Little Black Book of Algebra II Properties
Unit 2 - Polynomial Equations & Inequalities

2.1 *Laws of Exponents* - record the rules for adding, subtracting, multiplying and dividing quantities containing exponents, raising an exponent to a power, and using zero and negative values for exponents.

2.2 *Polynomial Terminology* – define and write examples of monomials, binomials, trinomials, polynomials, the degree of a polynomial, a leading coefficient, a quadratic trinomial, a quadratic term, a linear term, a constant, and a prime polynomial.

2.3 *Special Binomial Products* – define and give examples of perfect square trinomials and conjugates, write the formulas and the verbal rules for expanding the special products $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$, and explain the meaning of the acronym, FOIL.

2.4 *Binomial Expansion using Pascal's Triangle* – create Pascal's triangle through row 7, describe how to make it, explain the triangle's use in binomial expansion, and use the process to expand both $(a + b)^5$ and $(a - b)^5$.

2.5 *Common Factoring Patterns* - define and give examples of factoring using the greatest common factor of the terms, the difference of two perfect squares, the sum/difference of two perfect cubes, the square of a sum/difference ($a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$), and the technique of grouping.

2.6 *Zero–Product Property* – explain the zero–product property and its relevance to factoring: Why is there a zero–product property and not a property like it for other numbers?

2.7 *Solving Polynomial Equations* – identify the steps in solving polynomial equations, define double root, triple root, and multiplicity, and provide one reason for the prohibition of dividing both sides of an equation by a variable.

2.8 *Introduction to Graphs of Polynomial Functions* – explain the difference between roots and zeros, define end behavior of a function, indicate the effect of the degree of the polynomial on its graph, explain the effect of the sign of the leading coefficient on the graph of a polynomial, and describe the effect of even and odd multiplicity on a graph.

2.9 *Polynomial Regression Equations* – explain the Method of Finite Differences to determine the degree of the polynomial that is represented by data.

2.10 *Solving Polynomial Inequalities* – indicate various ways of solving polynomial inequalities, such as using the sign chart and using the graph. Provide two reasons for the prohibition against dividing both sides of an inequality by a variable.

Algebra II – Date

Simplify the following expression

$$(x^2)^3 + 4x^2 - 6x^3(x^5 - 2x) + (3x^4)^2 + (x+3)(x-6)$$

Unit 2, Activity 2, Expanding Binomials Discovery Worksheet

Name______________________________________ Date________________________

Expanding Binomials

Pascal's Triangle is an arithmetical triangle that can be used for some neat things in mathematics. Here's how to construct it:

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10   10  5   1
1   6  15  20   15  6   1
______________________________
```

(1) Find a pattern and write a rule to develop Pascal's triangle, then complete the next row.______

____

(2) Compare Pascal's Triangle to the expansions of the Bellringer problems. Determine which row is used in which expansion.

____

(3) Pascal's triangle only supplies the coefficients. Explain how to determine the exponents.

____

(4) Expand each of the following by hand:

a. $(a - b)^2$ ____

b. $(a - b)^3$ ____

c. How can the rule for a sum be modified to use with a difference?

____

(5) Expand each of the following using Pascal's triangle and then simplify each.

a. $(a + b)^6$

b. $(a - b)^7$

c. $(2x + 3y)^4$

Unit 2, Activity 2, Expanding Binomials Discovery Worksheet

Using Combinations to Expand Binomials

(1) How many subset combinations will there be of {3, 5, 9} if taken two at a time?

(2) Reviewing combinations learned in Algebra I, the symbol ${}_nC_r$ and $\binom{n}{r}$ mean the combination of *n* things taken *r* at a time. Write problem #1 using these symbols. ____________________

(3) Use the set {*a, b, c, d*} and list the sets which represent each of the following:

a. 4 elements taken 1 at a time or ${}_4C_1$

b. 4 elements taken 2 at a time or ${}_4C_2$

c. 4 elements taken 3 at a time or ${}_4C_3$

d. 4 elements taken 4 at a time or ${}_4C_4$

(4) What is the relationship between Pascal's triangle and combinations?

(5) Explain two ways that ${}_nC_r$ is used in this lesson?

(6) Calculator Activity: Locate the ${}_nC_r$ button on the graphing calculator and use it to check your last row on Pascal's triangle. Enter $y_1 = 7\ {}_nC_r\ x$ in the calculator. *(MATH , [PRB] 3:nCr on the TI–83/84 graphing calculator)* Set the table to start at 0 with increments of 1. Create the table and compare the values to Pascal's Triangle on the previous page.

Use this feature to expand $(a + b)^9$

Name______________________________________ Date_______________________

Expanding Binomials

Pascal's Triangle is an arithmetical triangle that can be used for some neat things in mathematics. Here's how to construct it:

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10   10   5   1
    1   6   15   20   15   6   1
  1   7   21   35   35   21   7   1
```

(1) Find a pattern and write a rule to develop Pascal's triangle, then complete the next row.______

The row always starts with 1. Then you add the two numbers above it. The numbers are also symmetric on both sides.

(2) Compare Pascal's Triangle to the expansions of the Bellringer problems. Determine which row is used in which expansion.

Each row is the coefficients of the terms. The power of the binomial is the 2nd term of the row.

(3) Pascal's triangle only supplies the coefficients. Explain how to determine the exponents.
The exponents of the first term start at the power of the binomial and decrease by one each time until 0. The exponents of the 2nd term start at 0 and increase by one until the power of the binomial. The sum of the exponents of a and b is the power of the binomial.

(4) Expand each of the following by hand: *(Teacher Note: The coefficient "1" is not necessary but is used to illustrate the numbers in the row of Pascal's Triangle.)*

- $(a - b)^2 = 1a^2 - 2ab + 1b^2$
- $(a - b)^3 = 1a^3 - 3a^2b + 3ab^2 - 1b^3$
- How can the rule for a sum be modified to use with a difference?
  The signs start with + then alternate.

(5) Expand each of the following, using Pascal's triangle and then simplify each.

- $(a + b)^6 = 1a^6 + 6a^5b + 15a^4b^2 + 20\,a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$
- $(a - b)^7 = 1a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - 1b^7$
- $(2x + 3y)^4 = 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

Unit 2, Activity 2, Expanding Binomials Discovery Worksheet with Answers

Using Combinations to Expand Binomials

(1) How many subset combinations will there be of {3, 5, 9} if taken two at a time?

three – {3, 5}, {3, 9}, {5, 9}

(2) Reviewing combinations learned in Algebra I, the symbol ${}_nC_r$ and $\binom{n}{r}$ mean the combination of *n* things taken *r* at a time. Write problem #1 using these symbols. ***${}_3C_2$ or*** $\binom{3}{2}$

(3) Use the set {*a*, *b*, *c*, *d*} and list the sets which represent each of the following:

a. 4 elements taken 1 at a time or ${}_4C_1$ ***{a}, {b}, {c}, {d}***

b. 4 elements taken 2 at a time or ${}_4C_2$ ***{a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}***

c. 4 elements taken 3 at a time or ${}_4C_3$ ***{a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}***

d. 4 elements taken 4 at a time or ${}_4C_4$ ***{a, b, c, d}***

(4) What is the relationship between Pascal's triangle and combinations?

The numbers of subsets in each are the numbers in the row of Pascal's triangle, 1, 4, 6, 4, 1

(5) Explain two ways that ${}_nC_r$ is used in this lesson?

It can be used to find the combination of terms in a set or to find the coefficients used in binomial expansion where n is the exponent of the binomial and r+1 is the number of terms in the expansion.

(6) Calculator Activity: Locate the ${}_nC_r$ button on the graphing calculator and use it to check your last row on Pascal's triangle. Enter $y_1 = 7\ {}_nC_r\ x$ in the calculator. *(MATH, [PRB] 3:nCr on the TI–83/84 graphing calculator)* Set the table to start at 0 with increments of 1. Create the table and compare the values to Pascal's Triangle on the previous page.

Use this feature to expand $(a + b)^9$

$$a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9$$

Name_____________________________________ Date_______________________

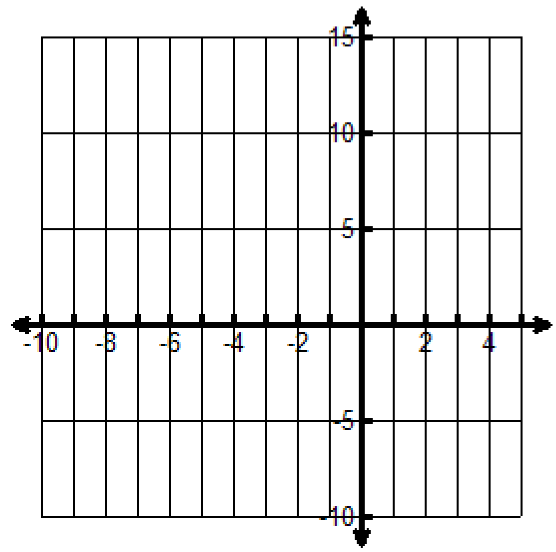
Investigating Graphs of Polynomials

Using a graphing calculator, graph each of the equations on the same graph in the specified window. Find the zeros and sketch the graph locating zeros and answer the questions.

(1) $y_1 = x^2 + 7x + 10$

$y_2 = 3x^2 + 21x + 30$,

$$y_3 = \frac{1}{2}x^2 + \frac{7}{2}x + \frac{10}{2}.$$



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

y_1 = ______________________________

y_2 = ______________________________

y_3 = ______________________________

(b) List the zeros: ______________________

(c) How many zeros? __________________ (d) How many roots? ______________________

(e) What is the effect of the constant factor on the zeros? ________________________________

(f) What is the effect of the constant factor on the shape of the graph? ______________________

____

(g) Discuss end behavior.____

(h) Graph $y_4 = 3y_1$ (On calculator, find y_1 under $\boxed{\text{VARS}}$, Y–VARS, 1: Function, 1: Y_1.) Looking at the graphs, which other equation is this equivalent to? _____

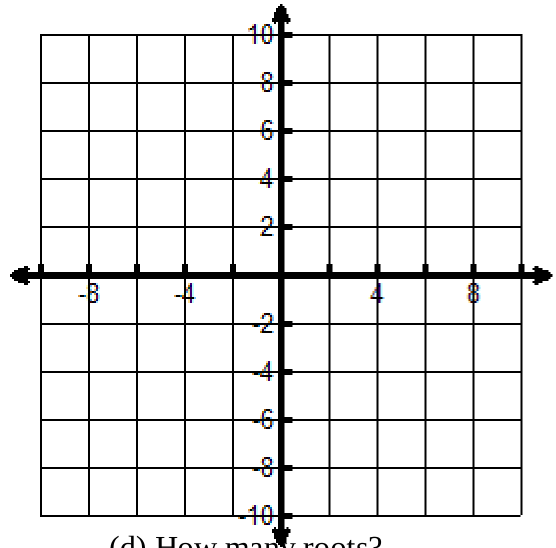
(i) This transformation is in the form *kf(x)*. How does k > 1 or 0<k<1 affect the graph? ________

____

____

(2) $y_1 = x^3 + 6x^2 + 8x$

$y_2 = -x^3 - 6x^2 - 8x$.



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

y_1 = ________________________________

y_2 = ________________________________

(b) List the zeros: ________________(c) How many zeros? _______(d) How many roots? ______

(e) What effect does a common factor of –1 have on the zeros? ____________________________

(f) What is the effect of a common factor of –1 on the end behavior? _______________________

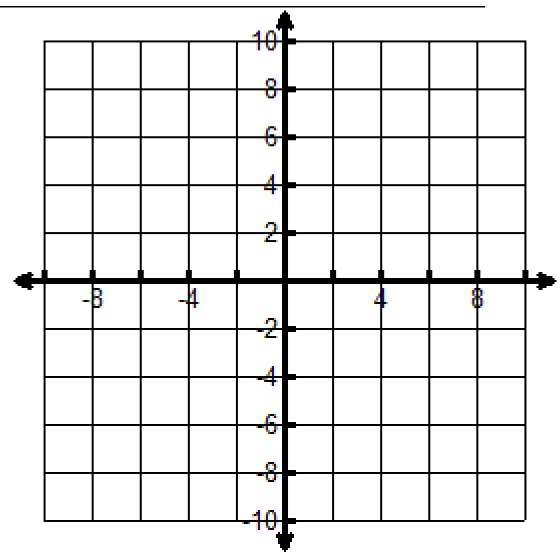
____

(g) Since this transformation is in the form *kf(x)*, how does k = –1 affect the graph? ___________

____

(3) $y_1 = x^2 - 6x + 9$

$y_2 = x^2 + 4x + 4$.



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

y_1 = ____

y_2 = ____

(b) List the zeros: y_1: ______ y_2:______(c) How many zeros in each?

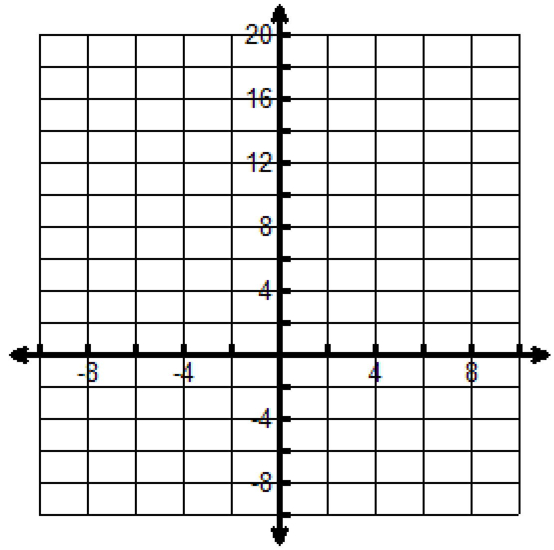
(d) How many roots? _______(e) Discuss multiplicity. ____________________________________

____

(f) What does the graph look like when there is a double root?____________________________

____

(4) $y_1 = x^3 - x^2 - 8x + 12$



(a) List the zeros: ______________________________

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

y_1 = _______________________________________

(c) How many zeros? ____________________________

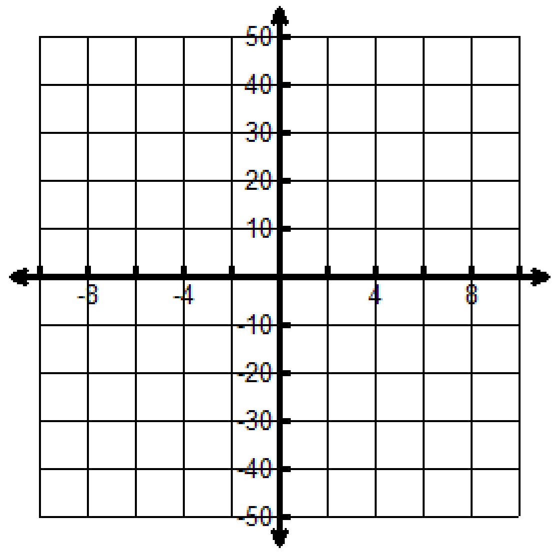
(d) How many roots? ____________________________

(e) Discuss multiplicity and its effect on the graph.

____

(f) Discuss end behavior. ____

(5) $y_1 = x^4 - 3x^3 - 10x^2 + 24x$



(a) List the zeros: ______________________

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

y_1 = _______________________________________

(c) How many zeros? ______________________________

(d) How many roots? ______________________________

(e) Looking at the number of roots in problems 1 through 4, how can you determine how many roots a polynomial has?____

(f) Discuss end behavior. ____

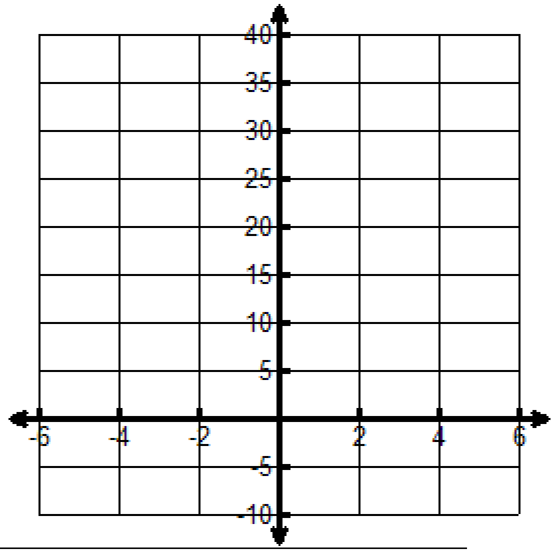
(g) Graph $y_2 = -y_1$. What is the effect on the zeros and the end behavior? ___________________

(h) Looking at the end-behavior in problems 1 through 4, how can you predict end behavior?

____

____

(6) $y_1 = x^4 + 2x^3 - 11x^2 - 12x + 36$



(a) List the zeros: ________________________________

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

y_1 = ____

(c) How many zeros? _____________________________

(d) How many roots? _____________________________

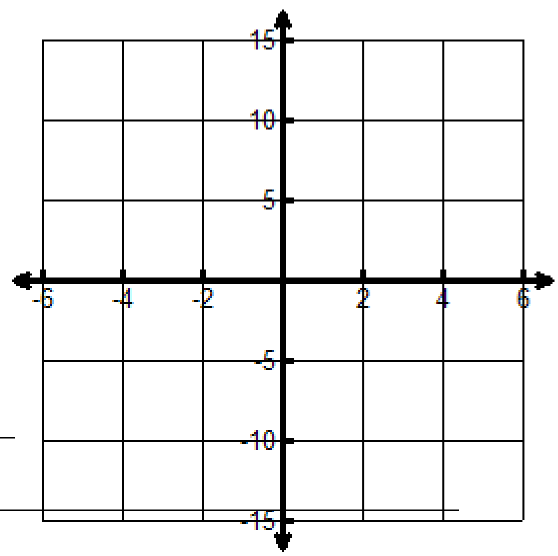
(e) Discuss multiplicity.____

____

(f) Discuss end behavior. ____

____

(7) $y_1 = x^5 - 6x^4 + 9x^3$



(a) List the zeros: ________________________________

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

y_1 = ____

(c) How many zeros? _______ (d) How many roots? ____

(e) Discuss multiplicity.________________________________

____

(f) Discuss end behavior. ____

(g) What is the difference in the looks of the graph for a double root and a triple root?_________

____

____

Unit 2, Activity 7, Graphing Polynomials Discovery Worksheet with Answers

Name_______________________________________ Date________________________

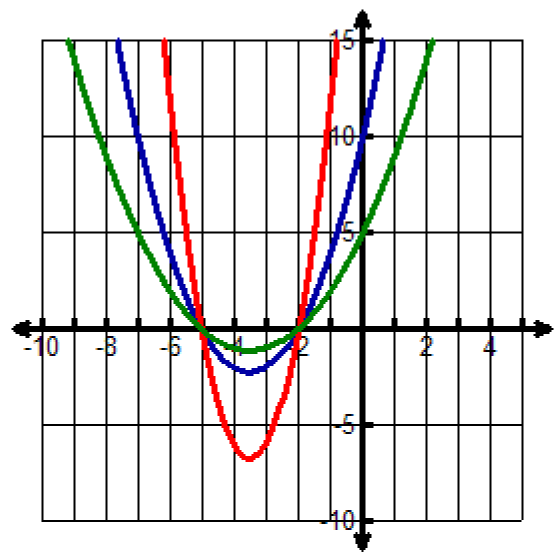
Investigating Graphs of Polynomials

Using a graphing calculator, graph each of the equations on the same graph in the specified window. Find the zeros and sketch the graph locating zeros and answer the questions.

(1) $y_1 = x^2 + 7x + 10$

$y_2 = 3x^2 + 21x + 30,$

$$y_3 = \frac{1}{2}x^2 + \frac{7}{2}x + \frac{10}{2}.$$



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

y_1 = ***(x + 5)(x + 2)***

y_2 = ***3(x + 5)(x + 2)***

y_3 = ½ ***(x + 5)(x + 2)***

(b) List the zeros: ***{–2, –5}***

(c) How many zeros? ***2*** (d) How many roots? ***2***

(e) What is the effect of the constant factor on the zeros? ***nothing***

(f) What is the effect of the constant factor on the shape of the graph? ***if constant is > 1 stretched vertically, more steep between zeros, < 1 wider***

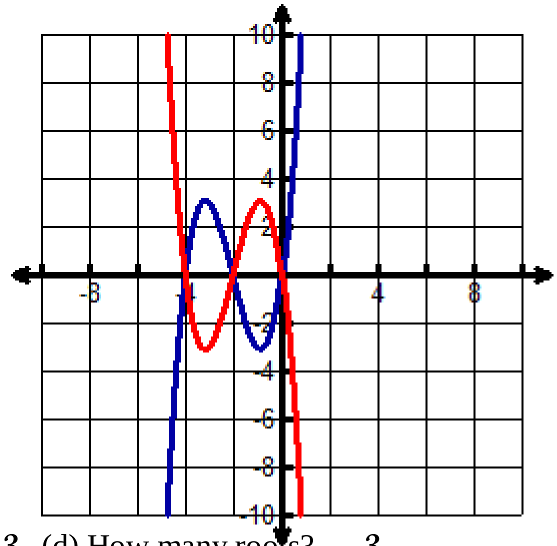
(g) Discuss end behavior. ***starts up and ends up***

(h) Graph $y_4 = 3y_1$ (On calculator, find y_1 under v, Y–VARS, 1: Function, 1: Y_1.) Looking at the graphs, which other equation is this equivalent to? ***y_2***

(i) This transformation is in the form *kf(x)*. How does k > 0 or 0<k<1 affect the graph? ***k > 0 does not affect the zeros but affects the y–values or the range. k>1 stretches the graph vertically and 0<k<1 compresses the graph vertically***

(2) $y_1 = x^3 + 6x^2 + 8x$

$y_2 = -x^3 - 6x^2 - 8x$



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

y_1 = ***x(x + 4)(x + 2)***

y_2 = ***–1 x(x + 4)(x + 2)***

(b) List the zeros: {–4, –2, 0} (c) How many zeros? ***3*** (d) How many roots? ***3***

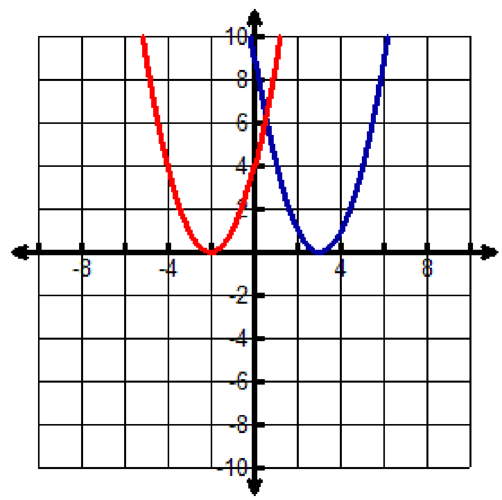
(e) What effect does a common factor of –1 have on the zeros? ***nothing***

(f) What is the effect of a common factor of –1 on the end-behavior? ***rotates the graph in space around the x-axis, y_1 started down and ended up, negative made y_2 start up and end down.***

(g) Since this transformation is in the form *kf(x)*, how does k = –1 affect the graph? ***rotates the graph through space around the x–axis, positive y-values are now negative and negative y-values are now positive***

(3) $y_1 = x^2 - 6x + 9$

$y_2 = x^2 + 4x + 4.$



(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

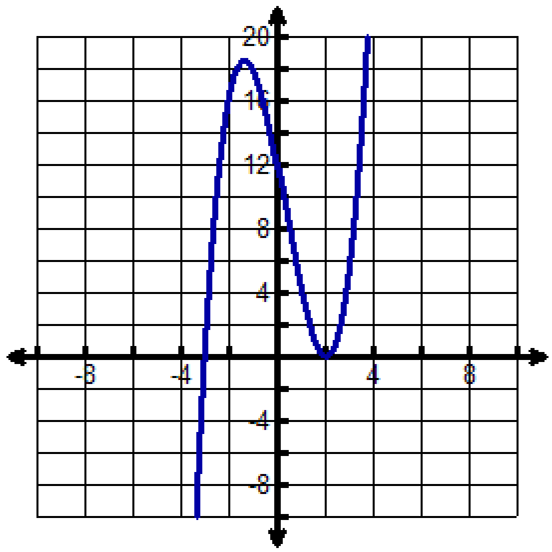
y_1 = ***$(x - 3)^2$***

y_2 = ***$(x + 2)^2$***

(b) List the zeros: y_1: {3} y_2: {–2} (c) How many zeros in each? ***one***

d) How many roots? ***two*** (e) Discuss multiplicity. ***There is one double root so we say there the root has a multiplicity of two – one zero, two roots***

(f) What does the graph look like when there is a double root? ***it skims off the x–axis***



(4) $y_1 = x^3 - x^2 - 8x + 12$

(a) List the zeros: ***{–3, 2}***

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

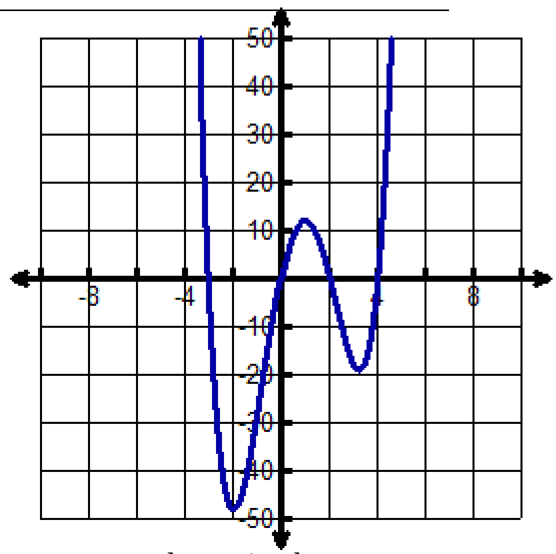
$y_1 =$ ***$(x + 3)(x - 2)^2$***

(c) How many zeros? ***2***

(d) How many roots? ***3***

(e) Discuss multiplicity and its effect on the graph. ***There is a single root and a double root. The double root is the location where the graph skims off the x–axis.***

(f) Discuss end behavior. ***starts down and ends up***



(5) $y_1 = x^4 - 3x^3 - 10x^2 + 24x$

(a) List the zeros: ***{–3, 0, 2, 4}***

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

$y_1 =$ ***$x(x + 3)(x - 2)(x - 4)$***

(c) How many zeros? ***4*** (d) How many roots? ***4***

(e) Looking at the number of roots in problems 1 through 4, how can you determine how many roots a polynomial has? ***highest exponent***

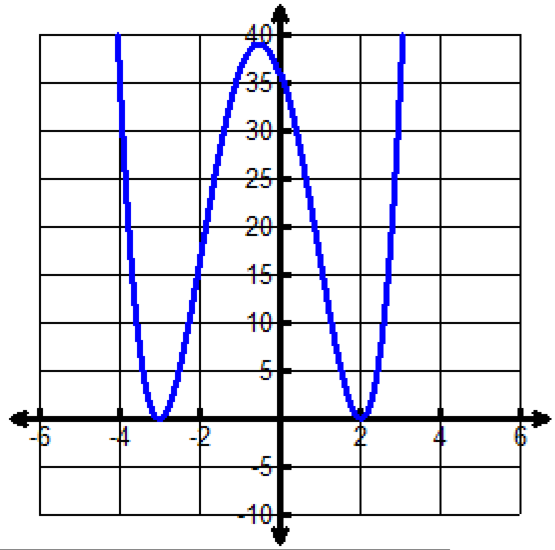
(f) Discuss end behavior. ***starts up and ends up***

(g) Graph $y_2 = -y_1$. What is the effect on the zeros and the end behavior? ***opposite***

(h) Looking at the end-behavior in problems 1 through 4, how can you predict end behavior?

- ***If the degree of the polynomial is odd and leading coefficient is positive - starts down ends up;***
- ***degree odd and leading coefficient negative - starts up and ends down;***
- ***degree even and leading coefficient positive – starts up and ends up;***
- ***degree even and leading coefficient negative – starts down and ends down***

(6) $y_1 = x^4 + 2x^3 - 11x^2 - 12x + 36$



(a) List the zeros: ***{–3, 2}***

(b) Use the zeros to write the equation in factored form and check graphs to see if both forms are equivalent.

$y_1 = (x + 3)^2 (x - 2)^2$

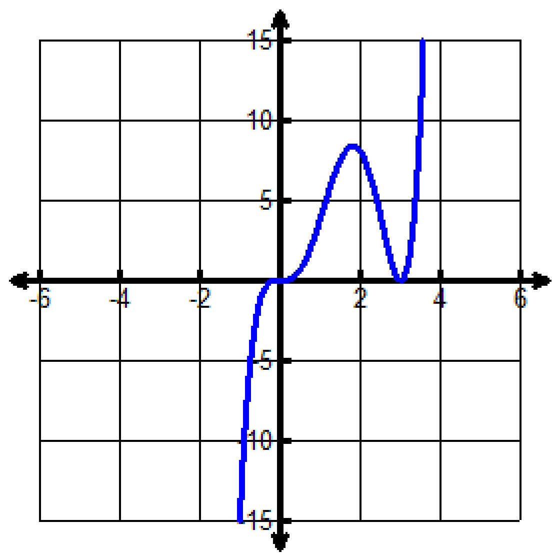
(c) How many zeros? ***2***

(d) How many roots? ***4***

(e) Discuss multiplicity. ***2 sets of double roots***

(f) Discuss end behavior. ***starts up and ends up***

(7) $y_1 = x^5 - 6x^4 + 9x^3$



(a) List the zeros: ***{0, 3}***

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

$y_1 = x^3(x - 3)^2$

(c) How many zeros? ***2*** (d) How many roots? ***5***

(e) Discuss multiplicity. ***one double root and one triple root***

(f) Discuss end behavior. ***starts down and ends up***

(g) What is the difference in the looks of the graph for a double root and a triple root? ***a double root or any root created by an even exponent skims off the x–axis while a triple root or any root created by an odd exponent flattens out and goes through the x–axis***

Unit 2, Activity 8, Data & Polynomial Functions Discovery Worksheet

Name_______________________________________ Date_________________________

Predicting Degree of Polynomial by Zeros

x	–3	–2	–1	0	1	2	3	4
y	–36	0	8	0	–12	–16	0	48

(1) Using the data above, what would be the least degree of a polynomial that would model the data and explain why.

____

(2) Predict the equation of the polynomial in factored form. ________________________________

(3) Plot the data in a graphing calculator and make a scatter plot. (To enter data on a TI–84 calculator: **STAT**, **1:Edit**, enter data into $\mathbf{L_1}$ and $\mathbf{L_2}$. To set up the plot of the data: **2**$^{\mathbf{nd}}$ **, STAT PLOT, 1:PLOT1, ENTER**, **On**, **Type**: , **Xlist:** $\mathbf{L_1}$, **Ylist:** $\mathbf{L_2}$, **Mark** (any). To graph the scatter plot: **ZOOM , 9: ZoomStat.)** Then enter the equation to see if it matches the data. Adjust the leading coefficient of the equation until the graph matches the data and write the final equation. _________

Method of Finite Differences

x	–3	–2	–1	0	1	2	3	4
y	–14	–11	–8	–5	–2	1	4	7

(4) Using the data above, what would be the least degree of a polynomial that would model the data and explain why you came to that conclusion.

____

(5) Use the Method of Finite Differences to twice subtract the *y*-values to get 0 to determine that the function is linear, a polynomial of first degree. Find the equation of the line. ________________

(6) Apply the method of finite differences to the first table several times and develop the guidelines for determining the degree of the polynomial:

y = c if ____

y = ax+ b if ____

y = ax^2 + bx + c if ____

y = ax^3 + bx^2 + cx + d if ____

(7) What are the limitations of using this method in evaluating real-life data?

____

Real Life Application

year	1994	1996	1998	2000	2002	2004
number	50	56	65	75	94	110

Because of improved health care, people are living longer. The above data relates the number of Americans (in thousands) who are expected to be over 100 years old for the selected years. (Source: US Census Bureau) Enter the data into the calculator, letting x = 4 correspond to 1994 and make a scatter plot. Then graph the following equations:

$y_1 = 6.057x + 20.4857$, $y_2 = 0.4018x^2 - 1.175x + 48.343$, $y_3 = -0.007x^3 + 0.5893x^2 - 2.722x + 52.1428$.

- Which polynomial best models the number of Americans over 100 years old? _______________
- Use the equation chosen to predict the number of Americans who will be over 100 years old in the year 2008. __________________________________

Name____ Date__________________________

Predicting Degree of Polynomial by Zeros

x	–3	–2	–1	0	1	2	3	4
y	–36	0	8	0	–12	–16	0	48

(1) Using the data above, what would be the least degree of a polynomial that would model the data and explain why.
3rd degree polynomial because there are three zeros

(2) Predict the equation of the polynomial in factored form. $y = x(x - 3)(x + 2)$

(3) Plot the data in a graphing calculator and make a scatter plot. (To enter data on a TI–84 calculator: **STAT**, **1:Edit**, enter data into **L_1** and **L_2** . To set up the plot of the data: **2nd , STAT PLOT, 1:PLOT1, ENTER**, **On**, **Type**: , **Xlist: L_1, Ylist: L_2, Mark** (any). To graph the scatter plot: **ZOOM , 9: ZoomStat.)** Then enter the equation to see if it matches the data. Adjust the leading coefficient of the equation until the graph matches the data and write the final equation. $f(x) = 2x(x - 3)(x + 2)$

Method of Finite Differences

x	–3	–2	–1	0	1	2	3	4
y	–14	–11	–8	–5	–2	1	4	7

(4) Using the data above, what would be the least degree of a polynomial that would model the data and explain why you came to that conclusion.
1st degree because the change in y over the change in x (slope) is constant

(5) Use the Method of Finite Differences to twice subtract the y-values to get 0 to determine that the function is linear, a polynomial of first degree. Find the equation of the line. $f(x) = 3x - 5$

(6) Apply the method of finite differences to the first table several times and develop the guidelines for determining the degree of the polynomial:

y = c if *the 1st order differences are 0*

y = ax+ b if *the 2nd order differences are 0*

$y = ax^2 + bx + c$ if *the 3rd order differences are 0*

$y = ax^3 + bx^2 + cx + d$ if *the 4th order differences are 0*

(7) What are the limitations of using this method in evaluating real-life data?
Real-world data is not exact

Real Life Application

year	1994	1996	1998	2000	2002	2004
number	50	56	65	75	94	110

Because of improved health care, people are living longer. The above data relates the number of Americans (in thousands) who are expected to be over 100 years old for the selected years. (Source: US Census Bureau) Enter the data into the calculator, letting x = 4 correspond to 1994 and make a scatter plot. Then graph the following equations:

$y_1 = 6.057x + 20.4857$, $y_2 = 0.4018x^2 - 1.175x + 48.343$, $y_3 = -0.007x^3 + 0.5893x^2 - 2.722x + 52.1428$.

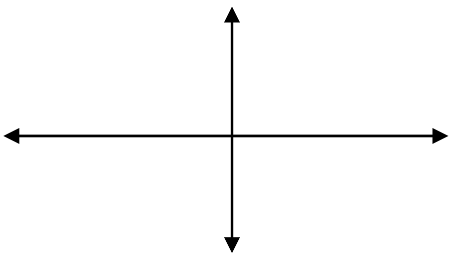
- Which polynomial best models the number of Americans over 100 years old: y_3 *(Teacher Note: y_2 and y_3 look very similar. Students should also consider end-behavior that fits the real world situation.)*
- Use the equation chosen to predict the number of Americans who will be over 100 years old in the year 2008. $y_3(18)$*=153,571 Americans*

Unit 2, Activity 9, Solving Polynomial Inequalities by Graphing

Name______________________________________ Date________________________

Polynomial Inequalities The equations in the Bellringer have only one variable. However, it is helpful to use a two-variable graph to quickly solve a one-variable inequality. Fast graph the following polynomial functions only paying attention to the x–intercepts and the end-behavior. Use the graphs to solve the one-variable inequalities by looking at the positive and negative values of y.

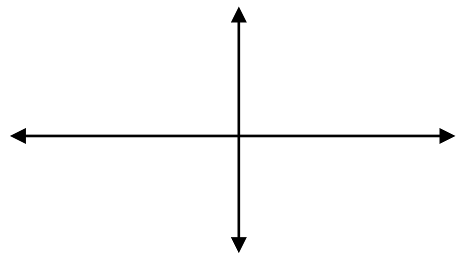
(1) Graph $y = -2x + 6$



Solve for x: $-2x + 6 > 0$

____

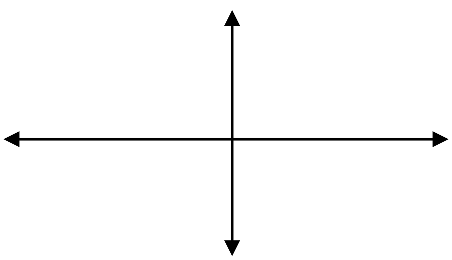
(4) Graph $y = (x - 3)(x + 4)(x - 7)$



Solve for x: $(x - 3)(x + 4)(x - 7) \geq 0$

____

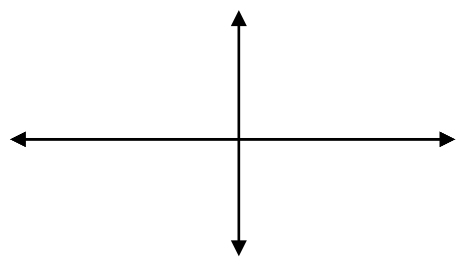
(2) Graph $y = x(x - 4)$



Solve for x: $x(x - 4) > 0$

____

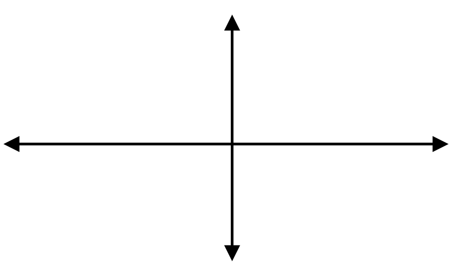
(5) Graph $y = x^2 - 9x + 14$



Solve for x: $x^2 - 9x < -14$

____

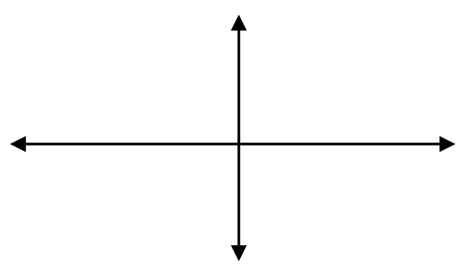
(3) Graph $y = x(x - 4)$



Solve for x : $x(x - 4) \leq 0$

____

(6) Graph $y = 5x^3 - 15x^2$



Solve for x: $5x^3 \leq 15x^2$ *(Hint: Isolate 0 first.)*

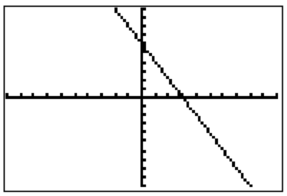
____

Unit 2, Activity9, Solving Polynomial Inequalities by Graphing with Answers

Name______________________________________ Date________________________

Polynomial Inequalities The equations in the Bellringer have only one variable. However, it is helpful to use a two-variable graph to quickly solve a one-variable inequality. Fast graph the following polynomial functions only paying attention to the x–intercepts and the end-behavior. Use the graphs to solve the one-variable inequalities by looking at the positive and negative values of y.

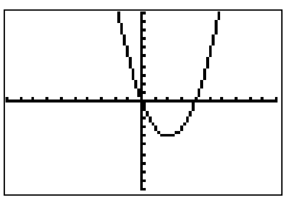
(1) Graph $y = -2x + 6$



Solve for x: $-2x + 6 > 0$

$(-\infty, 3)$

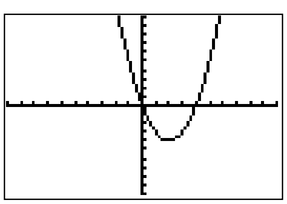
(2) Graph $y = x(x - 4)$



Solve for x: $x(x - 4) > 0$

$(-\infty, 0) \cup (4, \infty)$

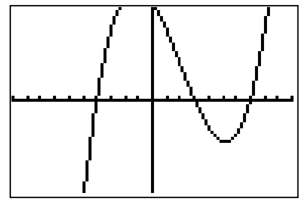
(3) Graph $y = x(x - 4)$



Solve for x: $x(x - 4) \leq 0$

$[0, 4]$

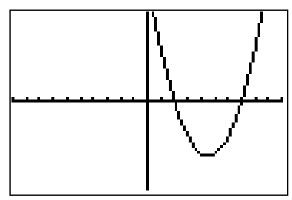
(4) Graph $y = (x - 3)(x + 4)(x - 7)$



Solve for x: $(x - 3)(x + 4)(x - 7) \geq 0$

$[-4, 3] \cup [7, \infty)$

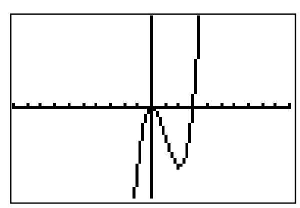
(5) Graph $y = x^2 - 9x + 14$



Solve for x: $x^2 - 9x < -14$ *(Hint : Isolate 0 first.)*

$(2, 9)$

(6) Graph $y = 5x^3 - 15x^2$



Solve for x: $5x^3 \leq 15x^2$ *(Hint: Isolate 0 first.)*

$(-\infty, 3]$

Unit 2, Activity10, Polynomial Identities Discovery Worksheet

Name_______________________________________ Date________________________

Polynomial Form of a Binomial Number A binomial number is a number of the form, $a^n \pm b^n$ where *a* and *b* and *n* are integers. Expand the following polynomials to create binomial numbers. Write the binomial number in the blank.

(1) ____________ $= (a - b)(a + b)$

(2) ____________ $= (a - b)(a^2 + ab + b^2)$

(3) ____________ $= (a - b)(a + b)(a^2 + b^2)$

(4) ____________ $= (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

(5) ____________ $= (a - b)(a + b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

(6) ____________ $= (a - b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$

(7) ____________ $= (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$

(8) ____________ $= (a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6)$

Describe any patterns you see. ____

____

____

Consider the sums below.

$$
\begin{aligned}
a^2 + b^2 &= a^2 + b^2 \\
a^3 + b^3 &= (a + b)\left(a^2 - a\,b + b^2\right) \\
a^4 + b^4 &= a^4 + b^4 \\
a^5 + b^5 &= (a + b)\left(a^4 - a^3\,b + a^2\,b^2 - a\,b^3 + b^4\right) \\
a^6 + b^6 &= \left(a^2 + b^2\right)\left(a^4 - a^2\,b^2 + b^4\right) \\
a^7 + b^7 &= (a + b)\left(a^6 - a^5\,b + a^4\,b^2 - a^3\,b^3 + a^2\,b^4 - a\,b^5 + b^6\right) \\
a^8 + b^8 &= a^8 + b^8 \\
a^9 + b^9 &= (a + b)\left(a^2 - a\,b + b^2\right)\left(a^6 - a^3\,b^3 + b^6\right) \\
a^{10} + b^{10} &= \left(a^2 + b^2\right)\left(a^8 - a^6\,b^2 + a^4\,b^4 - a^2\,b^6 + b^8\right).
\end{aligned}
$$

Describe any patterns you see: ____

____

Unit 2, Activity10, Polynomial Identities Discovery Worksheet

The Square of a Trinomial Prove the polynomial identity for $(a + b + c)^2$ three ways.

Proof #1: Rewrite $(a + b + c)^2 = (a + b + c)(a + b + c)$, expand and simplify.

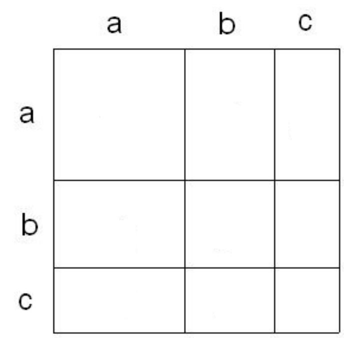
Proof #2: Rewrite $(a + b + c)^2 = ((a+b) + c)^2$ *and use the identity* $(x+y)^2 = x^2+2xy+y^2$

Proof #3: Prove geometrically.

Area of large rectangle = ________________________

Sum of the areas of the small rectangles =

____



Application Three integers a, b, and c that satisfy $a^2 + b^2 = c^2$ are called *Pythagorean Triples*. There are infinitely many such numbers and there also exists a polynomial identity to generate the triples. Let n and m be integers, $n>m$. Define $a = n^2 – m^2$, $b = 2nm$, $c = n^2+m^2$. Prove the three numbers $a, b,$ and c always form a Pythagorean triple by expanding each side of the following polynomial identity:

$$a^2 \quad + \quad b^2 \quad = \quad c^2$$
$$(n^2 - m^2)^2 \quad + \quad (2nm)^2 \quad = \quad (n^2+m^2)^2$$

Choose several numbers n and m to generate a Pythagorean triple (a, b, c) and test them.

Unit 2, Activity10, Polynomial Identities Discovery Worksheet with Answers

Name_____________________________________ Date_______________________

Polynomial Form of a Binomial Number A binomial number is a number of the form, $a^n \pm b^n$ where *a* and *b* and *n* are integers. Expand the following polynomials to create binomial numbers. Write the binomial number in the blank.

(1) $\underline{a^2 - b^2}$ $= (a - b)(a + b)$

(2) $\underline{a^3 - b^3}$ $= (a - b)(a^2 + ab + b^2)$

(3) $\underline{a^4 - b^4}$ $= (a - b)(a + b)(a^2 + b^2)$

(4) $\underline{a^5 - b^5}$ $= (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

(5) $\underline{a^6 - b^6}$ $= (a - b)(a + b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

(6) $\underline{a^7 - b^7}$ $= (a - b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$

(7) $\underline{a^8 - b^8}$ $= (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$

(8) $\underline{a^9 - b^9}$ $= (a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6)$

Describe any patterns you see. ***The first factor is always (a – b). For binomial numbers with odd exponents ($a^{2n+1} - b^{2n+1}$), the second factor has all sums with decreasing powers of a and increasing powers of b. Binomials with even exponents are two perfect squares and should first be factored using that identity, then apply odd exponent properties.***

Consider the sums below.

$$
\begin{aligned}
a^2 + b^2 &= a^2 + b^2 \\
a^3 + b^3 &= (a + b)\left(a^2 - a\,b + b^2\right) \\
a^4 + b^4 &= a^4 + b^4 \\
a^5 + b^5 &= (a + b)\left(a^4 - a^3\,b + a^2\,b^2 - a\,b^3 + b^4\right) \\
a^6 + b^6 &= \left(a^2 + b^2\right)\left(a^4 - a^2\,b^2 + b^4\right) \\
a^7 + b^7 &= (a + b)\left(a^6 - a^5\,b + a^4\,b^2 - a^3\,b^3 + a^2\,b^4 - a\,b^5 + b^6\right) \\
a^8 + b^8 &= a^8 + b^8 \\
a^9 + b^9 &= (a + b)\left(a^2 - a\,b + b^2\right)\left(a^6 - a^3\,b^3 + b^6\right) \\
a^{10} + b^{10} &= \left(a^2 + b^2\right)\left(a^8 - a^6\,b^2 + a^4\,b^4 - a^2\,b^6 + b^8\right).
\end{aligned}
$$

Describe any patterns you see: ***Answers may vary. Even exponent binomials that are powers of 2, cannot be factored. Exponents 6 and 10 start with the factor ($a^2 + b^2$). Odd exponent binomials start with (a+b).***

Unit 2, Activity10, Polynomial Identities Discovery Worksheet with Answers

The Square of a Trinomial Prove the polynomial identity for $(a + b + c)^2$ three ways.

Proof #1: Rewrite $(a + b + c)^2 = (a + b + c)(a + b + c)$, expand and simplify.

$=a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2$
$=a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Proof #2: Rewrite $(a + b + c)^2 = ((a+b) + c)^2$ *and use the identity* $(x+y)^2 = x^2+2xy+y^2$

$((a+b) + c)^2 = (a+b)^2+2(a+b)c+c^2 = a^2 +2ab+ b^2 + 2ac +2bc +c^2$
$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Proof #3: Prove geometrically.

Area of large rectangle = ***$(a + b + c)^2$***

Sum of the areas of the small rectangles =

$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

	a	b	c
a	a^2	ab	ac
b	ab	b^2	bc
c	ac	bc	c^2

Application Three integers a, b, and c that satisfy $a^2 + b^2 = c^2$ are called *Pythagorean Triples*. There are infinitely many such numbers and there also exists a polynomial identity to generate the triples. Let n and m be integers, $n > m$. Define $a = n^2 - m^2$, $b = 2nm$, $c = n^2 + m^2$. Prove the three numbers a, b, and c always form a Pythagorean triple by expanding each side of the following polynomial identity:

$$a^2 + b^2 = c^2$$
$$(n^2 - m^2)^2 + (2nm)^2 = (n^2+m^2)^2$$
$$n^4 - 2n^2m^2 + m^4 + 4n^2m^2 = n^4 + 2n^2m^2 + m^4$$
$$n^4 + m^4 + 4n^2m^2 = n^4+ 4n^2m^2 + m^4$$

$$n^4+ 4n^2m^2 + m^4 = n^4+ 4n^2m^2 + m^4$$

Choose several numbers n and m to generate a Pythagorean triple (a, b, c) and test them.
Answers may vary: Let $n = 5$ and $m = 4$. $a=5^2-4^2=9$, $b=2(5)(4)=40$, $c=5^2+4^2=41$
Test : $9^2 + 40^2 = 41^2$, $1681=1681$

Unit 2, Activity 7, Specific Assessment Graphing Polynomials

Name_____________________________________ Date_______________________

Graphs of Polynomials

Find the zeros and use the rules developed in the Graphing Polynomials Discovery Worksheet to sketch the following graphs without a calculator. Label accurately the zeros, end–behavior, and *y*–intercepts. Do not be concerned with minimum and maximum values between zeros.

a. $y = x^3 - 8x^2 + 16x$

b. $y = -2x^2 - 12x - 10$

c. $y = (x - 4)(x + 3)(x + 1)$

d. $y = -(x + 2)(x - 7)(x + 5)$

e. $y = (2 - x)(3 - x)(5 + x)$

f. $y = x^2 + 10 + 25$

g. $y = (x - 3)^2(x + 5)$

h. $y = (x - 3)^3(x + 5)$

i. $y = (x - 3)^3(x + 5)^2$

j. $y = (x - 3)^4(x + 5)$

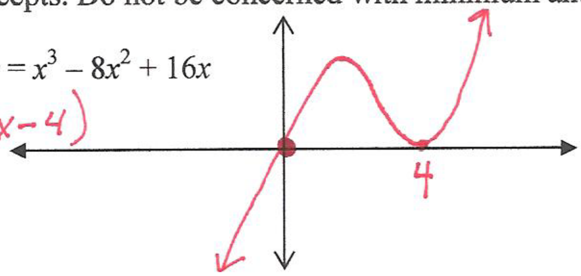
Name________________________________ Date______________________

Graphs of Polynomials

Find the zeroes and use the rules you developed in the Discovery Worksheet to sketch the following graphs without a calculator. The following must be labeled and accurate: zeroes, end behavior, and y intercepts. Do not be concerned with minimum and maximum values between zeroes.

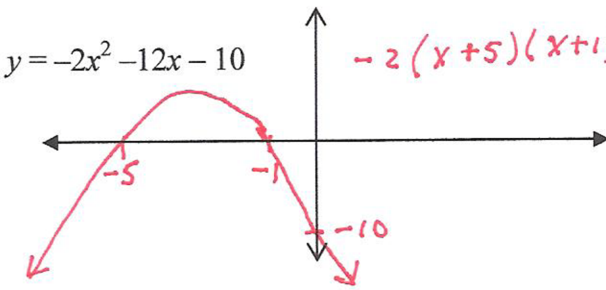
a. $y = x^3 - 8x^2 + 16x$

$x(x-4)(x-4)$

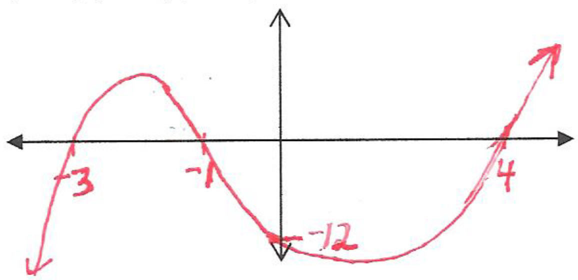


b. $y = -2x^2 - 12x - 10$

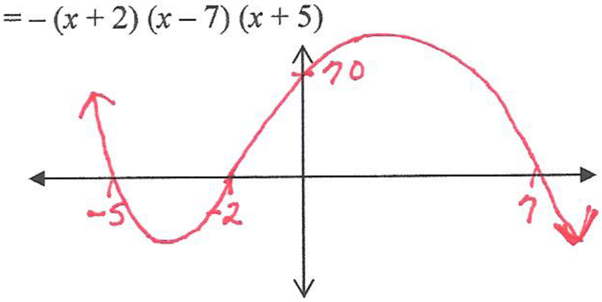
$-2(x+5)(x+1)$



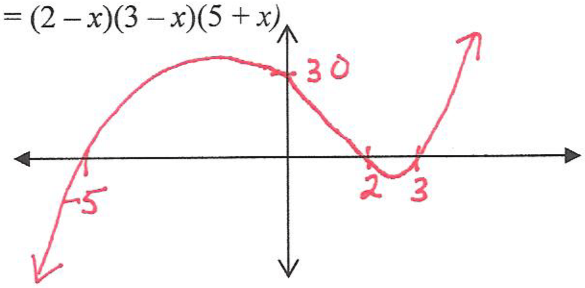
c. $y = (x-4)(x+3)(x+1)$



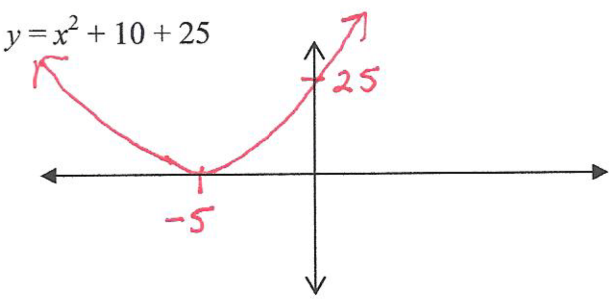
d. $y = -(x+2)(x-7)(x+5)$



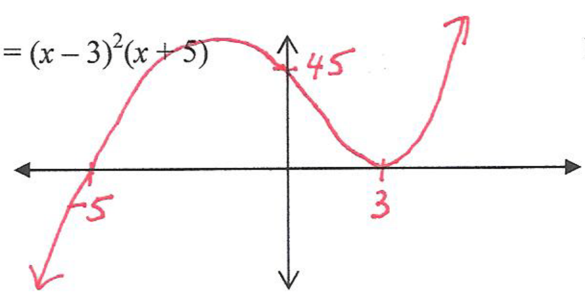
e. $y = (2-x)(3-x)(5+x)$



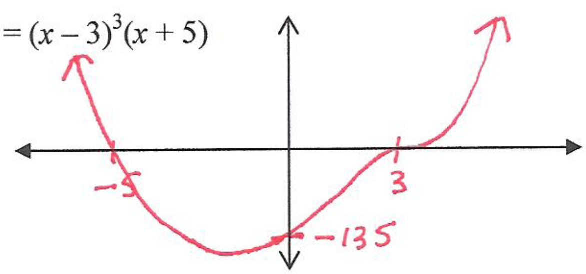
f. $y = x^2 + 10 + 25$



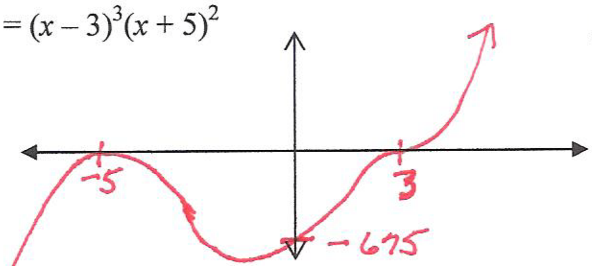
g. $y = (x-3)^2(x+5)$



h. $y = (x-3)^3(x+5)$



i. $y = (x-3)^3(x+5)^2$



j. $y = (x-3)^4(x+5)$