Geometry
Unit 2: Reasoning and Proof

Time Frame: Approximately two weeks

Unit Description

This unit introduces the development of arguments for geometric situations. Conjectures and convincing arguments are first based on experimental data, then are developed from inductive reasoning, and, finally, are presented using deductive proofs in two-column, flow patterns, paragraphs, and indirect formats.

Student Understandings

Students understand the basic role proof plays in mathematics. Students learn to distinguish proofs from convincing arguments. They understand that proof may be generated by first providing numerical arguments such as measurements, and then by replacing the measurements with variables.

Guiding Questions

1. Can students develop inductive arguments for conjectures and offer reasons supporting their validity?
2. Can students develop short algorithmic-based proofs that generalize numerical arguments?
3. Can students develop more general arguments based on definitions and basic axioms and postulates?

Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)

CCSS for Mathematical Content	
CCSS#	**CCSS Text**
Congruence	
G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.9-10.1b	Write arguments focused on discipline-specific content. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns.

Sample Activities

Activity 1: Daily Warm-ups with Vocabulary (CCSS: G.CO.1, RST.9-10.4)

Materials List: pencil, index cards

Teacher Note: This activity has not been changed because it already addresses the CCSS listed.

Using 3 x 5 or 5 x 7 index cards, teachers should have students create *vocabulary cards* (view literacy strategy descriptions) for each term listed below. *Vocabulary cards* help students see connections between words, examples of the word, and critical attributes associated with the word. Have students complete one or two vocabulary cards per day. All vocabulary cards should be completed by the beginning of Activity 5 so students will have reviewed the terms/concepts necessary for the remaining activities. Students should place the term in the middle of the card. It is best if the class develops a complete definition that is mathematically correct to avoid misconceptions. Have students use prior knowledge to offer their definitions in a whole group setting, and then have them create one definition as a class. In the upper left corner, the students should put a definition/explanation of the term. In the upper right corner, students should give examples of names and symbols for the term. In the lower left corner, students should give examples of real-life objects that could represent each term. In the lower right corner, students should draw an illustration of what this term would look like including how it is correctly labeled. See the example card which follows.

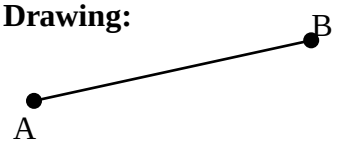
Definition/explanation: a part of a line with two endpoints; has measure

Ways to name:
$\overline{AB}$ or $\overline{BA}$ means Segment AB or Segment BA; also, points *A* and *B* are the endpoints of the segment

line segment

Real-life objects:
pencil, piece of dry spaghetti, flag pole

Drawing:



Students should be encouraged to use the cards to study and to refer to them as they encounter various symbols throughout the rest of the course. Students can buy rings to keep the cards together, or they can punch holes in the cards in order to keep them in their binders. Another option for organization is to have students keep the cards in a zippered bag which could also be kept in their binders. Remember, students should use these cards to help them study and when they are doing assignments, so it is important to check that the students are creating their cards correctly.

Have students create vocabulary cards for the following terms (please include other terms as necessary):

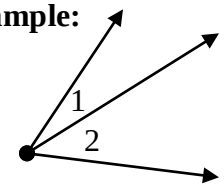
- point
- line
- plane
- line segment
- ray
- angle
- congruent
- parallel lines
- perpendicular lines
- adjacent angles
- vertical angles
- linear pair
- complementary
- supplementary

For the last five terms in the list, modify the vocabulary card to include examples, non-examples, and relationships between the angles. See the modified card below.

Teacher Note: On the card below, it would also be helpful to include a non-example that shows two non-adjacent angles that share a vertex.

Definition: two angles that share a common vertex and a common side, but no common interior points

Example:

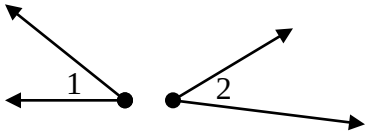


$\angle 1$ and $\angle 2$ are adjacent

adjacent angles

Relationship:
All linear pairs are adjacent angles.

Non-example:



Be sure the following relationships are included:

- Vertical angles are congruent.
- Two angles which form a linear pair are supplementary.
- Complementary angles which are also adjacent form a right angle.

Activity 2: Daily Warm-ups With Technology (GLEs: 9)

Materials List: computer drawing program, pencil, paper, rulers, protractors, compass, patty paper

Using a computer drawing program such as *The Geometer's Sketchpad*®, have students review basic terminology by constructing various figures such as angles, segments, segment bisectors, angle bisectors, etc. Give students characteristics of the figure they are to draw, such as the measure of the angle or segment. Have students measure segments and angles provided using the drawing program. When measuring segments, be sure students have the opportunity to measure in both English and metric units. It is also a good idea to have students use the drawing program to investigate the special angle pairs formed by two intersecting lines. If there is no access to a computer drawing program, use other materials such as rulers, protractors, and patty paper to construct and explore the same concepts.

Activity 3: Algebraic Proofs (GLE: 19)

Materials List: paper, pencil, Proof Puzzle BLM

In this activity, students will work in cooperative groups to correctly order the steps and reasons in an algebraic proof. Either copy the Proof Puzzle BLM or write out the steps (statements) and reasons (algebraic properties) used to solve an algebraic equation as shown on the Proof Puzzle BLM. Cut the statements and reasons on the dotted lines to separate all the pieces and place them in an envelope or baggie. Give the envelope to a group of students and ask them to arrange the steps in a logical order. Depending on the ability of the class, it may be easier if the statements are in one envelope and the reasons (algebraic properties) are in a separate envelope. Four algebraic proofs are provided in the Proof Puzzle BLM. Additional proofs may need to be created if the class has a large number of students or for additional practice. If students are given the statements and reasons in separate envelopes, more proofs should be created for this activity until students can sort out the statements and reasons on their own. This activity forces students to think through the process of solving an equation in order to arrange the steps in a logical order. Once students are satisfied with their ordering, they should copy the final result on paper.

Once groups have had the opportunity to complete their proofs, have them present the proofs to the class. Using a modified q*uestioning the content (QtC)* (view literacy strategy descriptions) technique, have students ask questions about the proofs to clarify their own understanding. The goal of *QtC* is to help students construct meaning from the content from which they are expected to learn. Instead of students asking the questions during reading, in this activity students will be asking questions after they have reviewed the proofs of other groups. Some possible questions might be:

- Does the flow of the proof make sense logically?
- Is the correct reason given for the statement presented?
- Are the statements and reasons necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

Direct students to think about their classmates' presentations and develop questions that may highlight incorrect logic or missing information. Set rules that create an environment conducive to this process. Students should ask and answer most of the questions; however, where necessary ask questions and offer explanations to clarify any misconceptions and incorrect answers. Reinforce to students that this questioning process should be used whenever they read others' proofs or write their own proofs.

Once students have adjusted to organizing the proofs, introduce proofs with unnecessary information. Require that students use only information that is relevant to the proof and organize the information into a logical order.

Activity 4: Finding Segment and Angle Measures Analytically (GLEs: 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper

Review the correct symbol for denoting the measure of a line segment. Be careful to point out the differences between the symbols ($\overleftrightarrow{AB}, \overline{AB}, \overrightarrow{AB}$ and AB) and their meanings. Introduce the Segment Addition Postulate which states, "If *A, B,* and *M* are collinear points and *M* is between *A* and *B*, then $AM + MB = AB$." Also introduce the Midpoint theorem which states, "If *M* is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$." Students should be given various opportunities to find the measures of segments involving algebraic expressions by employing the Segment Addition Postulate and the Midpoint theorem. For example:

> If *A* is between *C* and *T*, $CA = 2x + 5, AT = 5x - 2,$ and $CT = 8x - 2$, find *x* and *AT*. *Solution: Using the Segment Addition Postulate, we know that* $CA + AT = CT$ *, so* $x = 5,$ and $AT = 23$ *units.*

In addition to finding measures of segments analytically, students should work with the Angle Addition Postulate which states, "If *R* is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$." Students should work problems in which they are to find the measures of various angles with and without algebra. Also, have students use the definition of angle bisector (If $\overrightarrow{PQ}$ is an angle bisector of $\angle RPS$, then *Q* is in the interior of $\angle RPS$ and $\angle RPQ \cong \angle SQP$) to find angle measures using algebra.

Once students have mastered the segment and angle addition postulates, provide students with the opportunity to complete basic geometric proofs based on algebraic concepts (definition of congruence, angle and segment addition postulates, properties of equality). Use *questioning the author (QtC)* (view literacy strategy descriptions) employed in Activity 3 to promote higher order thinking and understanding. Some possible questions might be:

- Does the flow of the proof make sense logically?
- Is the correct reason given for the statement presented?
- Are the statements and reasons necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

Direct students to think about their classmates' presentations and develop questions that may highlight incorrect logic or missing information. Set rules that create an environment conducive to this process. Students should ask and answer most of the questions; however, where necessary ask questions and offer explanations to clarify any misconceptions and incorrect answers. Reinforce to students that this questioning process should be used whenever they read others' proofs or write their own proofs.

Activity 5: Proofs (GLE: 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper

The algebraic proofs were two column proofs, but some students find flow proofs or paragraph proofs easier to follow. Emphasis should be placed on providing a convincing, easy-to-follow argument with reasons rather than on using one particular format. The content for these proofs should focus on basic geometric concepts (segment and angle addition, congruent segments, and angles). While these proofs may be similar to those in Activity 7, these proofs are different because students have to come up with the statements and reasons as opposed to just arranging them in the correct order. Students must determine the arguments and reasons with their classmates. Facilitate students' work with proofs by having the students create a modified math *text chain* (view literacy strategy descriptions). A *text chain* typically has a group of students create a text based on content that has already been presented. Each member of the group adds a line to the text until the text is completed. In this activity, the *text chain* has been modified by having students complete a proof based on the algebraic and geometric concepts learned in this unit.

- Have students work in small groups of three to four students.
- In each group, each member should take a turn writing one statement and reason for the proof. The first member will write the first statement and reason. The second member will read the first person's statement and reason and decide if it is logical. Then he/she will add his/her own statement and reason. This process will continue until the entire proof is written. Each time a member receives the proof, he/she should read the entire proof to be sure he/she agrees with the logic and flow of the proof. If any person in the group has a concern with any of the previous information, he/she should help his/her classmate correct the statement then add his/her new information. Groups should be allowed to use a two-column proof, a paragraph proof, or a flow proof. In order to keep all students writing/reading, each group should be given multiple conjectures to be proven (one for each member).
- Look for correct proofs. When most groups have completed their proofs, encourage them to discuss their ideas with other groups. At this point, students should question each other if they feel as though there are errors in any of the work.
- Choose three different groups to write a particular (correct) proof on the board. As a class, discuss variations and similarities of the three proofs, and talk about extra steps that could be added or omitted.

Example *text chain*:

Given: $9x + 3(x - 2) = 5x - 7(1 - 4x)$

Prove: $= 1/21$

	$9x + 3(x - 2) = 5x - 7(1 - 4x)$	Given Equation
Student 1	$9x + 3x - 6 = 5x - 7 + 28x$	*Distributive Property*
Student 2	$12x - 6 = 33x - 7$	*Simplification*
Student 3	$12x - 6 + 6 = 33x - 7 + 6$	*Addition Property of Equality*
Student 4	$12x = 33x - 1$	*Simplification*
Student 1	$12x - 33x = 33x - 33x - 1$	*Subtraction Property of Equality*
Student 2	$-21x = -1$	*Simplification*
Student 3	$\frac{-21x}{-21} = \frac{-1}{-21}$	*Division Property of Equality*
Student 4	$x = \frac{1}{21}$	*Simplification*
Student 1	*Checks the answer.*	

Students should check the step completed prior to their receiving the proof and discuss any errors they discover as the proof progresses.

Activity 6: Fun with Angles (GLEs: 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper

Review the relationships among angles formed by the intersection of two parallel lines and a transversal that were learned in grade 8. Provide students with a graphic similar to Diagram 1 in which lines *a* and *b* are parallel. First, provide a number that represents the measure of angle 1. Have students find the measures of all the other numbered angles in the diagram and provide a justification for each measurement found (e.g., if the measure of angle 1 is 105^o, the measure of angle 5 is 105^o because angles 1 and 5 are corresponding angles).

Next, have students provide a convincing argument that pairs of angles are either congruent or supplementary (e.g., given that lines *a* and *b* are parallel, prove that angles 1 and 7 are supplementary), without using angle measures. (*Solution: If lines a and b are parallel, then angles 1 and 5 are congruent corresponding angles. Angles 5 and 7 are supplementary because they form a linear pair. If angles 5 and 7 are supplementary and angle 1 is congruent to angle 5, then angles 1 and 7 must also be supplementary since angles which are congruent can be substituted for one another*.)

Slightly more difficult proofs can be devised using diagrams similar to Diagram 2.

Use activities that require students to provide proofs or convincing arguments for answers throughout the year.

Diagram 1

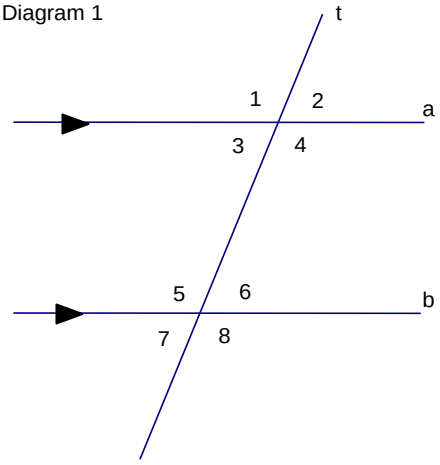
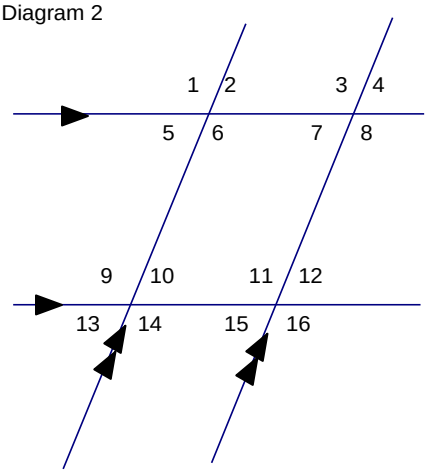


Diagram 2



Sample Assessments

General Assessments

- The student will answer prompts that include the following concepts in his/her math *learning logs* (view literacy strategy descriptions):
  - Compare two-column proofs, flow proofs, and paragraph proofs. What do you believe are the advantages and disadvantages of each? Which do you prefer to use? Why?
- The student will create a portfolio containing samples of his/her activities. For instance, the student will select the proof he/she understood the best and explain what was learned from the proof.
- The student will organize information for a proof and write his/her own proofs for basic algebraic and geometric concepts.

Activity-Specific Assessments

- Activity 3: The student will write proofs independently using basic algebra concepts. The student will explain the reasons for each step in the process of solving the problem.

- Activities 3 and 5: Provide students with proofs—some that are accurate and some that have flaws. The student will evaluate the proofs and identify and correct any flaws that exist.

- Activity 6: Provide students with one measurement in a diagram using parallel lines and transversals (possibly three parallel lines and one or two transversals). The student should find all the missing angle values in the diagram and provide an explanation of how each value was determined.