in question #1.
b. Draw the object that was created in question #2 and label the dimensions.
c. Draw the object that was created in question #4 and label the dimensions.
d. State the surface area and volume for each of the objects.
e. Identify which object has the greatest surface area and volume.
f. Explain what the difference is between the two objects that is responsible for making one larger than the other.

Provide additional sets of vertices for polygons and have students determine which solids are created from the new polygons. Be sure to make one set of vertices represent the vertices of a right triangle (i.e., (2,0),(0,4),(0,0)). Have students find the volume and surface area of the cone created by the right triangle. For a challenge, have students use their understanding of volume and surface area of cylinders and cones to find the volume and surface area of other solid figures produced by other polygons.

If students have trouble visualizing the rotation of a rectangle to form a cylinder or a right triangle to form a cone, use the websites listed below. These websites have an animation that shows rotation to create a cylinder and cone. The websites are in French, but the buttons are pretty easy to understand.

http://dmentrard.free.fr/GEOGEBRA/Maths/Espace/cylindre.html
http://dmentrard.free.fr/GEOGEBRA/Maths/Espace/Cone.html

Button list:
- Animer/Arreter – starts/stops the animation
- Remise à zero – resets the rectangle/triangle to “zero.”
- Animation plus rapide – will make the rotation go faster (it will actually start/stop the rotation as well; each time the rotation starts, it is faster than before)
- Animation moins rapide – will make the rotation go slower (it will actually start/stop the rotation as well; each time the rotation starts it is slower than before)
- Trace activée – will trace the rectangle around the surface of the cylinder/cone
- Trace effacée – will only show the outline of the cylinder/cone

There are also sliders on the site. For the cylinder, there is only one slider in yellow in the top left corner. This slider allows the student to rotate the rectangle manually. For the cone, there are two other sliders on the right. The top slider, “Rayon,” will change the measure of the radius (i.e., base of the right triangle) while the bottom slider, “h,” will change the height of the cone (i.e., the height of the right triangle).

**Activity 6: More with Volume and Surface Area (GLE: 7)**

Materials List: pencil, paper

Have students review the processes for finding the surface area of prisms and pyramids. They should generalize that to find the total surface area of a prism or pyramid, they need...
to find the sum of all the areas of the lateral faces and the base(s). The volume of the prisms can be generalized as the product of the area of the base and the height of the prism \((V = Bh)\). The volume of the pyramids should be generalized as finding \(\frac{1}{3}\) of the product of the area of the base and the height of the pyramid.

After these generalizations are made, have students practice finding the surface area and volume of prisms and pyramids with regular polygons as their bases.

**Activity 7: Cross Sections of Solids (CCSS: G.GMD.4)**

Materials List: paper, pencil, the Internet, modeling clay (optional), string (optional), solid shape models (optional)

For this activity, students will need to access the website [http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html](http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html). The website has an interactive activity for students to investigate cross sections of solids. Students will use the interactive tool to determine the different polygons that can be created when slicing a cube after different rotations of the cube. There are four problems for students to solve. Students should record their answers. There is a button for solutions for each problem. Direct students not to click on the solution buttons while they are completing the investigations.

Once students have had the opportunity to answer the problems and record their solutions, have students share their findings with the whole group. Problems 3 and 4 will be more difficult for students to visualize, so it will be helpful to have three-dimensional models or modeling clay and string to help them see the different slices that could be made on the given solids. For problem 4, the solution indicates that a hyperbola could be created by slicing the cone perpendicular to the base, not through the vertex. Technically, this is only half of a hyperbola as the true hyperbola is created by slicing a double-napped cone (two cones joined at their vertices). Students should be able to easily see the circle, the ellipse (some may call it an oval, so use the correct terminology), and a triangle (the cone sliced through the vertex at any angle and intersecting the circle in two places).

**2013-2014**  
**Activity 8: Efficient Packaging (CCSS: G.MG.3, WHST.9-10.2b)**

Materials List: Soda Cans activity sheet from NCTM Illuminations website, pennies or other congruent circular items, pencil, paper, calculator, the Internet

In this activity, students will investigate packaging efficiency in terms of area and volume. Begin with the NCTM Illuminations activity, Soda Cans, located at [http://illuminations.nctm.org/LessonDetail.aspx?id=L690](http://illuminations.nctm.org/LessonDetail.aspx?id=L690). This lesson leads students through an investigation of two different problems. The first problem involves arranging soda cans on a shelf, while in the second problem students will attempt to determine the
best possible package to carry more than one can of soda. Be sure to review the website prior to starting the lesson as the website provides valuable information about what students may ask and discover. This lesson focuses on the area covered by the soda cans when determining efficiency. Additionally, have students determine the amount of packaging material that would need to be used based on the arrangements they created (surface area). Also, have students determine the packaging efficiency based on the volume of the package and the space taken up inside of the package (for example, divide the volume of the soda cans by the volume of the total package to determine the percentage of space being occupied by the soda cans). At the end of the lesson, have the groups of students create a RAFT writing (view literacy strategy descriptions) assignment for their presentation to the class. The parts are defined as:

R – Role – concerned consumer group
A – Audience – any beverage company that produces soda cans (fictional or real)
F – Form – letter
T – Topic – explain why the design the group created is more efficient than the current design used by the beverage company and why that company should change to the group’s design.

In this letter, students should be able to explain their tool for determining the efficiency of the packaging. Students should also understand that while they are free to be creative, their letter should include the correct mathematics to convince the beverage company to accept their design. Once the RAFT writing is completed, have groups share their designs and their writings with the whole class. Students should listen for accurate information and sound reasoning in the RAFTs.

2013-2014
Activity 9: Other Design Problems (CCSS: G.MG.3)

Materials List: cylindrical container, water, irregular objects (like an egg-shaped paperweight), tube of toothpaste (with the box), ruler, centimeter cubes, paper, pencil, calculator

This activity builds on the knowledge and reasoning students developed in the previous activity. Provide students with a cylindrical object whose volume can be calculated and with markings to measure a predetermined amount of water (a beaker from a science class would do well). Ask them to place water in the cylinder but not to fill it to the top. Discuss with the students the volume of water in the beaker. Ask them to place an irregular object, like an egg-shaped paperweight, into the water, being careful not to spill any water. Note the displacement of the water and determine the volume of the paperweight. Depending on how the markings are listed on the beaker, the measurement of volume for this object is likely to be in milliliters. Discuss with students that 1 milliliter is the same as one cubic centimeter and when each measure should be used. Discuss what this information means for the volume of the irregular object placed into the water.
Next, repeat the activity with a tube of toothpaste. Have the box for the tube on hand as well. After students determine the volume of the tube of toothpaste, have them determine the volume of the box (it may be necessary for students to measure the box to determine the volume). Instruct students to determine what percent of the box is used by the toothpaste and what percent is empty space. Discuss with students whether they believe the toothpaste is packaged efficiently based on their understandings from Activity 8. Ask students to attempt to design a package that is more efficient for the tube of toothpaste. Students may wish to change the tube as well, as long as the amount of toothpaste does not change. Students should work in groups of 2 – 4 to work on a more efficient design. Then have groups present their ideas to the class and justify why they believe their design is more efficient than the current design. Additionally, students may vote on which design for the entire class is the most efficient.

**2013-2014**

**Activity 10: Density in Geometry (CCSS: G.MG.2, RST.9-10.1)**

Materials List: paper; pencil; the Internet; Population Density BLM; meter sticks, yardsticks, or measuring tapes

Begin by using student questions for purposeful learning (SQPL) (view literacy strategy descriptions). Present students with the statement, “The United States of America is overpopulated.” This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks. Students are not to decide whether they believe the statement is true or false; instead they are to develop a list of questions that they need to answer to determine whether the statement is factual. Allow students to ponder the statement for a moment and ask them to think of some questions they might have related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. When all of the pairs have developed their questions, have one member from each pair share their questions with the class. As the questions are read aloud, write them on the board or overhead. Students should also copy these in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in some way. Once all of the students’ questions have been shared, look over the list and determine if additional questions should be added. The list should include the following questions:

- What is the population of the USA?
- What does overpopulated mean?
- Why is overpopulation a concern?
- How does the population of the USA compare to the population of other major countries like China, India, and Canada?
- Are some areas of the USA more populated than others? Why?
- Can we compare populations by just looking at the number of people, or do we need to consider other measures?
At this point, be sure students have copied all of the questions in their notebooks and continue with the lesson as follows. The questions may be written on chart paper/posters that can be posted around the room so the questions are visible throughout the lesson. Tell students to pay attention as the material is presented to find the answers to the questions posted, focusing on those questions which have been starred or highlighted. Students should refer to these questions throughout the lesson as all questions may not be answered until the end of the lesson.

Ask students, “Do you believe our classroom is overpopulated? Why?” Students should think about their responses for a minute alone, then share their thoughts with the whole class. Based on their answers to the question, have students brainstorm their ideas about what the word *overpopulation* means. Have them share their ideas with the whole class and attempt to get a consensus from the students to develop a class definition of *overpopulation*. Then ask the students if anyone wishes to change his/her answer to the first question based on the definition created. Have students justify their reasoning if they do change their position. Then present students with the definition from Merriam-Webster: “the condition of having a population so dense as to cause environmental deterioration, and impaired quality of life, or a population crash.” Discuss with students how the class definition compares to the dictionary definition. One of the key words in the definition is *dense*. Ask students how they might determine the density of any specific population. Then have students visit the website [http://geography.about.com/od/populationgeography/a/popdensity.htm](http://geography.about.com/od/populationgeography/a/popdensity.htm) and read the information presented about Population Density. If access to the Internet is limited, the information can be printed from the website to be handed to each student. Instruct students to answer the questions below and cite evidence to defend their answers.

1. What is population density?
2. Is there a formula for population density? If so, what is it?
3. How might someone compute the population density of a school?
4. If Monaco has a total population of 32,000 people, why is the population density 43,000 people per square mile?
5. Based on the article, if we know that a country has a population density of 73 people per square mile, can we assume that each square mile of land area has 73 people living on it? Why?

After students have had time to read the information and answer the questions, have pairs of students discuss their answers. Then have the pairs of students make groups of four to discuss their answers further. In these discussions, students should be able to defend their answers citing evidence from the informational text. After allowing students time to discuss in groups, have groups share with the whole class.

Then ask students to estimate how much space each person has in the classroom. Have them explain their reasoning for their estimation. Then give each student a copy of the Population Density BLM. Have students work in pairs to answer the first three questions on the BLM. They will need meter sticks, yardsticks, or measuring tapes to find the length and width of the room. Students should know how to find the area of the room before this lesson. After students have had time to complete the first three questions, have
them share their findings with the class. Revisit the question about whether the class is overpopulated and have them use the information they have calculated to defend their answer. Then, have the students complete questions 4 and 5 individually. After students have completed questions 4 and 5, check the answers with the whole class. Lead students through a discussion about why some cities are more heavily populated than others. Use a website like http://projects.nytimes.com/census/2010/map to illustrate the population density and have students see where most people have settled.

To end the lesson, revisit the SQPL statement presented at the beginning of the activity for which students developed questions. Revisit all of the questions the students asked and answer as many as possible. There may be some questions that are not able to be answered through this activity but that may present an opportunity for further discussions/activities related to population density.

Continue the application of the concept of density through other problems that may deal with the size of a room to fit a set number of people, the size of a crop to be planted on a given acreage of land, the amount of fish that can be safely placed into an aquarium, etc.

Activity 11: Geometric Probability (GLE 21)

Materials List: pencil, paper, calculator

Students should be given problems that require them to find the area of a variety of shapes. This should include basic figures as well as figures within other figures, or combinations of figures. Ask students to find the probability of randomly selecting a point in a shaded region of the given figure.

Example:
Mark created a game consisting of 32 squares on a rectangular game board. The board measures 1-foot by 2-feet. 16 of the squares are 3-inches by 3-inches while the other 16 squares are 2-inches by 2-inches. He earns 3 points for hitting the board and not hitting a square, 5 points for hitting one of the larger squares, 10 points for hitting one of the smaller squares. What is the probability that he will earn 10 points with one throw of a dart? Solution: $\frac{6}{288} = \frac{1}{48} = 22\%$.

Ask students to apply geometric probability using the Length Probability Postulate—the probability of a point’s lying on a smaller portion of a segment is equal to the length of the smaller portion divided by the length of the entire segment.

Example:
A radio station will play the song of the day once during each hour. The 101st caller will win $100. If you turn on the radio at 2:35 p.m., what is the probability that you have missed the start of the song during the 2:00 p.m. to 3:00 p.m. hour? Solution: $\frac{\frac{1}{2}}{\frac{1}{2}} = 58\%$.
Sample Assessments

Performance and other types of assessments can be used to ascertain student achievement. Examples include:

General Assessments

- The student will complete learning log entries for this unit. Suggested topics include:
  - Explain why pyramids and cones have $\frac{1}{3}$ as a factor in their formulas for volume.
  - Show how to find the volume and surface area of a solid that combines a cylinder with a cone, or a prism with a pyramid. Be specific.
  - How might the population density of a school affect how much food is made in the cafeteria each day or how many books are in the library?
- The teacher will provide the student with three-dimensional models. The student will sketch diagrams and take appropriate measurements from actual objects needed to calculate volume and surface area. The student will label sketches with measurements and then show the process used to calculate volume and surface area. Since this task is time consuming, the student will be given no more than three objects, some type of prism, either a cone or pyramid, and a cylinder.

Activity-Specific Assessments

- **Activity 2**: The student will build a pyramid with a minimum surface area and minimum volume. The student will show the measurements of the bases, height of the faces, and the height of the pyramid based on the given minimum surface area and volume.

- **Activities 8 and 9**: The student will design a container to hold a specific volume of a specified product. Assign each student a different volume and specific shape or assign the volume and allow the student to choose the shape. The student will design a label for the container and determine the area of that label. The student will create a newspaper advertisement about the product to fit within a specified area.

- **Activity 10**: Assign each student a country. Have students conduct research to be able to calculate the population density of their assigned country and the population densities of the larger cities of the country. Students could create a population density map of the country similar to that of the one they saw of the USA or they could create some other representation to demonstrate how the population is dispersed throughout the country.