Unit 5, Activity 1, The Counting Principle

Directions: With a partner find the answer to the following problems.

1. A person buys 3 different shirts (Green, Blue, and Red) and two different pants (Khaki and Grey). How many different outfits can the person make out of the five pieces of clothing? Make a tree diagram showing all the possible ways this can be done.

2. Jim, Bob, Carol, and Sue are standing in line. What are all the possible ways they could arrange themselves? List them below.

3. Moesha has 6 different pairs of socks and 2 different pairs of sneakers. How many possible combinations of socks/sneakers are there if she picks one pair of socks and one pair of sneakers each day? Do this problem without drawing a tree diagram or making a list.

4. How many license plates are possible if four letters are to be followed by two digits and you can repeat letters and digits?

5. How many license plates are possible if four letters are to be followed by two digits and you cannot repeat letters or digits?

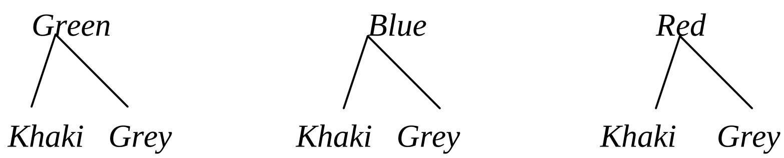
6. How many license plates are possible if two letters are to be followed by four digits and you can repeat letters and digits?

7. A 4-letter password is required to enter a computer file. How many passwords are possible if no letter is repeated?

Unit 5, Activity 1, The Counting Principle with Answers

Directions: With a partner find the answer to the following problems.

1. A person buys 3 different shirts (Green, Blue, and Red) and two different pants (Khaki and Grey). How many different outfits can the person make out of the five pieces of clothing? Make a tree diagram showing all the possible ways this can be done. *Answer: 6 outfits shown below:*



2. Jim, Bob, Carol, and Sue are standing in line. What are all the possible ways they could arrange themselves? List them below.

J, B, C, S	*B, C, S, J*	*C, B, S, J*	*S, B, J, C*
J, B, S, C	*B, C, J, S*	*C, B, J, S*	*S, B, C, J*
J, S, B, C	*B, J, S, C*	*C, J, B, S*	*S, C, J, B*
J, S, C, B	*B, J, C, S*	*C, J, S, B*	*S, C, B, J*
J, C, B, S	*B, S, J, C*	*C, S, B, J*	*S, J, B, C*
J, C, S, B	*B, S, C, J*	*C, S, J, B*	*S, J, C, B*

3. Moesha has 6 different pairs of socks and 2 different pairs of sneakers. How many possible combinations of socks/sneakers are there if she picks one pair of socks and one pair of sneakers each day? Do this problem without drawing a tree diagram or making a list.
   Answer: 6 x 2 = 12

4. How many license plates are possible if four letters are to be followed by two digits and you can repeat letters and digits?

   Answer: 26 x 26 x 26 x 26 x 10 x 10 = 45,697,600 plates

5. How many license plates are possible if four letters are to be followed by two digits and you cannot repeat letters or digits?

   Answer: 26 x 25 x 24 x 23 x 10 x 9 = 32,292,000 plates

6. How many license plates are possible if two letters are to be followed by four digits and you can repeat letters and digits?

   Answer: 26 x 26 x 10 x 10 x 10 x 10 = 6,760,000 plates

7. A 4-letter password is required to enter a computer file. How many passwords are possible if no letter is repeated?

   Answer: 26 x 25 x 24 x 23 = 358,800 passwords

Student Note: When you find the number of possible arrangements or orderings in a counting problem, when different orderings of the same items are counted separately, you have what is referred to as a *PERMUTATION* problem. For example, if you are looking at filling officers for a President and Vice-President, then the two names George (G) and Susan (S) have two possibilities or arrangements. Hence, GS and SG are two different arrangements since George could be President or Vice-President, and the same goes for Susan.

Example: What are all the possible arrangements for the letters A, B, and C?
In this example, the orderings are important and taken into account. ABC is a different ordering from ACB, BCA, BAC, CAB, and CBA. All in all, there are six arrangements possible by making a list. If you use the counting principle, you have the following:

There are 3 slots to fill: _______ _______ ________
In the first slot we could put any of the three letters: __3__ _______ _______
In the second, we could put one of the other two letters: __3__ __2__ _____
In the third slot, we only have one letter to fill the slot: __3__ __2__ __1__
Multiplying we get: 3 x 2 x 1 = 6 ways to arrange the letters A, B, C.

When that order is taken into consideration, we have a PERMUTATION. If order is not important, you have what is referred to as a COMBINATION (this will be studied next).

Directions: Use the Counting Principle to determine the answers to the following permutation problems.

1. You have just been hired to determine the programming for a television network. When selecting the shows to be shown on Monday night, you find there are 27 shows to choose from, but only 4 slots to fill for that night. How many different sequences of 4 shows are possible?

2. How many different 6-letter license plates can be made using only the letters of the alphabet if no letter can be used more than once on any plate?

3. In a 9-horse race, how many possibilities are there for the horses to place 1^{st}, 2^{nd}, and 3^{rd} in the race?

4. A disc jockey can play seven songs in one time slot. In how many different orders can the 7 songs be played?

5. A car dealer has 38 used cars to sell. Each day two cars are chosen for advertising specials. One car appears in a television commercial, and the other appears in a newspaper advertisement. In how many ways can the two cars be chosen on any given day?

6. Melody has nine bowling trophies to arrange in a horizontal line on a shelf. How many arrangements are possible?

7. In how many ways can a president, a vice president, and a treasurer be chosen from a group of 15 people?

Permutation Notation: One way to write a permutation mathematically is using the notation, nPr, where *n* is the number of objects you have to choose from and *r* is the number of slots to be filled. For example, if you have 10 people to choose from and 3 slots (such as president, vice president, and secretary) to fill, this could be denoted as ${}_{10}P_{3}$ and the answer would be found by: 10 x 9 x 8 = 720 ways to choose. Solve the following problems using that notation, and make up a real-world problem that fits that particular permutation problem.

8. ${}_{5}P_{3}$

9. ${}_{10}P_{10}$

10. ${}_{8}P_{2}$

Student Note: When you find the number of possible arrangements or orderings in a counting problem, when different orderings of the same items are counted separately, you have what is referred to as a *PERMUTATION* problem. For example, if we are looking at filling officers for a President and Vice-President, then the two names George (G) and Susan (S) have two possibilities or arrangements. Hence, GS and SG are two different arrangements since George could be President or Vice-President, and the same goes for Susan.

Example: What are all the possible arrangements for the letters A, B, and C?
In this example, the orderings are important and taken into account. ABC is a different ordering from ACB, BCA, BAC, CAB, and CBA. All in all, there are six arrangements possible by making a list. If you use the counting principle we have the following:

There are 3 slots to fill: _______ _______ ________
In the first slot we could put any of the three letters: __3__ _______ _______
In the second, we could put one of the other two letters: __3__ __2__ _____
In the third slot, we only have one letter to fill the slot: __3__ __2__ __1__
Multiplying we get: 3 x 2 x 1 = 6 ways to arrange the letters A, B, C.

When that order is taken into consideration, we have a PERMUTATION. If order is not important, you have what is referred to as a COMBINATION (this will be studied next).

Directions: Use the Counting Principle to determine the answers to the following permutation problems.

1. You have just been hired to determine the programming for a television network. When selecting the shows to be shown on Monday night, you find there are 27 shows to choose from, but only 4 slots to fill for that night. How many different sequences of 4 shows are possible?

   Answer: 27 x 26 x 25 x 24 = 421,200

2. How many different 6-letter license plates can be made using only the letters of the alphabet if no letter can be used more than once on any plate?

   Answer: 26 x 25 x 24 x 23 x 22 x 21 = 165,765,600 plates

3. In a 9-horse race, how many possibilities are there for the horses to place 1^{st}, 2^{nd}, and 3^{rd} in the race?
   Answer: 9 x 8 x 7 = 504

4. A disc jockey can play seven songs in one time slot. In how many different orders can the 7 songs be played?
   Answer: 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040 ways

Unit 5, Activity 2, Permutations with Answers

5. A car dealer has 38 used cars to sell. Each day two cars are chosen for advertising specials. One car appears in a television commercial and the other appears in a newspaper advertisement. In how many ways can the two cars be chosen on any given day?

   Answer: 38 x 37 = 1406 ways

6. Melody has nine bowling trophies to arrange in a horizontal line on a shelf. How many arrangements are possible?

   Answer: 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 9! or 362,880 arrangements

7. In how many ways can a president, a vice president, and a treasurer be chosen from a group of 15 people?

   Answer: 15 x 14 x 13 = 2730 ways

Permutation Notation: One way to write a permutation mathematically is using the notation, ${}_{n}P_{r}$, where *n* is the number of objects you have to choose from and *r* is the number of slots to be filled. For example, if you have 10 people to choose from and 3 slots (such as president, vice president, and secretary) to fill, this could be denoted as ${}_{10}P_{3}$ and the answer would be found by: 10 x 9 x 8 = 720 ways to choose. Solve the following problems using that notation, and make up a real-world problem that fits that particular permutation problem.

8. ${}_{5}P_{3}$

   Answer: 60; see student problems that match the solution.

9. ${}_{10}P_{10}$

   Answer: 3,628,800; see student problems that match the solution.

10. ${}_{8}P_{2}$

   Answer: 56; see student problems that match the solution.

Unit 5, Activity 3, Combinations

Student Note: When you intend to select ***r*** items from ***n*** different items but DO NOT TAKE ORDER INTO ACCOUNT, you are really concerned with possible *COMBINATIONS* rather than permutations. That is, ***when different orderings of the same items are counted separately, you have a permutation, but when different orderings of the same items are not counted separately, you have a combination problem.***

For example: At a local pizza parlor, there are 9 toppings to choose from. If a person gets a 3-topping pizza, how many different 3-topping pizzas can be made?

If ordering were important you would get: 9 x 8 x 7 = 504 possible pizzas. However, in this situation ordering is not important, i.e., Hamburger (H), Pepperoni (P), and Sausage(S) or HPS is the same as HSP, PSH, PHS, SPH, and SHP. In fact, every 3-topping pizza has 6 arrangements of the same toppings, and all of these are counted in this 504 answer. In a combination situation, you simply find the answer to the permutation problem, and then divide by the number of arrangements of the number of slots you are filling. In the above problem, there were three slots to fill, so you divide by 3 x 2 x 1 = 6.

Therefore, to answer the original question, you would have to divide 504 by 6 to get the true solution to this problem.
Thus, 504 ÷ 6 = 84 (there are really only 84 different 3-topping pizzas possible).

Directions: Solve the following problems below. Determine if order is important or unimportant, and solve each using what you have learned about permutations and combinations.

1. Susie wants to bring 3 books with her to read on her trip. If she has 8 books to choose from, how many choices does she really have to do this?

2. Five people are selected from a panel of 10 to serve on a committee. In how many ways can the committee be chosen?

3. If there are 8 different types of flowers to choose from at a local florist, in how many ways can Mrs. Harris pick 3 types of flowers to make a bouquet?

4. A team of nine players is to be chosen from 15 available players. In how many ways can this be done?

5. In math class, there are 25 students. The teacher picks 4 students to serve on the bulletin board committee. How many different committees of 4 are possible?

6. A combination lock has 30 numbers on it. How many different 3-number combinations are possible if no numbers may be repeated?

7. A person has a full deck of cards and shuffles it. He then pulls one card from the deck and then his friend pulls another card from the deck. How many possible outcomes could there be when they pull the two cards if order is important? If order is not important? Explain how you got your answer.

8. A combination problem can be shown by using ${}_nC_r$ notation (just as in permutation problems). Find the value of ${}_{10}C_3$ and come up with a real-world situation that would model this type of problem.

9. A home alarm system has a 3-digit code that can be used to deactivate the system. If the homeowner forgets the code, how many different codes might the homeowner have to try if digits can be repeated?

10. At a fair, there is a game in which a person rolls a single die and then flips a coin. How many possible outcomes are there in this game? Explain how you got your answer.

Unit 5, Activity 3, Combinations with Answers

Student Note: When you intend to select ***r*** items from ***n*** different items but DO NOT TAKE ORDER INTO ACCOUNT, you are really concerned with possible *COMBINATIONS* rather than permutations. That is, ***when different orderings of the same items are counted separately, you have a permutation, but when different orderings of the same items are not counted separately, you have a combination problem.***

For example: At a local pizza parlor, there are 9 toppings to choose from. If a person gets a 3-topping pizza, how many different 3-topping pizzas can be made?

If ordering were important you would get: 9 x 8 x 7 = 504 possible pizzas. However, in this situation ordering is not important, i.e., Hamburger (H), Pepperoni (P), and Sausage(S) or HPS is the same as HSP, PSH, PHS, SPH, and SHP. In fact, every 3-topping pizza has 6 arrangements of the same toppings, and all of these are counted in this 504 answer. In a combination situation, you simply find the answer to the permutation problem, and then divide by the number of arrangements of the number of slots you are filling. In the above problem, there were three slots to fill, so you divide by 3 x 2 x 1 = 6.

Therefore, to answer the original question, you would have to divide 504 by 6 to get the true solution to this problem.
Thus, 504 ÷ 6 = 84 (there are really only 84 different 3-topping pizzas possible).

Directions: Solve the following problems below. Determine if order is important or unimportant, and solve each using what you have learned about permutations and combinations.

1. Susie wants to bring 3 books with her to read on her trip. If she has 8 books to choose from, how many choices does she really have to do this?

   Answer: $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$ *or 56 ways.*

2. Five people are selected from a panel of 10 to serve on a committee. In how many ways can the committee be chosen?

   Answer: $\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$ *or 252 ways*

3. If there are 8 different types of flowers to choose from at a local florist, in how many ways can Mrs. Harris pick 3 types of flowers to make a bouquet?

   Answer: $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$ *or 56 ways*

4. A team of nine players is to be chosen from 15 available players. In how many ways can this be done?
   Answer: $\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$ *or 5005 ways.*

5. In math class, there are 25 students. The teacher picks 4 students to serve on the bulletin board committee. How many different committees of 4 are possible?
   Answer: $\frac{25 \times 24 \times 23 \times 22}{4 \times 3 \times 2 \times 1}$ *or 12650 ways.*

6. A combination lock has 30 numbers on it. How many different 3-number combinations are possible if no numbers may be repeated?
   Answer: 30 x 29 x 28 = 24360 combinations

7. A person has a full deck of cards and shuffles it. He then pulls one card from the deck, and then his friend pulls another card from the deck. How many possible outcomes could there be when they pull the two cards if order is important? If order is not important? Explain how you got your answer.
   Answer: 52 x 51 = 2652 (if order of the cards is important)
   (52 x 51) ÷ 2 = 1326 (if order of the cards is not important)

8. A combination problem can be shown by using nCr notation (just as in permutation problems). Find the value of ${}_{10}C_{3}$ and come up with a real-world situation that would model this type of problem.

   Answer: 120; Check student problems.

9. A home alarm system has a 3-digit code that can be used to deactivate the system. If the homeowner forgets the code, how many different codes might the homeowner have to try if digits can be repeated?

   Answer: 10 x 10 x 10 = 1000 codes.

10. At a fair, there is a game in which a person rolls a single die and then flips a coin. How many possible outcomes are there in this game? Explain how you got your answer.

    Answer: 6 x 2 = 12; could have drawn a tree diagram or used counting principle.

Unit 5, Activity 5, More Complex Probability

Student Note: The counting techniques talked about thus far are sometimes used to determine the probability of an event.

For example: What is the probability that a random selection of 3 names from a field of 10 horses will yield the winner, the second place, and the third place horses in the order they were selected?

First, determine the total number of outcomes. In this situation, since order matters, it is a permutation: 10 x 9 x 8 = 720 possible outcomes.
Since in this situation there is only one way to win, the theoretical probability is: P (picking the three places) = 1/720. This can be expressed as a fraction, a decimal (.00138) or a percent (.14%).

Directions: Determine the theoretical probability for the following:

1. The letters M, A, T, H, E, M, A, T, I, C, and S are written on separate sheets of paper and put into a bag. A letter is pulled from the bag. Determine the probability of the following:
   a. P(A) = _______ b. P(M) = _________ c. P (F) = __________
   d. P (not A) = __________ P (a letter) = _________

2. Jan was born in 1973. If security codes are composed of 4 different digits, what is the probability that a security code issued to Jan would be the year of her birth?

3. A spinner numbered 1 through 10 is spun. Each outcome is equally likely. Write each probability as a fraction, decimal, and percent.
   a. P (9) = ________, _________, ________
   b. P (multiple of 4) = _______ , ________, ________

4. Susie wants to bring 3 books with her to read on her trip. If she has 8 books on a shelf to choose from, and one of the books is *Little Women*, what is the probability one of the books she packs will be *Little Women* if she chooses the books randomly from her shelf?

5. A typical combination lock is opened with the correct sequence of three numbers between 0 and 49 inclusive (including 0 and 49). If a number can be used more than once, what is the probability of guessing those three numbers and opening the lock on the first try? Write your answer as a fraction and as a percent.

6. Jerry attended a conference. There were 100 people (including Jerry) at the conference. All of the names of the people attending were placed into a box, and three names were pulled for door prizes. What is the probability that Jerry will win a door prize? Write the answer as a fraction and percent.

Unit 5, Activity 5, More Complex Probability with Answers

Student Note: The counting techniques talked about thus far are sometimes used to determine the probability of an event.

For example: What is the probability that a random selection of 3 names from a field of 10 horses will yield the winner, the second place, and the third place horses in the order they were selected?

First, determine the total number of outcomes. In this situation, since order matters, it is a permutation: 10 x 9 x 8 = 720 possible outcomes.
Since in this situation there is only one way to win, the theoretical probability is: P (picking the three places) = 1/720. This can be expressed as a fraction, a decimal (.00138) or a percent (.14%).

Directions: Determine the theoretical probability for the following:

1. The letters M, A, T, H, E, M, A, T, I, C, and S are written on separate sheets of paper and put into a bag. A letter is pulled from the bag. Determine the probability of the following:
   a. P(A) = __*2/11*___ b. P(M) = __*2/11*_____c. P (F) = ___*0*____
   d. P (not A) = ____*9/11*____ P (a letter) = ___*1*___

2. Jan was born in 1973. If security codes are composed of 4 different digits, what is the probability that a security code issued to Jan would be the year of her birth?
   Answer: 1/5040

3. A spinner numbered 1 through 10 is spun. Each outcome is equally likely. Write each probability as a fraction, decimal, and percent.
   a. P (9) = __*1/10*____, ____*.1*___, ____*10%*___
   b. P (multiple of 4) = _*2/10 or 1/5*__ , __*.2*___, __*20%*___

4. Susie wants to bring 3 books with her to read on her trip. If she has 8 books on a shelf to choose from, and one of the books is *Little Women*, what is the probability one of the books she packs will be *Little Women* if she chooses the books randomly from her shelf?
   Answer: 1/56

5. A typical combination lock is opened with the correct sequence of three numbers between 0 and 49 inclusive (including 0 and 49). If a number can be used more than once, what is the probability of guessing those three numbers and opening the lock on the first try? Write your answer as a fraction and as a percent.
   Answer: 1/117649 or 0.00085%

6. Jerry attended a conference. There were 100 people (including Jerry) at the conference. All of the names of the people attending were placed into a box, and three names were pulled for door prizes. What is the probability that Jerry will win a door prize? Write the answer as a fraction and percent.
   Answer: 1/161700 or 0.0006184%

Unit 5, Activity 7, Probability Based on Sample Data with Answers

Directions: Solve the following problems. Be ready to discuss your solution with your classmates.

1. A worker at a fruit juice plant inspects 800 bottles of juice.
   a. If 10 of the bottles are improperly filled, what is the probability that a randomly chosen bottle has too much or too little juice in it?

   b. If the company produces 20,000 bottles of juice in a week, predict the number of bottles of juice that will be improperly filled each week.

2. A bakery inspects samples of its pastries each day. On a certain day, a worker inspected 650 pastries and found 15 that were defective.
   a. Based upon this data, what is the percent (rounded to the nearest tenth-percent) of defective pastries that the plant produced that day?

   b. If 3000 pastries are made, how many would you expect to be defective?

   c. After altering one of the machines, the next day the inspector checked a sample of 750 pastries and found that 17 were defective. Did the percentage of defective pastries improve or worsen? Explain how you know.

3. A T-shirt manufacturer produced 10,000 shirts last week. Three different inspectors took samples of the shirts produced, and the results are shown in the table below.

Inspector	# shirts sampled	# defective
A	500	21
B	300	17
C	200	12

   a. What is the probability that a shirt is defective out of the whole batch if only Inspector A's results are used to determine the probability?

   b. What is the probability that a shirt is defective if all of the inspectors' results are tabulated?

   c. How many shirts would you expect to be defective out of the whole batch produced using the results you found in part b?

Unit 5, Activity 7, Probability Based on Sample Data with Answers

Directions: Solve the following problems. Be ready to discuss your solution with your classmates.

1. A worker at a fruit juice plant inspects 800 bottles of juice.
   a. If 10 of the bottles are improperly filled, what is the probability that a randomly chosen bottle has too much or too little juice in it?
   Answer: 10/800 or 1/80 or 1.25%

   b. If the company produces 20,000 bottles of juice in a week, predict the number of bottles of juice that will be improperly filled each week.
   Answer: 250 bottles.

2. A bakery inspects samples of its pastries each day. On a certain day, a worker inspected 650 pastries and found 15 that were defective.
   a. Based upon this data, what is the percent (rounded to the nearest tenth- percent) of defective pastries that the plant produced that day?
   Answer: 2.3%

   b. If 3000 pastries are made, how many would you expect to be defective?
   Answer: 69

   c. After altering one of the machines, the next day the inspector checked a sample of 750 pastries and found that 17 were defective. Did the percentage of defective pastries improve or worsen? Explain how you know.
   Answer: The percentage was slightly lower but basically remained the same as before (about 2.3% when rounded to the nearest tenth-percent).
3. A T-shirt manufacturer produced 10,000 shirts last week. Three different inspectors took samples of the shirts produced and the results are shown in the table below.

Inspector	# shirts sampled	# defective
A	500	21
B	300	17
C	200	12

   a. What is the probability that a shirt is defective out of the whole batch if only Inspector A's results are used to determine the probability?
   Answer: 21/500 or 4.2%

   b. What is the probability that a shirt is defective if all of the inspectors' results are tabulated?
   Answer:1/20 or 5%

   c. How many shirts would you expect to be defective out of the whole batch produced using the results you found in part b?
   Answer: 500