Algebra II
Unit 2: Polynomial Equations and Inequalities

Time Frame: Approximately four weeks

Unit Description

This unit develops the procedures for factoring polynomial expressions in order to solve polynomial equations and inequalities. It introduces the graphs of polynomial functions using technology to help solve polynomial inequalities.

Student Understandings

Even in this day of calculator solutions, symbolically manipulating algebraic expressions is still an integral skill for students to advance to higher mathematics. However, these operations should be tied to real-world applications so students understand the relevance of the skills. Students need to understand the reasons for factoring a polynomial and determining the correct strategy to use. They should understand the relationship of the Zero–Product Property to the solutions of polynomial equations and inequalities, and connect these concepts to the zeroes of a graph of a polynomial function.

Guiding Questions

1. Can students use the rules of exponents to multiply monomials?
2. Can students add and subtract polynomials and apply to geometric problems?
3. Can students multiply polynomials and identify special products?
4. Can students expand a binomial using Pascal's triangle?
5. Can students factor expressions using the greatest common factor, and can they factor binomials containing the difference in two perfect squares and the sum and difference in two perfect cubes?
6. Can students factor perfect square trinomials and general trinomials?
7. Can students factor polynomials by grouping?
8. Can students select the appropriate technique for factoring?
9. Can students prove polynomial identities and use them to describe numerical relationships?
10. Can students apply multiplication of polynomials and factoring to geometric problems?
11. Can students factor in order to solve polynomial equations using the Zero–Product Property?
12. Can students relate factoring a polynomial to the zeroes of the graph of a polynomial?
13. Can students relate multiplicity to the effects on the graph of a polynomial?

14. Can students determine the effects on the graph of factoring out the greatest common constant factor?
15. Can students predict the end-behavior of a polynomial based on the degree and sign of the leading coefficient?
16. Can students sketch a graph of a polynomial in factored form using end–behavior and zeros?
17. Can students solve polynomial inequalities by the factor/sign chart method?
18. Can students solve polynomial inequalities by examining the graph of a polynomial using technology?

Unit 2 Grade-Level Expectations (GLEs)

Teacher Note: The individual Algebra II GLEs are sometimes very broad, encompassing a variety of functions. To help determine the portion of the GLE that is being addressed in each unit and in each activity in the unit, the key words have been underlined in the GLE list, and the number of the predominant GLE has been underlined in the activity. Some Grade 9 and Grade 10 GLEs have been included because of the continuous need for review of these topics while progressing in higher level mathematics.

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
2.	Evaluate and perform basic operations on expressions containing rational exponents (N-2-H)
Algebra	
4.	Translate and show the relationships among non-linear graphs, related tables of values, and algebraic symbolic representations (A-1-H)
5.	Factor simple quadratic expressions including general trinomials, perfect squares, difference of two squares, and polynomials with common factors (A-2-H)
6.	Analyze functions based on zeros, asymptotes, and local and global characteristics of the function (A-3-H)
7.	Explain, using technology, how the graph of a function is affected by change of degree, coefficient, and constants in <u>polynomial</u>, rational, radical, exponential, and logarithmic functions (A-3-H)
9.	Solve quadratic equations by factoring, completing the square, using the quadratic formula, and graphing (A-4-H)
10.	Model and solve problems involving <u>quadratic</u>, <u>polynomial</u>, exponential, logarithmic, step function, rational, and absolute value equations using technology (A-4-H)
Geometry	
16.	Represent translations, reflections, rotations, and dilations of plane figures using <u>sketches</u>, <u>coordinates</u>, vectors, and matrices (G-3-H)
Data Analysis. Probability, and Discrete Math	
19.	Correlate/match data sets or graphs and their representations and classify

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
	them as exponential, logarithmic, or polynomial functions (D-2-H)
Patterns, Relations, and Functions	
22.	Explain the limitations of predictions based on organized sample sets of data (D-7-H)
24.	Model a given set of real-life data with a non-linear function (P-1-H) (P-5-H)
25.	Apply the concept of a function and function notation to represent and evaluate functions (P-1-H) (P-5-H)
27.	Compare and contrast the properties of families of polynomial, rational, exponential, and logarithmic functions, with and without technology (P-3-H)
28.	Represent and solve problems involving the translation of functions in the coordinate plane (P-4-H)
29.	Determine the family or families of functions that can be used to represent a given set of real-life data, with and without technology (P-5-H)
CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Arithmetic with Polynomials and Rational Expressions	
A.APR.4	Prove polynomial identities and use them to describe numerical relationships.
ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.11-12.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.11-12.2d	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.

Sample Activities

Ongoing: Little Black Book of Algebra II Properties

Materials List: black marble composition book, Unit 2 - Little Black Book of Algebra II Properties BLM

- Have students continue to add to the Little Black Books they created in Unit 1 which are modified forms of *vocabulary cards* (view literacy strategy descriptions). When students create *vocabulary cards*, they see connections between words, examples of the word, and the critical attributes associated with the word, such as a mathematical formula or theorem. *Vocabulary cards* require students to pay attention to words over time, thus improving their memory of the words. In addition, *vocabulary cards* can become an easily accessible reference for students as they prepare for tests, quizzes, and other activities with the words. These self-made reference books are modified versions of *vocabulary cards* because, instead of creating cards, the students will keep the vocabulary in black marble composition books (thus the name "Little Black Book" or LBB). Like *vocabulary cards*, the LBBs emphasize the important concepts in the unit and reinforce the definitions, formulas, graphs, real-world applications, and symbolic representations.

- At the beginning of the unit, distribute copies of the Little Black Book of Algebra II Properties BLM for Unit 2. These are lists of properties in the order in which they will be learned in the unit. The BLM has been formatted to the size of a composition book so students can cut the list from the BLM and paste or tape it into their composition books to use as a table of contents.
- The student's description of each property should occupy approximately one-half page in the LBB and include all the information on the list for that property. The student may also add examples for future reference.
- Periodically check the Little Black Books and require that the properties applicable to a general assessment be finished by the day before the test, so pairs of students can use the LBBs to quiz each other on the concepts as a review.

Polynomial Equations & Inequalities

2.1 *Laws of Exponents* - record the rules for adding, subtracting, multiplying, and dividing quantities containing exponents, raising an exponent to a power, and using zero and negative values for exponents.

2.2 *Polynomial Terminology* – define and write examples of monomials, binomials, trinomials, polynomials, the degree of a polynomial, a leading coefficient, a quadratic trinomial, a quadratic term, a linear term, a constant, and a prime polynomial.

2.3 *Special Binomial Products* – define and give examples of perfect square trinomials and conjugates, write the formulas and the verbal rules for expanding the special products $(a + b)^2$, $(a – b)^2$, $(a + b)(a – b)$, and explain the meaning of the acronym, FOIL.

2.4 *Binomial Expansion using Pascal's Triangle* – create Pascal's triangle through row 7, describe how to make it, explain the triangle's use in binomial expansion, and use the process to expand both $(a + b)^5$ and $(a – b)^5$.

2.5 *Common Factoring Patterns* – define and give examples of factoring using the greatest common factor of the terms, the difference of two perfect squares, the sum/difference of two perfect cubes, the square of a sum/difference ($a^2 + 2ab + b^2$, $a^2 – 2ab + b^2$), and the technique of grouping.

2.6 *Zero–Product Property* – explain the Zero–Product Property and its relevance to factoring: Why there is a zero–product property and not a property like it for other numbers.

2.7 *Solving Polynomial Equations* – identify the steps in solving polynomial equations, define double root, triple root, and multiplicity, and provide one reason for the prohibition of dividing both sides of an equation by a variable.

2.8 *Introduction to Graphs of Polynomial Functions* – explain the difference between roots and zeros, define end-behavior of a function, indicate the effect of the degree of the polynomial on its graph, explain the effect of the sign of the leading coefficient on the graph of a polynomial, and describe the effect of even and odd multiplicity on a graph.

2.9 *Polynomial Regression Equations* – explain the Method of Finite Differences to determine the degree of the polynomial that is represented by data.

2.10 *Solving Polynomial Inequalities* – indicate various ways of solving polynomial inequalities such as using the sign chart and using the graph. Provide two reasons for the prohibition against dividing both sides of an inequality by a variable.

Activity 1: Multiplying Binomials and Trinomials (GLEs: 2, 19; CCSS: WHST.11-12.2d)

Materials List: paper, pencil, large sheet of paper for each group, graphing calculator

This activity has not changed because it already incorporates this CCSS. The students will apply the simple operations of polynomials learned in Algebra I to multiply complex polynomials.

Math Log Bellringer:

Simplify the following expression: $(x^2)^3 + 4x^2 - 6x^3(x^5 - 2x) + (3x^4)^2 + (x + 3)(x - 6)$ and write one mathematical property, law, or rule that you used.

Solution: $3x^8 + x^6 + 12x^4 + 5x^2 - 3x - 18$. Answers for property will vary but could include any of the following: laws of exponents, distributive property, commutative property, associative property, combining like terms, polynomial rule of listing terms in descending order.

Activity:

- Overview of the Math Log Bellringers:
  - As in Unit 1, each in-class activity in Unit 2 is started with an activity called a Math Log Bellringer that either reviews past concepts to check for understanding (i.e., reflective thinking about what was learned in previous classes or previous courses) or sets the stage for an upcoming concept (i.e., predictive thinking for that day's lesson).
  - A math log is a form of a *learning log* (view literacy strategy descriptions) that students keep in order to record ideas, questions, reactions, and new understandings. Documenting ideas in a log about content's being studied forces students to "put into words" what they know or do not know. This process offers a reflection of understanding that can lead to further study and alternative learning paths. It combines writing and reading with content learning. The Math Log Bellringers will include mathematics done symbolically, graphically, and verbally.
  - Since Bellringers are relatively short, blackline masters have not been created for each of them. Write them on the board before students enter class, paste them into an enlarged *Word*® document or *PowerPoint*® slide, and project using a TV or digital projector, or print and display using a document or overhead projector. A sample enlarged Math Log

Bellringer *Word*® document has been included in the blackline masters. This sample is the Math Log Bellringer for this activity.

- ➢ Have the students write the Math Log Bellringers in their notebooks, preceding the upcoming lesson during beginning–of–class record keeping, and then circulate to give individual attention to students who are weak in that area.

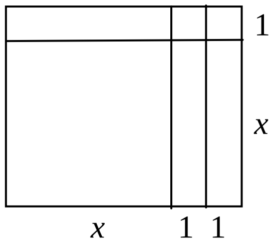
- When students have completed the Bellringer, have them use *discussion* (view literacy strategy descriptions) in the form of Think-Pair-Square-Share. It has been shown that students can improve learning and remembering when they participate in a dialog about class. In Think-Pair-Square-Share, after being given an issue, problem, or question, students are asked to think alone for a short period of time and then pair up with someone to share their thoughts. Then have pairs of students share with other pairs, forming, in effect, small groups of four students. It highlights students' understanding of what they know, as well as what they still need to learn, in order to fully comprehend the concept.
  - ➢ Have each student write one mathematical property, law or rule that he/she used to simplify the expression in the Bellringer. The property, law or rule should be written in a sentence describing the process used.
  - ➢ Pair students to first check the correctness of their Bellringer and properties, laws and rules. If they have written the same property, law or rule, have the pair write an additional property, law or rule.
  - ➢ Divide the students into groups of four to compare their properties, laws and rules. Have the group write their combined properties, laws, and rules on large sheets of paper and tape them to the board to compare with other groups.
  - ➢ In addition to the laws of exponents, look for the commutative, associative, and distributive properties, FOIL, combining like terms, and arranging the terms in descending order.

- With students still in groups, review the definitions of monomial, binomial, trinomial, polynomial, degree of polynomial, and leading coefficient. Have each group expand $(a + b)^2$, $(a – b)^2$, and $(a + b)(a – b)$ and write the words for finding these special products, again comparing answers with other groups and voting on the best verbal explanation. Define the word conjugate.

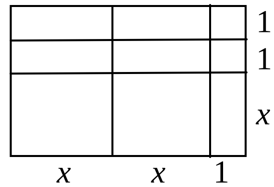
- Have students expand several binomial and trinomial products.

- Application:
  (1) The length of the side of a square is $x + 3$ cm. Express the perimeter and the area as polynomial functions using function notation.
  (2) A rectangular box is $2x + 3$ feet long, $x + 1$ feet wide, and $x – 2$ feet high. Express the volume as a polynomial in function notation.
  (3) For the following figures, write an equation showing that the area of the large rectangle is equal to the sum of the areas of the smaller rectangles.

(a)



(b)



Solution:
(1) $p(x) = 4x + 12$ *cm,* $A(x) = x^2 + 6x + 9$ *cm*^{2}
(2) $V(x) = 2x^3 + x^2 - 7x - 6$
(3a) $(x + 2)(x + 1) = x^2 + 1x + 1x + 1x + 1 + 1 = x^2 + 3x + 2$
(3b) $(2x + 1)(x + 2) = x^2 + x^2 + 1x + 1x + 1x + 1x + 1x + 1 + 1 = 2x^2 + 5x + 2$

Activity 2: Using Pascal's Triangle to Expand Binomials (GLEs: 2, 27)

Materials List: paper, pencil, graphing calculator, 5 transparencies or 5 large sheets of paper, Expanding Binomials Discovery Worksheet BLM

The focus of this activity is to find a pattern in coefficients in order to quickly expand a binomial using Pascal's triangle, and to use the calculator ${}_nC_r$ button to generate Pascal's triangle.

Math Log Bellringer: Expand the following binomials:
(1) $(a + b)^0$
(2) $(a + b)^1$
(3) $(a + b)^2$
(4) $(a + b)^3$
(5) $(a + b)^4$
(6) Describe the process you used to expand #5
Solutions:
(1) 1, (2) $a + b$*,(3)* $a^2 + 2ab + b^2$*,(4)* $a^3 + 3a^2b + 3ab^2 + b^3$*,*
(5)$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$*, (6) Answers will vary.*

Activity:

- Have five of the students each work one of the Bellringer problems on a transparency or large sheets of paper, while the rest of the students work in their notebooks. Have the five students put their answers in front of the class and explain the process they each used. Compare answers to check for understanding of the FOIL process.

- Write the coefficients of each Bellringer problem in triangular form (Pascal's triangle) and have students find a pattern.

- Expanding Binomials Discovery Worksheet:
  - On this worksheet, the students will discover how to expand a binomial using both Pascal's triangle and combinations. Distribute the Expanding Binomials Discovery Worksheet BLM and have students work in pairs on the Expanding Binomials section of the worksheet. Circulate to check for understanding and stop after this section to check for correctness.
  - Allow students to complete the section on Using Combinations to Expand Binomials and check for correctness.

- Administer the Activity–Specific Assessment to check for understanding expanding a binomial.

Activity 3: Factoring Special Polynomials (GLEs: 2, 5, 10, 24, 27)

Materials List: paper, pencil, graphing calculator

In this activity, students will factor a polynomial containing common factors, a perfect square trinomial, and binomials that are the difference of two perfect squares, the sum of two perfect cubes, or the difference of two perfect cubes.

Math Log Bellringer:

Find the greatest common factor and describe the process you used:
(1) 24, 36, 60
(2) $8x^2y^3$, $12x^3y$, $20x^2y^2$

Solutions: (1) 12, (2) $4x^2y$, (3) descriptions of the processes used will vary

Activity:

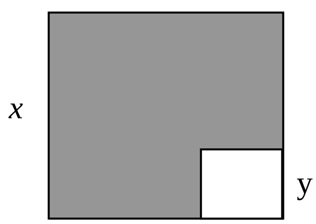
- Use the Bellringer to review the definition of factor and discuss the greatest common factors (GCF) of numbers and monomials. Have students factor common factors out of several polynomials.
- Have students examine the first four trinomials below and use the verbal rules written in Activity 1 to determine how to rewrite the trinomials in factored form. Then have students apply the rules to more complicated trinomials (problems #5 and #6).
  (1) $3a + 6a^2 + 3a^3$
  (2) $a^2 + 6a + 9$
  (3) $s^2 - 8s + 16$
  (4) $16h^2 - 25$
  (5) $9x^2 + 42x + 49$
  (6) $64x^2 - 16xy + y^2$

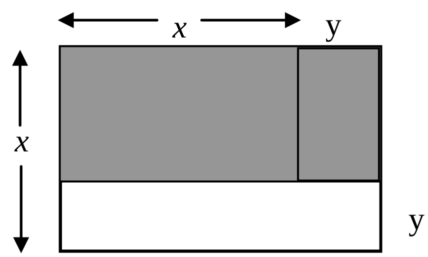
  Solutions:
  (1) $3a(1 + 2a + a^2)$ or $3a(1 + a)^2$
  (2) $(a + 3)^2$
  (3) $(s - 4)^2$
  (4) $(4h - 5)(4h + 5)$
  (5) $(3x + 7)^2$
  (6) $(8x - y)^2$

- Have students expand the following (a + b)(a^2 – ab + b^2) and (a – b)(a^2 + ab – b^2) and write two verbal rules that will help them factor $a^3 - b^3$ and $a^3 + b^3$.

- Have students work numerous factoring problems factoring out common factors, perfect square trinomials, difference of two perfect squares, and sum and difference of two perfect cubes.

- Application:
  (1) The area of a rectangle can be represented by $25x^2 - 16$. What is a binomial expression for each side?
  (2) A small square of plastic is to be cut from a square plastic box cover. Express the area of the shaded form in factored form and show that it is equal to the area of the shaded region in the second figure.





Solution: (1) $(5x–4)(5x+4)$
(2) $x^2 - y^2 = (x + y)(x - y)$

Activity 4: Factoring Quadratic Trinomials (GLEs: 2, <u>5</u>, 9, 10)

Materials List: paper, pencil, graphing calculator

Students will expand and factor expressions to find the relationships necessary to factor quadratic trinomials.

Math Log Bellringer:

Expand the following:
(1) $(x + 4)(x + 3)$
(2) $(x - 4)(x - 3)$
(3) $(x + 4)(x - 3)$
(4) $(x - 4)(x + 3)$
(5) Discuss what makes the middle and last terms of the answer + or –.

Solutions:
(1) $x^2 + 7x + 12$
(2) $x^2 - 7x + 12$
(3) $x^2 + x - 12$
(4) $x^2 - x - 12$

(5) The sign of the middle term in the trinomial is determined by the sign of the larger second number of the binomials. When the signs in the two binomials are the same, the sign of the last term in the trinomial is positive. When the signs in the two binomials are different, the sign of the last term in the trinomial is negative.

Activity:

- Use the Bellringer to discuss the relationships between the middle term as being the sum of the inner and outer terms and the last term as the product of the last terms. Discuss the signs and have students write a rule based on their discussions in #5.
- Have students factor numerous quadratic trinomials including trinomials with leading coefficients other than 1.
- Application:
  (1) A rectangular window has an area ($x^2 + 8x + 15$) sq. meters. Find the factors that represent the sides of the window.
  (2) The area of a rectangular lot is ($5x^2 – 3x – 2$) sq. feet. What is the perimeter of the lot?
  (3) Write a quadratic trinomial that can be used to find the side of the square if the area less the side is twenty. (Hint: Isolate zero and factor in order to find the possible lengths of the side.*)*

  Solutions:
  (1) $(x + 5)(x+3)$
  (2) $12x+2$
  (3) $s^2 - s = 20 \Rightarrow s^2 - s - 20 = 0 \Rightarrow (s-5)(s+4) = 0 \Rightarrow s = 5 \text{ or } s = -4$
  A side must be positive; therefore, s = 5 feet.

Activity 5: Factoring by Grouping (GLEs: 2, 5, 10)

Materials List: paper, pencil, graphing calculator

Students will review all methods of factoring and factor a polynomial of four or more terms by grouping terms.

Math Log Bellringer:

Factor completely and explain which special process you used in each:
(1) $2x^2y^3 + 6xy^2 + 8x^3y$
(2) $4x^2 + 4x + 1$
(3) $16x^2 – 36y^2$
(4) $1 + 8x^3$
(5) $9x^2 – 12x +4$
(6) $3x^2 +6x$

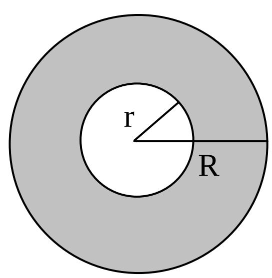
Solutions:
(1) $2xy(xy^2 + 3y + 4x^2)$, Factor out a common factor.
(2) $(2x + 1)^2$, This is a perfect square binomial.
(3) $(4x – 6y)(4x + 6y)$, This is the difference in two perfect squares.
(4) $(1 + 2x)(1 – 2x + 4x^2)$, This is the sum of two perfect cubes.
(5) $(3x – 2)^2$ This is a perfect square trinomial.
(6) $3x(x + 2)$ Factor out a common factor.

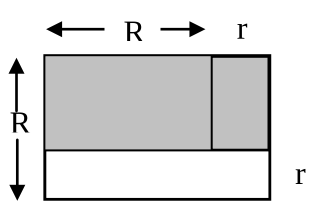
Activity:

- Have the students use the Bellringer to develop the steps for factoring a polynomial completely: (1) factor out GCF, (2) if the polynomial is a binomial, look for special products such as difference of two perfect squares or the sum/difference of two perfect cubes, and (3) if the polynomial is a trinomial, look for a perfect square trinomial or FOIL.

- Give the students a polynomial made of four monomials such as $6x^3 + 3x^2 – 4x – 2$. Allow them to work in pairs to brainstorm possible methods of factoring and possible ways to group two monomials in order to apply one of the basic factoring patterns. Develop factoring by grouping and add to the list. Provide students with guided practice problems.
  Solution: $(3x^2 – 2)(2x + 1)$

- Application:
  (1) The area of a rectangle is $xy + 2y + x + 2$ ft^2 . Find the possible lengths of the sides.
  (2) Prove that the ratio of the area of the circular shaded region below to the rectangular shaded region equals π .





Solution:
(1) $(y + 1)$ and $(x + 2)$
(2) $Area_1 = \pi(R^2 – r^2)$ and $Area_2 = (R – r)(R + r)$

$$\frac{\text{Area}_1}{\text{Area}_2} = \pi$$

Activity 6: Solving Equations by Factoring (GLEs: 2, 5, 6, 9, 24)

Materials List: paper, pencil, graphing calculator

In this activity, the students will develop the Zero–Product Property and use it and their factoring skills to solve polynomial equations.

Math Log Bellringer:

Solve for x:
(1) $2x = 16$
(2) $2x^2 = 16x$
(3) Explain the property used to get the answer to #2.

Solutions:
(1) $x = 8$
(2) $x = 8$ and $x = 0$
(3) There will be several explanations. See the activity below to guide students to the correct explanation.

Activity:

- Determine how many students got both answers in Bellringer problem #2 and use this to start a discussion about division by a variable – do not divide both sides of an equation by a variable because the variable may be zero. Define division as $\frac{a}{b} = c$ if and only if $bc = a$ and have students explain why division by zero is "undefined."

- Have a student who worked problem #2 correctly, write the problem on the board showing his/her work. (He/she should have isolated zero and factored.) Have the students develop the Zero–Product Property of Equality. Make sure students substitute to check their answers. Review the use of *and* and *or* in determining the solution sets in compound sentences. Compare the solution for problem #2 with the solution to the problem $x(x + 2) = 8$ solved incorrectly as {8, 6}. Have students substitute solutions to check answers and discuss why there is no "Eight Product Property of Equality" or any other number except zero. Use guided practice to allow students to solve several more quadratic polynomial equations using factoring.

- Have the students solve the following and discuss double and triple roots and multiplicity. Multiplicity occurs when the same number is a solution more than once.
  (1) $(x - 4)(x - 3)(x + 2) = 0$
  (2) $y^3 - 3y^2 = 10y$
  (3) $x^2 + 6x = -9$
  (4) $(x^2 + 4x + 4)(x + 2) = 0$

  Solutions:
  (1) {–2, 3, 4}
  (2) {0, 5, –2}
  (3) {–3}, There is one solution with multiplicity of 2; therefore, the solution is called a double root.
  (4) {–2}, There is one solution with multiplicity of 3; therefore, the solution is called a triple root.

- Have students develop the steps for solving an equation by factoring:
  Step 1: Write in Standard Form (Isolate zero)
  Step 2: Factor

Step 3: Use the Zero–Product Property of equality
Step 4: Find the solutions
Step 5: Check

- Application:
  Divide the students in groups to set up and solve these application problems:
  (1) The perimeter of a rectangle is 50 in. and the area is 144 in^2. Find the dimensions of the rectangle.
  (2) A concrete walk of uniform width surrounds a rectangular swimming pool. Let x represent this width. If the pool is 6 ft. by 10 ft. and the total area of the pool and walk is 96 ft^2, find the width of the walk.
  (3) The longer leg of a right triangle has a length 1 in. less than twice the shorter leg. The hypotenuse has a length 1 in. greater than the shorter leg. Find the length of the three sides of the triangle.
  Solutions:
  (1)16 in. by 9 in , (2) 1 foot, (3) 2.5 in., 2 in., and 1.5 in.

Activity 7: Investigating Graphs of Polynomial Functions (GLEs: 2, 4, 5, 6, 7, 9, 10, 16, 19, 25, 27, 28)

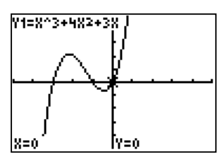
Materials List: paper, pencil, graphing calculator, Graphing Polynomials Discovery Worksheet BLM

In this activity, students will use technology to graph polynomial functions to find the relationship between factoring and finding zeros of the function. They will also discover end–behavior and the effects of a common constant factor, even and odd degrees, and the sign of the leading coefficient on the graph of a function.

Math Log Bellringer:

(1) Factor $x^3 + 4x^2 + 3x = 0$ and solve.
(2) Discuss what factoring properties you used to get the solution.
(3) Graph $y = x^3 + 4x^2 + 3x$ on your calculator and find the zeros.

Solutions:
(1) $x(x + 3)(x + 1) = 0$, $\{0, -3, -1\}$
(2) First I factored out the common factor, then used the reverse of FOIL.
(3)



Activity:

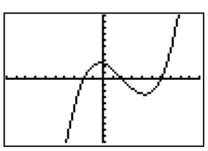
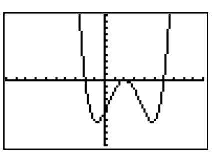
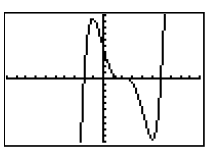
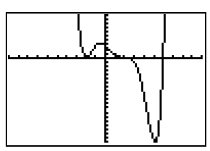
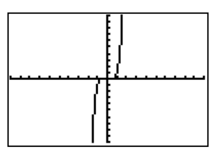
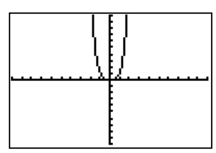
- Use the Bellringer to review calculator skills for finding zeros, adjusting the window to show a comprehensive graph displaying both intercepts and the maximum and minimum points. Have students determine why the solutions to the equation are the zeros of the graph. Discuss the end–behavior.

- Have the students graph $f(x) = x^3 - 3x^2 - 10x + 24$ on the graphing calculator. Find the zeros and use them to write the equation in factored form, then graph both the expanded and factored form on the graphing calculator to determine if they are the same equation. Use the calculator to find *f(4)* and *f(2)*.
  Solution: $f(x) = (x-4)(x+3)(x-2)$, $f(4) = 0$, $f(2) = 0$

- Have students graph $y = (x-2)^2(x+6)$ and find the zeros. Discuss the difference between root and zero: Zeros are x–intercepts where $y = 0$ indicating there must be a two-variable equation. Roots are solutions to one-variable equations.
  Solution: two zeros {2, –6}, three roots: 2 is a double root and -6 is a single root

- Discovering Graphs of Polynomials:
  - Divide students into groups of three and distribute the Graphing Polynomials Discovery Worksheet BLM. On this worksheet, the students will use their graphing calculators to discover the shapes of graphs, zeros, roots, end–behavior, translations, rotations and dilations.
  - Stop the groups after completing #1 to make sure they have a clear understanding of the answers and list their concepts in (i) on the board, then allow them to complete the remainder of the worksheet.
  - When the worksheet is complete, assign each group one problem in which to lead the discussion and list any general conclusions on the board.

- When students have completed going over the worksheet, enact the *professor know–it–all* strategy (view literacy strategy descriptions). Draw graphs similar to the following on the board, and tell the students that each group will come to the front of the class to be a team of Math Wizards (or any relevant and fun name) to answer questions concerning a particular graph. Students and the teacher should hold the Math Wizards accountable for their answers to the questions by assigning a point for all correct answers. Before they start, each group should come up with 3 questions that they will ask the Math Wizards about the graph. When the wizards are in front of the class, they can confer before answering the questions, but the speaking role should rotate among members of the group. Some sample questions are:
  1. What are the domain and range?
  2. Is the degree of the polynomial even or odd? Why?
  3. How many zeros are there?
  4. What is the smallest degree the polynomial can have? Why?
  5. What is the smallest number of roots this graph may have? Why?
  6. What is *f(0)*?
  7. Write the polynomial in factored form.

Activity 8: Modeling Real-Life Data with a Polynomial Function (GLEs: 2, 4, 5, 6, 7, 10, 16, 19, 22, 24, 25, 27, 28, 29)

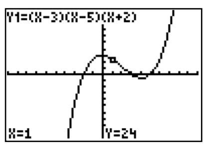
Materials List: paper, pencil, graphing calculator, Data & Polynomial Functions Discovery Worksheet BLM

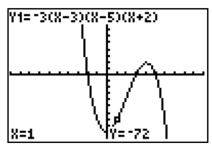
The students will plot data in a scatter plot and will determine what type of polynomial function best describes the data. They will create an equation based on the zeros.

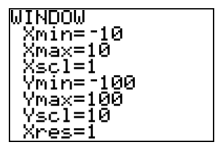
Math Log Bellringer: Make a rough sketch of the graphs of the following equations without a calculator:

(1) $f(x) = (x - 3)(x - 5)(x + 2)$
(2) $g(x) = -3(x - 3)(x - 5)(x + 2)$
(3) Locate $f(1)$ and $g(1)$ on the graphs.
(4) Explain the differences in the graphs and why?

Solutions:

(1) 

(2) 



(3) $f(1) = 24$, $g(1) = -72$

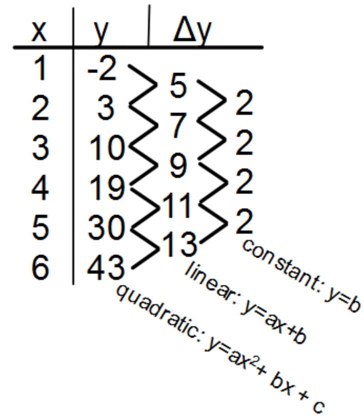
(4) The graphs have different end-behaviors. #1 starts down and ends up while #2 starts up and ends down because of the negative leading coefficient. $g(x)$ is also steeper between zeros or stretched in the vertical direction.

Activity:

- Use the Bellringer to review the graphing procedure learned in Activity 7. Have students determine zeros, *y*-intercepts, end–behavior and the effect of the leading constant, reinforcing that *x*-values can always be evaluated to get a better shape.

- Data and Polynomial Functions:
  - In this activity, the students will use two ways to determine which polynomial will best model given data.
  - Distribute the Data & Polynomial Functions Discovery Worksheet BLM and have students work in pairs to complete the worksheet stopping after each section to ascertain comprehension.
  - In the first section, Predicting Degree of Polynomial by Zeroes, the students should realize that the data exhibits three zeros; therefore, the polynomial should have a degree of at least three. Lead a discussion concerning possible double and triple roots or zeros not in the discrete data. If students are unfamiliar with plotting data in the graphing calculator, the steps to do this are in problem #3 on the Data & Polynomial Functions Discovery Worksheet BLM.
  - In the 2nd section, Method of Finite Differences, review slope and ask if the data is linear and why or why not. Name the process of twice subtracting the *y*–values to get 0 (if the

change in x is constant), the Method of Finite Differences, and refresh the students' memories of this method that was discussed in the Algebra I curriculum.
(Teacher Note: In the Method of Finite Differences, if the increments of x are equal, then repeated calculations of the differences in y will determine the degree of the polynomial. In the example to the right, y is a quadratic function because it took two iterations of differences to get to constant values. More examples:



x	1	2	3	4	5	6
y_1	3	4	9	18	31	48
y_2	9.5	45	126.5	272	499.5	827
y_3	-2	1	4	7	10	13
y_4	-11856	-8568	-4590	0	5100	10584

Solution: y_1 – quadratic, y_2 – cubic, y_3 – linear, y_4 – quartic

- Discuss the limitation of using this method in evaluating real-life data. The finite differences in real-life data will get close to constant to indicate a trend, but because the data is not exact, the differences usually will not become constant.

Activity 9: Solving Polynomial Inequalities (GLEs: 2, 4, 5, 6, 7, <u>9</u>, 10, 16, 19, 25, 27, 28)

Materials List: paper, pencil, graphing calculator, Solving Polynomial Inequalities by Graphing BLM

In this activity, students will solve single variable polynomial inequalities using both a sign chart and Cartesian graph.

Math Log Bellringer:

Solve for x:

(1) $-2x + 6 > 0$
(2) $x(x-4) > 0$
(3) $x(x-4) \le 0$
(4) $x(x-4) = 0$
(5) Explain the property used in #4.

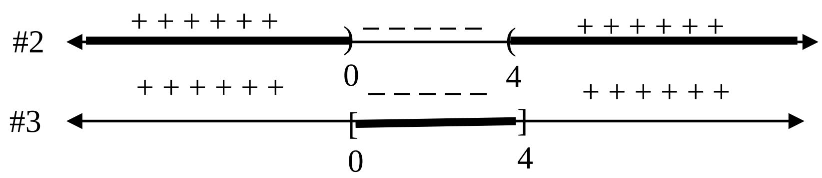
Solutions:

(1) $x < -3$
(2) $x < 0$ or $x > 4$
(3) $0 \le x \le 4$
(4) $\{0, 4\}$
(5) Zero–Product Property of Equality in which an answer of zero requires that one of the factors must equal zero.

Activity:

- Have students state the Zero–Product Property of Equality. Most students will solve Bellringer problem #2 incorrectly, forgetting about the negative–times–negative solution. Ask students if $x = -5$ is a solution. Use the Bellringer to generate the discussion concerning the following inequality properties:
  - (1) $ab > 0$ if and only if $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$. (Review compound sentence use of *and* and *or*.)
  - (2) $ab < 0$ if and only if $a < 0$ and $b > 0$ or $a > 0$ and $b < 0$.

- Solving Inequalities Using a Sign Chart:
  Have students draw a number line and locate the zeros for Bellringer problem #2 and #3.



  Reinforce that the zeros are the values that divide the number line into intervals and satisfy the equation. Test values in each interval and write + and – signs above that interval on the number line. Write the solution in set notation and interval notation. Repeat with problem #3.

  Solutions:

  #2 – interval notation: $(-\infty, 0) \cup (4, \infty)$, *set notation* $\{x : x < 0 \text{ or } x > 4\}$

  #3 - interval notation: $[0, 4]$, *set notation* $\{x: 0 \le x \le 4\}$

- Have students solve the following after discussing isolating 0 and not dividing by the variable. *(Teacher Note: Students should not only not divide by a variable because the variable may be zero, but if the variable is negative, the inequality sign will change.)* Use guided practice for more polynomials. Write answers in set notation.
  - (1) $(x - 3)(x + 4)(x - 7) \ge 0$
  - (2) $x^2 - 9x < -14$
  - (3) $5x^3 \le 15x^2$

  Solutions: (1) $-4 \le x \le 3$ *or* $x \ge 7$, (2) $2 < x < 7$, (3) $x \le 3$

- Solving Polynomials by Graphing:
  - Distribute the Solving Polynomial Inequalities by Graphing BLM. Have students work in pairs in this activity to discover an alternate method for solving polynomial inequalities using the graphs of polynomial functions.
  - Graph the first equation together and guide the students in understanding how a two–variable graph can assist in solving a one-variable inequality.
  - Allow students to finish the worksheet, and then check for individual understanding by assigning the Activity-Specific Assessment.

2013-14

Activity 10: More Polynomial Identities and Applying Them to Numerical Relationships (CCSS: A.APR.4, RST.11-12.4, WHST.11-12.2d)

Materials List: paper, pencil, Polynomial Identities Discovery Worksheet BLM

In this activity, the students will delineate the differences in polynomial expressions, equations, functions, and identities they have studied in previous activities and develop additional polynomial identities.

Math Log Bellringer: Classify the following:

(1) $x^2 + 6x + 9$
(2) $x^2 = -6x - 9$
(3) $y = x^{-3} + x^2 + 4$
(4) $(x+y)^2 = x^2 + 2xy + y^2$
(5) $f(x) = x^2 + 6x + 9$
(6) $x^2 + 6x + 9 > 0$
(7) $y = x^2 + 6x + 9$

(A) polynomial expression
(B) polynomial equation
(C) polynomial inequality
(D) polynomial function
(E) polynomial identity
(F) not a polynomial

Solutions: (1) A, (2) B, (3) F, (4) E, (5) D, (6) C, (7) D

Activity:

- When students have completed the Bellringer, have them apply a modified form of *GISTing* (view literacy strategy descriptions). *GISTing* is an excellent strategy for helping students paraphrase and summarize essential information. Students are required to limit the gist of a paragraph to a set number of words.
  - Have students write a paragraph using good mathematical vocabulary to describe the differences in polynomial expressions, equations, functions and identities referring to previous activities in the unit.
  - Put students in pairs to compare and rewrite the answers using fewer than 15 words for each definition. Answers should look something like this:

    (1) A polynomial <u>expression</u> has variables and constants with whole number exponents added, subtracted or multiplied. *(Discussion: Polynomial expressions follow the definition of a polynomial reviewed ion Activity 1 and have no equal signs.)*

    (2) A polynomial <u>equation</u> has a polynomial expression set equal to another polynomial expression. *(Discussion: In a polynomial equation, students are finding a finite number of values for the variable that make both sides of the equation true.)*

    (3) A polynomial <u>function</u> equates a polynomial expression to *f(x)* or *y*. *(Discussion: In a polynomial function, students are inputting a value for the independent variable to determine the value of the dependent variable.)*

    (4) A polynomial <u>identity</u> is an equation where both sides represent the same polynomial in different forms. *(Discussion: In a polynomial identity, any number or numbers replacing the variables on the left will create the same value when replacing the variables with these numbers on the right side.)*

- Have 4 volunteers write one of their definitions on the board and allow all groups to refine.
- Have students list several more examples of polynomial identities they have used in previous activities.

- Polynomial Identities:
  - Distribute the Polynomial Identities Discovery Worksheet BLM on which the students will discover additional identities and create rules for factoring them. To save time, assign different polynomials to different groups and share answers before groups discover the patterns.
  - In the 2nd section, students will prove the square of a trinomial three ways: expanding in two different ways and using the geometric interpretation similar to Activity 3.
  - In the 3rd section, students will examine a polynomial identity that will generate Pythagorean triples.

- Mental Math for fun: Have students use the polynomial identity $(a + b)^2 = a^2 + 2ab + b^2$ to square two digit numbers mentally. Start with easy numbers.
  - Square any two digit number like $17^2 = (10 + 7)^2$ by squaring the left number (100), double the product of the two numbers (140), square the right number (49) and add (100+140 + 49 =289).

Sample Assessments

General Assessments

- Use Bellringers as ongoing informal assessments.
- Collect the Little Black Books of Algebra II Properties and grade for completeness at the end of the unit.
- Monitor student progress using small quizzes to check for understanding during the unit on such topics as the following:
  (1) expanding and factoring the difference of two perfect squares and cubes
  (2) factoring trinomials
  (3) solving polynomial equations by factoring
  (4) factoring and graphing polynomials
  (5) solving polynomial inequalities
- Administer two comprehensive assessments:
  (1) factoring polynomial expressions
  (2) solving polynomial equations and inequalities and graphing

Activity-Specific Assessments

- Activity 2:

  Draw Pascal's triangle to the row beginning with 5, then expand the following binomials:
  (1) $(x - y)^5$
  (2) $(4x + y)^3$

  Solutions:
  (1) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
  (2) $64x^3 + 48x^2y + 12xy^2 + y^3$

- Activity 3:

  Factor the following polynomials:
  (1) $9 - a^2$
  (2) $27 - x^3$
  (3) $8 + y^3$
  (4) $x^2 + 8x + 16$
  (5) $y^2 - 10y + 25$
  (6) $7t^2 + 14t$

  Solutions:
  (1) $(3 - a)(3 + a)$
  (2) $(3 - x)(9 + 3x + x^2)$
  (3) $(2 + y)(4 - 2y + y^2)$
  (4) $(x + 4)^2$
  (5) $(y - 5)^2$
  (6) $7t(t + 2)$

- Activity 4:

  Factor the following trinomials:
  (1) $x^2 + 7x + 10$, (2) $x^2 - 7x + 10$, (3) $y^2 + 2y - 15$, (4) $t^2 - 2t - 8$, (5) $5x^2 + 28x + 15$
  (6) Discuss the difference in the way you factored #1 and #2 and why.

  Solutions:
  (1) $(x + 2)(x + 5)$
  (2) $(x - 2)(x - 5)$
  (3) $(y - 3)(y + 5)$
  (4) $(t - 4)(t + 2)$,
  (5) $(5x + 3)(x + 5)$
  (6) Both signs have to be the same to get +10, but in #1 both signs have to be positive and in #2 both signs have to be negative.

- <u>Activity 6</u>:

  Solve the following equations by factoring:
  (1) $x^2 + 10x + 16 = 0$
  (2) $x^2 - 25 = 0$
  (3) $x^3 - x^2 = 6x$
  Solutions: (1) {–2, –8}, (2) {±5}, (3) {0, 3, –2}

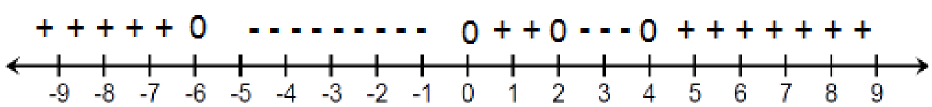
- <u>Activity 7</u>: Specific Assessment Graphing Polynomials BLM

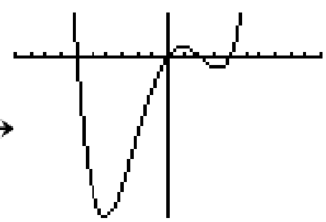
- <u>Activity 9</u>:

  Solve the following inequalities by both the sign chart method and the graphing method without using a graphing calculator showing all your work.
  (1) $x(x-4)(x+6)(x-2) > 0$
  (2) $-2x(x-3)^2(x+4) \le 0$
  (3) Discuss which method you prefer and why.

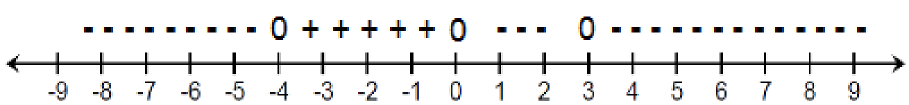
  Solutions:

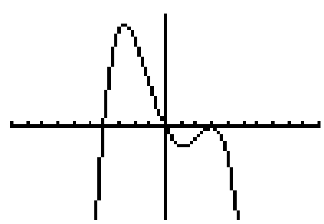
  (1) $(-\infty, -6) \cup (0, 2) \cup (4, \infty)$





  (2) $(-\infty, -4] \cup [0, \infty)$





  (3) Answers will vary.