Algebra I – Part 1

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The Louisiana Department of Education issued the first version of the Comprehensive Curriculum in 2005. The 2012 Louisiana Transitional Comprehensive Curriculum is aligned with Grade-Level Expectations (GLEs) and Common Core State Standards (CSSS) as outlined in the 2012-13 and 2013-14 Curriculum and Assessment Summaries posted at http://www.louisianaschools.net/topics/gle.html. The Louisiana Transitional Comprehensive Curriculum is designed to assist with the transition from using GLEs to full implementation of the CCSS beginning the school year 2014-15.

**Organizational Structure**
The curriculum is organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. Most activities in the curriculum were designed to be taught in 2012-13 and continued through 2013-14. Activities labeled 2013-14 align with CCSS that are to be implemented in that 2013-14. CCSS to be implemented in 2014-15 are not included in activities in this document.

**Implementation of Activities in the Classroom**
Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Transitional Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the CCSS associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

**Features**
*Content Area Literacy Strategies* are an integral part of approximately one-third of the activities. Strategy names are italicized. The link (view literacy strategy descriptions) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at http://www.louisianaschools.net/lde/uploads/11056.doc.

*Underlined standard numbers* on the title line of an activity indicate that the content of the standards is a focus in the activity. Other standards listed are included, but not the primary content emphasis.

A *Materials List* is provided for each activity and *Blackline Masters (BLMs)* are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for the course.

The *Access Guide to the Comprehensive Curriculum* is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities. This guide is currently being updated to align with the CCSS. Click on the *Access Guide* icon found on the first page of each unit or access the guide directly at http://sda.doe.louisiana.gov/AccessGuide.
Algebra I–Part 1
Unit 1: Variables and Numeric Relationships

Time Frame: Approximately five weeks

Unit Description

This introductory unit consists of a thorough review of math topics from earlier grades. Topics include work with subsets of the set of real numbers including how to plot them on a number line and perform operations on them; estimating square roots; evaluating expressions; and representing real-life situations with numerical models and graphs.

Student Understandings

Students can use the order of operations and work with rational and irrational numbers. Students write, evaluate, and simplify algebraic expressions in real-life situations and in mathematical formulas. They also recognize simple patterns in graphical, numerical, tabular and verbal forms.

Guiding Questions

1. Can students identify, classify, and order numbers in the real number systems (both rational and irrational numbers)?
2. Can students use order of operations and the basic properties when performing computations in numeric expressions?
3. Can students correctly evaluate numeric and algebraic expressions involving rational numbers and exponents?
4. Can students analyze functions in graphical, numerical, tabular, and verbal forms?

Unit 1 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>Grade-Level Expectations</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Number Relations</strong></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Identify and describe differences among natural numbers, whole numbers, integers, rational numbers, and irrational numbers (N-1-H) (N-2-H) (N-3-H)</td>
</tr>
<tr>
<td>2.</td>
<td>Evaluate and write numerical expressions involving integer exponents (N-2-H)</td>
</tr>
<tr>
<td>4.</td>
<td>Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)</td>
</tr>
<tr>
<td></td>
<td>Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)</td>
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<tr>
<td><strong>Algebra</strong></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)</td>
</tr>
<tr>
<td><strong>Data Analysis, Probability, and Discrete Math</strong></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)</td>
</tr>
</tbody>
</table>

| **CCSS for Mathematical Content** |
| **CCSS #** | **CCSS Text** |
| **Use Properties of Rational and Irrational Numbers** |
| N-RN.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. |

| **ELA CCSS** |
| **Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12** |
| WHST.9-10.1b | Write arguments focused on discipline-specific content. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience’s knowledge level and concerns. |
| WHST.9-10.10 | Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. |

**Sample Activities**

**Activity 1: Relationships in the Real Number System (GLE: 1)**

Materials List: paper, pencil, Where do I Belong? BLM.

Review the real number system and discuss with students what the natural numbers, whole numbers, integers, rational and irrational numbers are. Draw a Venn diagram showing how the various sets of numbers within the real number system are related. After fully reviewing the real number system, make copies of the modified word grid (view literacy strategy descriptions) activity, Where do I Belong? BLM. A word grid is a learning strategy that allows the user to relate characteristic features among terms or items and helps the learner to compare and contrast similarities and differences between the items in a list. Let students get in pairs to complete this activity, and then discuss the results as a class.
Activity 2: Understanding Rational and Irrational Numbers (GLEs: 1, 4)

Materials List: paper, pencil, scientific calculators, four-function calculators (variety), Internet (optional)

Using calculators, let students explore the difference between rational and irrational numbers. To begin, have students input several rational numbers using the division key and discuss why some rational numbers are finite (e.g., \( \frac{4}{5} \), \( \frac{3}{8} \), and \( \frac{15}{32} \)), while other rational numbers have non-terminating decimals that repeat (e.g., \( \frac{11}{12} \) and \( \frac{7}{11} \)). Ask students to investigate which fractions will terminate and which will repeat by looking for a pattern. Students should see that fractions that terminate have denominators with factors of 2 and 5 only. Any rational number having factors other than 2 or 5 will result in non-terminating repeating decimal numbers.

Next, have students input several irrational numbers and let them see that although the numbers appear to terminate on the calculator, the calculator is actually rounding off the last digit. Students need to understand that irrational numbers don’t terminate or repeat when converted to a decimal. There are some computer sites that show irrational values to many places. The website http://www.mathsisfun.com/irrational-numbers.html is one such site. It also explains in detail what an irrational number is along with values for \( \pi \) and \( e \) and other “famous” irrational numbers.

Finally, discuss how different calculators handle numbers that do not terminate. The goal here is to help students see that calculators can sometimes introduce calculation errors and how to handle this situation as it arises. For example, show students what error results from calculating with rounded values (i.e., multiply a number by 0.67 and then multiply it by 2/3). Present other examples of calculation error associated with rounding using calculators. Emphasize that in most cases, it is better not to round until the last step.

Activity 3: Estimating the Value of Square Roots (GLEs: 1, 4)

Materials List: paper, pencil, scientific calculators

Discuss with students how to estimate the value of irrational numbers involving square roots. Have the students determine which two whole numbers a particular square root would fall between. For example, if students know the square root of 49 is 7 and the square root of 64 is 8, then the square root of 51 would be between these two values (it would actually be closer in value to 7 than to 8, so an even better estimate might be 7.1). Once a thorough discussion takes place about estimation techniques, give students an opportunity to use their estimation skills by providing students with 10 square roots that are irrational and have them determine their approximate values. After students obtain their approximate values, have students share their reasoning with a partner first and then explain their reasoning to the class. Have students check their estimates with a calculator using the square root key.
Activity 4:  Naming Numbers on a Number Line (GLE: 1)

Materials List: paper, pencil

As a class (with the teacher modeling and students working at their desks individually), construct a number line showing the integers from –4 to +4. Teacher and students should then identify and label the halfway points between each pair of integers (e.g., $-3\frac{1}{2}$, $-2\frac{1}{2}$). Next, identify and label where the following numbers would be placed on the number line they created: $-\pi$, $-\sqrt{3}$, $-\sqrt{2}$, $\sqrt{2}$, $\sqrt{3}$, $\pi$. First, allow students to place the numbers on their own number lines. When students have completed their work, call individual students to the front of the room to place a number on a class number line and explain why he/she chose this particular position. Discuss the difference between an exact answer (such as the square root of 3) and an approximate answer (such as 1.73). Use this activity to reinforce the previous work done by having students name and identify the natural numbers, whole numbers, integers, rational, and irrational numbers. Use this opportunity to emphasize to students that a number line is made up of an infinite number of points. Many students think a number line consists only of integers; however, they need to understand that between any two integers are “infinitely many” points. For example, between 2 and 3, there are points such as 2.000001, 2.000002, and so on.

Activity 5:  Many Ways to Solve a Problem (GLEs: 4, 5; CCSS: WHST.9-10.10)

Materials List: paper, pencil, scientific calculators, teacher-made worksheets on operations with rational numbers, What Method Should I Use? BLM

Review paper and pencil operations with rational numbers (addition, subtraction, multiplication, and division) and include in the discussion how the calculations could be done using calculators. Also, use this opportunity to discuss estimation and mental strategies with respect to operations with rational numbers. This work should be a review for students from 7th and 8th grades. Provide opportunities for students to develop proficiency in solving problems involving operations with rational numbers.

Afterwards, make copies of and have students work in pairs on What Method Should I Use? BLM. In this activity, students are given a problem in which they have to decide whether they should solve a problem using estimation, mental math, paper/pencil, or calculator. Students write their decisions in the math learning log (view literacy strategy descriptions) section of the BLM to defend the choice they made. Learning logs are a literacy strategy designed to force students to put into words what they know or do not know. It offers students the opportunity to reflect on their understanding which can lead to further study and alternative learning paths. Learning logs are generally kept in a journal type of book, but because of the nature of Activity 5, the log is integrated into the BLM. Throughout the year, have students maintain a learning log in a central location, such as in a special place in a notebook or binder, in which to record new learning experiences. For Activity 5, have the students tape or paste the BLM into the log when completed.
After students have written their math learning logs, they should exchange math logs with another student to analyze one another’s work and provide feedback to another student. Once this has taken place, the teacher should lead a discussion of the worksheet and discuss what students believe to be the best approach to each of the problems.

**Activity 6: Using Exponents in Prime Factorization (GLE: 2)**

Materials List: paper, pencil, teacher-made worksheet on using exponents or problems from a math textbook

Review the prime factorization process with whole numbers and include in the discussion how exponents can be used to rewrite a particular prime factorization. For example, ask students to rewrite 136 as the product of primes. This should be a review for students from middle school work. Allow students the opportunity to prime factor different numbers using factor trees, and then have them write the numbers in factored form using exponents.

Include in the discussion the use of negative exponents (i.e., explain how $\frac{1}{2}$ could be written as $2^{-1}$ or $\frac{1}{16}$ could be written as $2^{-4}$). Since this is the first time that students have exposure to negative exponents, develop the concept in the following way: Have students investigate a pattern starting with $2^2=4$; $2^1=2$; $2^0=1$; $2^{-1}=\ ?$. Since each value is $\frac{1}{2}$ the previous value, then $2^{-1}$ would need to be $\frac{1}{2}$. Continue the process by using exponents of -2, -3, and so on, until the idea of using negative exponents is developed thoroughly. Once this is done, introduce the “rule” associated with negative exponents as is typically done in an algebra class. The goal of this activity is to help students to become comfortable with use of integer exponents. Provide a worksheet for practice as necessary for the particular class or use the math textbook as a resource for additional work on this topic.

**Activity 7: What Do You Get? (CCSS: N-RN.3)**

Materials List: paper, pencil, calculators

Have students get in groups of 3 to 4. Several questions will be presented in this activity to the groups, and the goal is to have each group discuss the questions and then come up with reasonable explanations and be able to defend their views. The teacher’s role here is to moderate the discussion/debate.

**Question 1:** Will the sum of two rational numbers be rational or irrational? Why or why not?

**Question 2:** Will the product of two rational numbers be rational or irrational? Why or why not?
Question 3: Will the sum of a rational number and an irrational number be rational or irrational? Why or why not?

Question 4: Will the product of a nonzero rational number and an irrational number be rational or irrational? Why or why not?

Present each individual question to students one at a time and discuss each individually. Allow students free reign to defend/debate their position on each question and to ask questions of the other groups. Remember, the teacher’s role is to moderate the discussion, not to jump in with the correct answers. At the end of the debate, wrap up the discussion with the culminating correct solution/answer for each question presented. Students should realize that the sum or product of two rational numbers will always result in a rational number, and that the sum or product of a rational (non-zero) number and an irrational number will always result in an irrational number.

An example of a possible explanation may be as simple as this:

“Since an irrational number goes on and on forever, if you add a rational number to it, such as the number 1, the number has increased by 1 but it still goes on and on forever, thus it’s going to be irrational”

An example of a more rigorous mathematical solution to the same question may be:

“If we use algebraic notation to represent a rational number using the letter R, and an irrational number using the letter I, if we find the sum we get R + I. If we assume this is some rational number we can use R₂ to represent this sum. Thus we get R + I = R₂. Using the idea of if we have an equation and subtract the same number from each side, we should still get a balanced equation. Utilizing this idea, we could subtract R from each side. Doing so results in the following equation: I = R₂ - R. This equation says that one side is irrational, I, and one side is rational, R₂ - R. This makes no sense. So the assumption that the addition of R + I was equal to some rational number is false, thus it must mean that the sum of a rational and an irrational number HAS to be irrational.”

Activity 8: Order of Operations Activity (GLEs: 2, 5, 8)

Materials List: paper, pencil, calculators – more than one kind for comparison, math textbook

Review the order of operations with numerical applications. Explain why order of operations is necessary. For example, present the problem 4 + 5 x 9 to students, and let them obtain an answer on their own. In this case, if students work from left to right they get a solution of 81, which is not correct. If the multiplication is done first and then the addition, an answer of 49 is obtained, which is correct. This sets up a situation that in math is not okay…two solutions when there really should only be one. Students need to see the importance of having an order to follow when calculating. Also, students should use different calculators, if possible, to show how certain calculators perform the order of operations differently. Include in the discussion the importance of using parentheses when inputting data. For example, (4+5)/(6+2) would be a different result than 4+5÷6+2. Also, include how to enter exponents in their calculators (e.g., 3³- 4(3 + 6)). After the discussion, provide students with practice in using the order of operations with problems from their math textbook. Make sure problems include the use of parentheses and exponents.
Once students become proficient at the skill of using order of operations, utilize professor know-it-all (view literacy strategy descriptions). Professor know-it-all is a strategy designed to reinforce content that has already been learned (in this case order of operations) and is meant to be used once coverage of particular content has been completed. It’s an effective review strategy because it positions students as “experts” on review topics to inform and be challenged by their peers, as well as be held accountable by them. In this particular use of the strategy, form groups of 3 to 4 students. Direct one group to come to the board and act as the “professors” and invite the other students at their desks to create problems that involve using order of operations and present these to the “professors” to come up with the solutions. Allow them to huddle before sharing their answer. Insist that the other students hold the “professors” accountable for their responses and explain how they obtained their solution. Groups should each take a turn being the “professors.” This is a fun way to review a concept and also to see if the students really have grasped the material being presented.

Activity 9: Writing and Simplifying Algebraic Expressions (GLE: 8, 9)

Materials List: paper, pencil, math textbook

Review with students how to write and simplify algebraic expressions for different real-life situations. For example, have students write an expression for the total weight of 24 cans of soft drink if each weighs \( k \) ounces (Answer: 24\( k \)); write an expression for the distance someone would travel if he/she went 40 miles per hour for \( t \) hours (Answer: 40\( t \)). Other examples can be found in any algebra textbook. Discuss simplifying algebraic expressions and combining like terms. For example, if a square has sides that are \( p \) units long, the perimeter can be expressed as \( p + p + p + p \) or 4\( p \) units in length. The goal here is for students to become proficient at both writing expressions and simplifying them.

Activity 10: Patterns in the Real World (GLEs: 2, 8, 9, 15; CCSS: WHST.9-10.10)

Materials List: paper, pencil, Patterns in the Real World BLM

In this activity, expose students to number patterns found in real-life examples, including patterns involving exponents. Have students describe the pattern in words, and then have them write an algebraic expression which could be used to express the \( n \)th term.

For example, suppose a new pizza shop opens in a shopping center. At the end of each day, a running total is kept for the total number of pizzas sold since the shop opened. On the first day, the shop sells 1 pizza. On the second day, the shop sells 3 more pizzas. On the third day, the shop sells 5 more pizzas. On the fourth day, the shop sells 7 more pizzas. On the fifth day, the shop sells 9 more pizzas. Have students describe the pattern in words, and then have them write an algebraic expression which could be used to express the number of pizzas sold on the \( n \)th day. Also, have students determine an algebraic expression which could be used to find the total number of pizzas sold since the shop opened by the end of the \( n \)th day. \( \{\text{Solution: To represent} \)
the number of pizzas sold on the nth day, the expression 2n-1 could be used. To represent the total number of pizzas sold since the shop opened the expression n^2 could be used.

After doing several examples as a whole class, give students the opportunity to do some on their own. Afterwards, put students in groups of three and assign Patterns in the Real World BLM. When students have completed their work, have them write a math learning log (view literacy strategy descriptions) entry about a solution to a problem they did from the worksheet. They should explain how they did the problem, what approach they took to finding the solution, what was the hardest part in finding the solution (what gave them the most trouble when attempting the solution), and how they overcame that. Discuss both the problems and the writing as a class.

Activity 11: Matching Real-life Situations and Their Graphs (GLE: 15; CCSS: WHST.9-10.1b)

Materials List: paper, pencil, Internet access, math textbook

One of the hardest things to help students visualize is the relationship between distance, time, and speed since it is hard to do this in an actual classroom. However, through the use of technology, students can better understand these relationships and how they can be interpreted in graphs. Begin this activity by using the interactive software found at http://mste.illinois.edu/users/Murphy/MovingMan/MovingMan.html. This computer simulation uses a context familiar to students, and the technology allows them to analyze the relationships more deeply because of the ease of manipulating the environment and observing the changes that occur. In the activity found on the website, a stick figure can be manipulated and the motion is graphed on either a distance/time graph, velocity/time graph, or acceleration/time graph.

Once students become familiar with the program and understand how the graph correlates to the motion of the stick figure, have students attempt to create a motion that would correlate to a particular graph (such as a horizontal line; increasing straight line; decreasing straight line; vertical line—which of course is impossible; quadratic type graph; etc.). Before actually doing the simulation, have students pair up to discuss what they think needs to be done to produce the graph for each situation presented. Then, try out the instructions using the computer simulation. This technique of having students hypothesize is a form of a strategy called SPAWN (view literacy strategy descriptions) writing. SPAWN is used as a way to create writing prompts in order to elicit student writing and thinking. In this particular use of SPAWN, students are being asked to figure out the solution to a problem (P) and what will happen next (N). Since this is not formal writing, it should not be graded as such. Afterwards, have students create their own graphs and write a story to match the graph they have created.

As an extension of this activity, provide students with various numberless graphs (possibly from their math textbooks) and real-life situations that correspond to each graph. Have students match the graph with the situation. For example, have graphs of distance/time, and relate the act of moving toward home and away from home on a given day in reference to the time during the
day. Provide students with many different situations and graph types which will require students to use analytical thinking.

**Activity 12: Matching a Table of Values with a Graph (GLE: 9, 15; CCSS: WHST.9-10.10)**

Materials List: paper, pencil, Tables to Graphs BLM

Provide students with copies of Tables to Graphs BLM. In this activity, students are presented with a table of values resulting from a real-life situation: the price to rent a moving truck in relation to the number of miles it was driven. Students will complete the table by determining the missing data. After completing the table, students will determine which graph (chosen from the graphs provided with the activity) best fits the data shown in the table. Students will then write a math learning log (view literacy strategy descriptions) entry about why the graph they chose is the only graph that fits the data. Finally, students will come up with an expression or equation which represents the cost to rent a truck driving $n$ miles.

**Sample Assessments**

**General Guidelines**

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

**General Assessments**

- The student will make a portfolio containing samples from various activities.
- The student will keep a math learning log (view literacy strategy descriptions) about the ideas and processes that are taught in class. The student can use the log as information or as a study guide, but it is also a good source of feedback to the teacher concerning questions the student has on a particular topic. Each week, the teacher picks up the learning log and examines it.
- For selected activities, the student will show his/her work, and use the work for assessment purposes.
- Use paper/pencil tests to assess the student’s ability to compute, write expressions, or evaluate expressions based upon the GLE’s presented in this unit.

**Activity-Specific Assessments**

- **Activity 4**: The student will put 15 numbers on a number line ranging from $-10$ to $10$. The teacher will provide the list of numbers with values of each type (natural, whole, integer, rational, and irrational) for the student to graph and have the student identify which subsets each number belongs to.
• **Activity 5**: The student will write his/her own problems that could best be solved using each technique (paper and pencil, estimation, technology, and mental math) along with an explanation of why this would be the best approach.

• **Activity 9**: The student will write verbal explanations (mathematical and verbal explanations) to a real-world algebraic expression, and then write an algebraic expression giving a verbal explanation.

• **Activity 11**: Have students create a numberless graph giving a verbal explanation of a motion problem.
Algebra I–Part 1
Unit 2: Measurement

**Time Frame:** Approximately three weeks

**Unit Description**

This unit is an advanced study of measurement. It includes the topics of significant digits and error in measurement. The study of how computations on measurements are dealt with, as well as the investigation of absolute and relative error and how they each relate to a measurement, is also included.

**Student Understandings**

Students find the number of significant digits in a measurement and they calculate and use significant digits to solve problems. Students view error as the uncertainty approximated by an interval around the true measurement. Students will also determine the appropriateness of units and scales when dealing with measurement in real-life situations.

**Guiding Questions**

1. Can students determine the number of significant digits in a measurement?
2. Can students calculate using the ideas of significant digits?
3. Can students understand the effects of error in measurement and how to calculate this error?
4. Can students differentiate between absolute error and relative error in measurement?
5. Can students apply the ideas of significant digits, and determine error in real-life situations?
6. Can students identify the appropriate units to use in any given measurement situation?

**Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)**

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
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<tbody>
<tr>
<td>4.</td>
<td>Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)</td>
</tr>
<tr>
<td>5.</td>
<td>Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)</td>
</tr>
</tbody>
</table>
Measurement

19. Use significant digits in computational problems (M-1-H) (N-2-H)

20. Demonstrate and explain how relative measurement error is compounded when determining absolute error (M-1-H) (M-2-H) (M-3-H)

21. Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>RST.9-10.1</td>
<td>Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.</td>
</tr>
<tr>
<td>RST.9-10.4</td>
<td>Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.</td>
</tr>
</tbody>
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ELA CCSS

<table>
<thead>
<tr>
<th>Activity 1: What are Significant Digits? (GLEs: 4, 19; CCSS: RST.9-10.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials List: paper, pencil, various measurement devices (rulers, scales, protractors, thermometers, etc.)</td>
</tr>
</tbody>
</table>

This unit on measurement will have several new terms to which students have not yet been exposed. Have students maintain a vocabulary self-awareness (view literacy strategy descriptions) chart for this unit. Vocabulary self-awareness is valuable because it highlights students’ understanding of what they know, as well as what they still need to learn, in order to fully comprehend the concept. Students indicate their understanding of a term/concept (using + for complete understanding of concept; a? for understanding of a concept to some degree but not completely; and – to indicate no understanding of a concept), but then adjust or change the marking as the lesson is taught to reflect their change in understanding. The objective is to have all terms marked with a + at the end of the unit. A sample self-awareness chart is shown below.
Relative error

Absolute error

Significant digits

<table>
<thead>
<tr>
<th>Term</th>
<th>+ (understand concept completely)</th>
<th>? (understand concept to some degree but still have questions)</th>
<th>- (don’t understand concept)</th>
<th>Example</th>
<th>Definition of term</th>
</tr>
</thead>
</table>

Be sure to allow students the opportunity to revisit their *vocabulary self-awareness* charts often (throughout this unit) to monitor their developing knowledge about important concepts. Sample terms to use include significant digits, absolute error, and relative error.

After students fill out their chart, discuss with students what significant digits are and how they are used in measurement. Find out what students might know about significant digits (they may have some background with this concept from their science classes). Build on their understanding and clear up any misconceptions they may have.

Significant digits are defined as all the digits in a measurement one is certain of plus the first uncertain digit. Significant digits are used because all instruments have limits, and there is a limit to the number of digits with which results are reported. All measurements are approximations—no measuring device can give perfect measurements without experimental uncertainty. For example, a mass measured to be approximately 13.2 g is said to have been measured to the nearest 0.1 g. In other words, it is somewhat uncertain about that last digit—it could be a "2"; then again, it could be a "1" or a "3." This can easily be seen if students measure a line segment, for instance, with a ruler. Have students measure a line segment of some sort with a ruler (to the nearest tenth centimeter) and then see what answers they have. What measurement each person may get depends on the ruler being used and the degree of precision/accuracy of the measurement tool as well as the reader of the measurement. If a person measured the line segment to be 3.4 centimeters, the uncertainty of the measurement (assuming the person making the measurement has been careful in their measurements) could be somewhere between 3.3 to 3.5 centimeters. A mass measured to be 13.20 g indicates the measurement has been taken to the nearest hundredth of a gram, but this too is an approximation of that last digit (the last digit not being not …it could be somewhat less or somewhat more). Demonstrate and discuss the process of measuring in relation to the use of significant digits using a variety of measurement tools (rulers, scales, protractors, thermometers, etc.). After students have an understanding of what is meant by the term “significant digits,” discuss and demonstrate the process of determining the number of significant digits in any given number. Explain to students that it is necessary to know how to determine the significant digits so that when performing measurement calculations with numbers, they will understand how to state the answer in the correct number of significant digits.
Rules for Significant Digits

1. Digits from 1-9 are always significant.
2. Zeros between two other significant digits are always significant.
3. One or more additional zeros to the right of both the decimal place and another significant digit are significant.
4. Zeros used solely for spacing the decimal point (placeholders) are not significant.

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. Discuss many examples of numbers/measurements and the number of significant digits there are according to the “rules”. Present each number and then have students get in pairs to decide how many digits are significant, and then discuss as a group.

Ex: The number 13.2 is said to have 3 significant figures.
Ex: The number 13.20 is said to have 4 significant figures.
Ex: 1002 kg has 4 significant figures.
Ex: 0.001 °C has only 1 significant figure,
Ex: 0.0230 mL has 3 significant figures

Activity 2: Significant Digits with Scientific Notation (GLE: 5, 19)

Materials List: paper, pencil

Present the following question to students: “How many significant digits are there in the measurements 190 miles and 50600 calories?” Utilize the discussion (view literacy strategy descriptions) strategy known as Think-Pair-Square-Share. Students remember best when they participate in dialog about class topics. This strategy is one way to encourage students to think deeper about a topic, and explain their thinking with their peers. In this particular strategy, after giving students the question, ask students to think alone about the problem for a short period of time, then have each student pair up with another student to share their thoughts. After they have had the opportunity for discussion, have pairs of students share with other pairs, forming small groups of four. Once this process has taken place, gather oral responses to the question for a full class discussion of the problem/solution to take place. The goal of discussion is to provide a deeper processing of content and rehearsal of newly learned content.

Once the discussion has taken place, talk about the different answers/viewpoints to the problem presented. There should have been some debate about the answers. The fact is that when a number ends in zeros that are not to the right of a decimal point, the zeros are not necessarily significant. The 190 miles may be 2 or 3 significant figures, depending on the situation. For example, perhaps the measurement was taken to the nearest mile, and so the measurement could be between 189 and 191 miles (remember the last digit is uncertain), so in that case the 0 here is significant resulting in 3 significant digits. However, suppose the person taking the measurement was expressing the mileage to the nearest ten miles; in this case, the measurement could have been 180 to 200 miles, so the 0 is not significant and thus 190 would only have 2 significant digits. This same argument can be made for the 50,600 calories (this could be 3, 4, or 5 significant digits depending on the situation—talk about this with students). Explain to students that because of this potential ambiguity, this can be avoided by the use of "scientific notation."
For example, depending on whether the number of significant figures is 3, 4, or 5, the number 50,600 calories could be written as:

- $5.06 \times 10^4$ calories (3 significant figures)
- $5.060 \times 10^4$ calories (4 significant figures)
- $5.0600 \times 10^4$ calories (5 significant figures).

By writing a number in scientific notation, the number of significant figures is clearly indicated by the number of numerical figures in the “digit” term as shown by these examples. This approach is a reasonable convention to follow anytime there is the problem of trailing zeros. This is the approach students should take in dealing with numbers of this type. If the zeros are included in the scientific notation, they will be considered significant, but if they are not included, they are considered not significant. A general rule of thumb that scientists use is that if there are numbers with trailing zeros, such as 5400, they are considered not significant (5400 would only have 2 significant digits). If the zeros are meant to be significant, then the number must be written in scientific notation to indicate this. For example, if the number was written as $5.40 \times 10^3$ this would indicate the measurement was measured to the nearest ten accounting for 3 significant digits. Discuss this idea with students fully. Provide students with additional examples before moving to the next activity.

*Teacher Note:* Students should be familiar with scientific notation from earlier grades as well as science classes, but a quick review in writing numbers using scientific notation may be needed for some students.

**Activity 3: How Many Significant Digits Are There? (GLEs: 19)**

Materials List: paper, pencil, How Many Significant Digits Are There BLM

In this activity, students are given the opportunity to show what they have learned about significant digits so far. It is a formative assessment opportunity—a chance to find out what students understand and the chance to clarify any lingering misconceptions about determining the number of significant digits.

Provide copies of the BLM for this activity to all students and have students work with a partner as they go through each of the examples. In the BLM, students are given an example of a number/measurement and are asked to identify the number of significant digits for each. Students then have to write the rule or comment on how they determined their answer to each example. When work is completed, go over the BLM as a class, clarifying any misconceptions students may still be having. Additional examples may have to be provided to assure student mastery of this skill.
Activity 4: Measuring the Utilities You Use (GLE: 19)

Materials List: paper, pencil, utility meters around students’ households, utility bills

Have students find the various utility meters (water, electricity) for their households. Have them record the units and the number of places found on each meter. Have the class get a copy of their family’s last utility bill for each meter they checked. Have students answer the following questions: What units and number of significant digits are shown on the bill? Are they the same? Why or why not? Does your family pay the actual “true value” of the utility used or an estimate? If students do not have access to such information, produce sample drawings of meters used in the community and samples of utility bills so that the remainder of the activity can be completed.

Activity 5: Calculating with Significant Digits—Addition and Subtraction (GLEs: 4, 19)

Materials List: paper, pencil, calculators

Begin this activity by presenting the following problem to students:

Daniel and Marquette were measuring the perimeter of their rectangular room. Daniel measured the length to be 7.30 meters while Marquette measured the width of the room to be 5.649 meters. They found the perimeter in the following way:

\[
\begin{array}{r}
7.30 \text{ meters} \\
7.30 \text{ meters} \\
5.649 \text{ meters} \\
+ \ 5.649 \text{ meters} \\
\hline
25.718 \text{ meters}
\end{array}
\]

Now that the concept of significant digits in the context of measurement has been discussed, do you think this is a proper way to express this answer? Why or why not?

Utilize the discussion (view literacy strategy descriptions) strategy Think-Pair-Square-Share. Have students work alone thinking about the problem first, then after a short time, have each student pair up with another student to discuss their ideas/views on the question. After several minutes, have each pair of students share with other pairs, forming small groups of four to discuss their views. Once this process has taken place, gather oral responses to the question for a full class discussion of the problem/solution. Use student responses to guide the instruction. The goal of discussion is to provide a deeper processing of content by the student.

Depending on the class, students may or may not agree with leaving the answer for the perimeter as 25.718 meters. If students do think it’s okay, ask, “What does it mean when you write the perimeter in this way?” Students should understand that it implies that the perimeter was measured to the thousandth of a meter (which is not the case, although the width of the room was, the length was not). Because of this, the final perimeter in this situation should not be written this way because of its false implications. Explain that when situations considered in the context of using significant digits, students must be aware of these types of discrepancies, particularly when applying measurement in the sciences.
Use this problem to discuss with students how to use significant digits when making calculations with addition and subtraction. There are different rules for how to round calculations in measurement depending on whether the operations involve addition/subtraction or multiplication/division. When adding (or subtracting), such as in finding the perimeter, the answer can be no more PRECISE than the least precise measurement (i.e., the perimeter must be rounded to the same decimal place as the least precise measurement). The final answer should reflect rounding to the least precise decimal place—it has really nothing to do with the number of significant digits. In the example presented to the class, the proper way to express the perimeter, using the rules of addition with significant digits is to round the answer to the nearest hundredth of a meter since that was the least precise measurement used. The perimeter would then be expressed as 25.72 meters.

Additional example: If one of the measures is 15 ft and another is 12.8 ft, then the perimeter of a rectangle (55.6 ft) would need to be rounded to the nearest whole number of feet (56 ft), since it cannot be assumed that the 15 foot measure was also made to the nearest tenth based on the information available. The same rule applies should the difference between the two measures be needed. Provide additional examples for students to apply the rules for calculating using addition and subtraction, and go over these as a class.

Activity 6: Calculating with Significant Digits—Multiplication and Division (GLEs: 4, 19; CCSS: RST.9-10.1; WHST.9-10.1b)

Materials List: paper, pencil, calculators, Multiplication and Division Using Significant Digits BLM

Make copies of the Multiplication and Division Using Significant Digits BLM and provide each student a copy. In this activity, the use of questioning the content (view literacy strategy descriptions) will be utilized. Questioning the Content is a strategy designed to have the learner process content being learned using written text. In this strategy, the focus is on presenting informational text to the learner, whereby students have to “read to learn.” Questions are presented based on the informational text, and students have to correctly apply what they have learned through reading to answer questions based on the content of the text. In this particular use of the strategy, students will read about the rules for computing multiplication and division problems using significant digits found on the BLM. Students are then to apply what they have read about the rules to solve problems based on the informational text.

Give students the opportunity to read the text (BLM) alone first, and then have students work in pairs on solving the problems presented on the BLM. Monitor the class as they go through this process. DO NOT answer the questions for them—students must learn the skill of reading to learn and must understand they actually build success in doing so. After everyone has answered the questions based on what they read, go over student answers and provide feedback as necessary to clarify any misconceptions they may have, but make sure always to refer to the text when doing so. Students need to see how the text can be used to justify answers in this activity.
Ultimately, students should understand that when multiplying, such as in finding the area of a rectangle, the answer must have the same number of **significant digits** as the measurement with the fewest number of significant digits. Notice, this is a different “rule” than for adding and subtracting (which was based on rounding the final answer to indicate the least precise measurement—not on the number of significant digits). For example, if one side of a rectangle were measured to be 12.8 cm (3 significant figures) and the other side measured 15 cm (2 significant figures), although the area is calculated to be 192 square centimeters, if the rules for multiplication using significant digits are applied, the answer should only have 2 significant digits in it. Therefore, students need to round to 190 square centimeters, which translates to $1.9 \times 10^2$ square centimeters using scientific notation to express the answer. The same rule applies for any problems involving division. Provide additional examples as needed to ensure student mastery of this concept.

**Activity 7: Calculating with Significant Digits—Review of Skills (GLEs: 4, 19)**

Materials List: paper, pencil, calculators

To wrap up student understanding on adding, subtracting, multiplying, and dividing with significant digits, begin by having students find the area and perimeter for another rectangle whose sides measure 9.7 cm and 4.2 cm. Allow students the opportunity to work on the problem, then go over answers. The calculated area is $(9.7\text{cm})(4.2\text{cm}) = 40.74 \text{ sq. cm}$ but should be rounded to 41 sq cm (two significant digits). The perimeter of 27.8 cm would not need to be rounded because both lengths are to the same precision (tenth of a cm).

After fully discussing calculating with significant figures, have students work computational problems (finding area, perimeter, circumference of 2-D figures) dealing with the topic of calculating with significant digits. Create problems for students to practice the skills learned in the previous activities.

**Activity 8: Applying Significant Digits in Measurement and Geometry (GLEs: 4, 5, 19; CCSS: WHST.9-10.10)**

Materials List: paper, pencil, calculators, rulers, string, meter sticks; real life objects (flat objects, rectangular prisms, and cylinders) for students to measure perimeter, area, volume and surface area

This activity has two parts:
1. **Review of student’s previous work with formulas used in middle school geometry**
2. **Hands-on measurement applying the use of the formulas to real-life objects, utilizing the ideas**

Begin the review by having students provide information on what they remember from previous work with geometry, including formulas for areas and perimeters of various 2-D figures (rectangles, triangles, circles, etc). Also, ask students to provide correct mathematical names for
various 3-D figures by presenting real-life examples, such as a cereal box (rectangular prism) or an oatmeal box (cylinder). Ask students to give the correct geometric name for each face (as in circle, rectangle, etc.) for each of the 3-D objects. Facilitate the review by having students recall facts that they remember about formulas needed to find surface area and volume for rectangular solids and cylinders which were learned in grade 8. As part of the review, include an analysis of the derivation of the formulas (i.e., the surface area of a cylinder is \(2 \pi r^2 + Ch\) because the faces are composed of two circles, \(2 \pi r^2\), and a rectangle whose dimensions are the circumference of the circle and the height of the cylinder, \(Ch\)). This review should encompass all ideas related to perimeter/circumference, area, surface area, and volume for rectangular prisms and cylinders.

Next, set up stations around the room and provide various 2-D (cut-out rectangles, circles, etc.) and 3-D objects (cereal boxes or cans) for students to measure. Students should be required to select the appropriate formulas and make the appropriate measurements to determine the perimeter or area (for the 2D figures) and surface area or volume (for the 3D figures). Allow students to work in pairs or small groups on this part of the activity. Students should be required to show all measurements and calculations made and utilize the rules of calculating with significant digits. Allow calculators to be used on these problems, although all work should be shown. Work should be handed in and checked by the teacher for correctness.

Once students have completed their work, and work is checked, the teacher should provide appropriate feedback to students. Follow this up with a full discussion about which measurements were taken in order to correctly determine the perimeter, area, surface area, or volume for each of the figures.

Finally, have students write a math learning log (view literacy strategy descriptions) entry about three things they learned while doing this activity that they didn’t know how to do before their work. Remember that learning log entries should be kept in a specific place, such as a binder or the back section of the student’s math notebook. Read the logs and provide feedback as necessary. The log should help you understand what the students actually learned through the activity.

**Activity 9: Absolute Error (GLEs: 20)**

**Materials List:** paper, pencil, calculators, Absolute Error in Measurement BLM

In any measurement, there will be a certain amount of error associated with the calculations. Two types of error that students must be familiar with are absolute error and relative error. In this activity, students need to understand and learn how to determine the absolute error related to a measurement. Absolute error is the amount of physical error in a measurement, period. For example, a meter stick is used to measure a given distance. The error is rather hastily made, but it is good to ±1mm. This is the absolute error of the measurement. The absolute error shows how large the error actually is. Absolute errors do not always give an indication of how important the error may be. If you are measuring a football field and the absolute error is 1 cm, the error is virtually irrelevant. But if you are measuring something small in length, such as something
about 3 cm for example, an absolute error of ±1 cm is very significant. The formula for calculating absolute error is:

\[
\text{Absolute Error} = |\text{Observed Value} - \text{Actual Value}|
\]

Review absolute value with students and explain to them that since the absolute value of the difference is taken, the order of the subtraction will not matter.

Present the following problems to students for a class discussion:

**Example 1:** Luis measures his pencil, and he gets a measurement of 12.8 cm but the actual measurement is 12.5 cm. What is the absolute error of his measurement?

\[
\text{Absolute Error} = 12.8 - 12.5 = .3 \text{ cm}
\]

**Example 2:** A student experimentally determines the specific heat of copper to be 0.3897 °C. Calculate the student's absolute error if the accepted value for the specific heat of copper is 0.38452 °C.

\[
\text{Absolute Error} = |0.3897 - 0.38452| = 0.00518
\]

**Example 3:** Many times, geometry students use an approximation for \(\pi\) as 3.14. Use your calculator \(\pi\) key to determine the absolute error related to this approximation and carry out this value to as many places as your calculator will allow.

\[
\text{(Answer is approximately } \frac{1}{64}\text{)}
\]

Present additional problems to students until you are convinced that students understand what absolute error is and how to determine it in any given situation. The important thing is that students understand that it simply is the difference between the actual or “true” value and the measured value. Without knowing the “true” value, it’s not possible to state what the absolute error is.

Next, provide copies of the Absolute Error in Measurement BLM to each student, and let them work on the BLM in pairs. After students have completed the BLM, go over the answers as a class. Discuss each question fully.

**Activity 10: Relative Error (GLEs: 4, 5, 20)**

Materials List: paper, pencil, calculators

The second type of error students need to be familiar with is relative error. Explain to students that in some cases, a very tiny absolute error can be very significant, while in others, a large absolute error can be relatively insignificant. It is often more useful to report accuracy in terms of relative error. Relative error gives an indication of how good a measurement is relative to the size of the thing being measured. For example, suppose two students measure two objects with a meter stick. One student measures the height of a room and gets a value of 3.215 meters ±1 mm (0.001 m). Another student measures the height of a small cylinder and measures 0.075 meters ±1 mm (0.001 m). Clearly, the overall accuracy of the ceiling height is much better than that of the 7.5 cm cylinder, although the absolute error is the same for both measurements. The comparative accuracy of these measurements can be determined by looking at their relative errors. Thus, relative error is a comparative measure. Relative error shows how large the error is in relation to the correct value, thus it’s the ratio between the absolute error and the actual or “true” value. The formula for relative error is:
Relative Error = \( \frac{\text{Absolute Error}}{\text{Actual value}} \times 100 \)

Discuss how to determine the relative error for the ceiling height:

\[
\text{Relative error (ceiling height)} = \frac{0.001m}{3.215m} \times 100 = 0.031\%
\]

Compare this to the relative error associated with the cylinder height:

\[
\text{Relative error (cylinder height)} = \frac{0.001m}{0.075m} \times 100 = 1.33\%
\]

In looking at the relative errors in each case, it is obvious that the percent error is much greater in the cylinder measurement than in the ceiling measurement. Discuss the meaning of this with students and compare each type of error measurement.

At this point, utilize a modified GISTing (view literacy strategy descriptions) strategy designed to help each student process content more effectively. GISTing is a way to focus student attention on key ideas, in this case having students to explain the difference between absolute error and relative error. It requires the student to summarize (in an organized way) what has been learned in a few short, discreet sentences (or even a few words). In this case, have students write in their own words the difference between absolute error and relative error using as many words as they would like. For example, the student might write, “Absolute error is the total amount of error in a measurement while relative error is actually the error in relationship to the actual measurement or percent of error.” Once students have written their statement, lead students to progressively refine their statement to use fewer and fewer words until the statement is summarized to its basic understanding. For example in this case, if the final GIST statement were limited to 10 words, the statement could be written as “Absolute = total error; Relative = error compared to measurement.” The goal is to have students understand in a very concise manner, what the two things (absolute and relative error) are and to fully develop their understanding through the use of this summarization strategy. After students have written their summary statements, pick up the students’ GISTS and use them to see who fully has understood the lesson. Use this to guide your instruction on the rest of the activity.

Next, present the following problems to students:

**Example 1:** Jeremy ordered a truckload of dirt to fill in some holes in his yard. The company told him that one load of dirt is 5 tons. The company actually delivered 4.955 tons. Chanelle wants to fill in a flowerbed in her yard. She buys a 50-lb bag of soil at a gardening store. When she gets home, she finds the contents of the bag actually weigh 49.955 lbs.

Which error is bigger?

*Answer:* The relative error for Jeremy is 0.9%. The relative error for Chanelle is 0.09%. This tells you that measurement error is more significant for Jeremy’s purchase.

**Example 2:** In an experiment to measure the acceleration due to gravity, Ronald’s group calculated it to be 9.96 m/s². The accepted value for the acceleration due to gravity is 9.81 m/s². Find the absolute error and the relative error of the group’s calculation. (*Answers: Absolute error is .15 m/s², relative error is 1.529%)*

Provide students with more opportunity for practice with calculating absolute and relative error.
Activity 11: Which Unit of Measurement? (GLEs: 5, 21)

Materials List: paper, pencil, centimeter ruler, meter stick, ounce scale, bathroom scale, quarter, cup, gallon jug, bucket, water

Divide students into groups. Provide students with a centimeter ruler and have them measure the classroom and calculate the area of the room in centimeters. Then provide them with a meter stick and have them calculate the area of the room in meters. Discuss with students which unit of measure is most appropriate to use in their calculations. Ask students if they were asked to find the area of the school parking lot, which unit would they definitely want to use. What about their entire town? In that case, kilometers would probably be better to use. Provide opportunities for discussion and/or examples of measurements of mass (weigh a quarter on a bathroom scale or a food scale) and volume (fill a large bucket with water using a cup or a gallon jug) similar to the linear example of the area of the room. Use concrete examples for students to visually explore the most appropriate units and scales to use when solving measurement problems.

Sample Assessments

General Assessments

- Portfolio Assessment: The student will create a portfolio divided into the following sections:
  1. Significant digits
  2. Absolute error
  3. Relative error
In each section of the portfolio, the student will include an explanation of each, examples of each, artifacts that were used during the activity, and sample questions given during class. The portfolio will be used as an opportunity for students to demonstrate an understanding of each concept.

- The student will complete entries in their math learning logs using such topics as these:
  - Darla measured the length of a book to be $11 \frac{1}{4}$ inches with her ruler and $11 \frac{1}{2}$ inches with her teacher’s ruler. Can Darla tell which measurement has more error associated with it? Why or why not? *(She cannot tell unless she knows which ruler is closer to the actual standard measure.)*
  - When would it be important to measure something to three or more significant digits? Explain your answer.

Activity-Specific Assessments

- Activity 1: The student will determine the number of significant digits for various measurements.
  - *Ex:* Find the number of significant digits for 3.400 and justify your reasoning.
• **Activity 2**: The student will explain how a measurement such as 3400 meters could have 2, 3, or 4 significant digits and, for each instance, write the number appropriately using scientific notation.

• **Activity 10**: The student will solve sample test questions, such as this:
  Raoul measured the length of a wooden board that he wants to use to build a ramp. He measured the length to be 4.2 m, but his dad told him that the board was actually 4.3 m. His friend, Cassandra, measured a piece of molding to decorate the ramp. Her measurement was .25 m, but the actual measurement was .35. Use relative error to determine whose measurement was more accurate. Justify your answer.

• **Activity 11**: The student will be able to determine the most appropriate unit and/or instrument to use in both English and Metric units when given examples such as these:
  - How much water a pan holds
  - Weight of a crate of apples
  - Distance from New Orleans to Baton Rouge
  - How long it takes to run a mile
  - Length of a room
  - Weight of a Boeing 727
  - Weight of a t-bone steak
  - Thickness of a pencil
  - Weight of a slice of bread
Algebra I–Part 1
Unit 3: Solving Equations and Real-life Graphs

**Time Frame:** Approximately three weeks

**Unit Description**

This unit focuses on using algebraic properties to solve algebraic equations. The relationship between a symbolic equation, a table of values, a graphical interpretation, and a verbal explanation is also established.

**Student Understandings**

Students can solve linear equations graphically, from tables, with symbols, and through verbal and/or mental mathematics sequences. Students identify independent and dependent variables, slope as a “rate of change,” and inverse and direct relationships in graphs based on data from real-life situations.

**Guiding Questions**

1. Can students perform specified real-number calculations and relate their solutions to properties of operations?
2. Can students solve equations using addition, subtraction, multiplication, and division with variables?
3. Can students solve linear equations with rational (integral, decimal, and fractional) coefficients and relate the solutions to symbolic, graphical, and tabular/numerical representations?
4. Can students solve problems involving proportions and percentages?
5. Can students identify data as being directly or inversely related?
6. Can students distinguish the difference between independent and dependent variables in a real-life situation?
7. Can students understand how slope of a graph relates to a rate of change in a real-life situation?
## Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

### Grade-Level Expectations

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<td>Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)</td>
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<td>Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)</td>
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<td>Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H) (D-2-H) (P-5-H)</td>
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<td>Graph a line when the slope and a point or when two points are known (G-3-H)</td>
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<td>Explain slope as a representation of “rate of change” (G-2-H) (A-1-H)</td>
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<td>Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)</td>
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### CCSS for Mathematical Content

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<td>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
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<td>Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.</td>
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<tr>
<td>RST.9-10.4</td>
<td>Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.</td>
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<td><strong>Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12</strong></td>
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<tr>
<td>WHST.9-10.2d</td>
<td>Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.</td>
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</table>
Sample Activities

Activity 1: Review of Basic Concept of Solving Equations (GLE: 11; CCSS: A-REI.1; CCSS: RST.9-10.3; WHST.9-10.2d)

Materials List: paper, pencil, math textbook

Students coming into 9th grade should be very familiar with solving simple, one-step algebraic equations mentally. Review with students the basic premise behind solving simple equations building on the idea of equations as a “balance scale,” and discuss keeping both sides of the equation balanced. Have students solve a variety of real-life problems involving simple algebraic equations using their math textbook as a resource.

Have students do SPAWN writing (view literacy strategy descriptions). Have students explain the steps they would use to solve an algebraic equation for an unknown value when there are variables on both sides of an equation. This writing incorporates the use of the P (Problem Solving) part of the strategy as described below.

SPAWN is a strategy which uses higher-level thinking prompts to elicit student writing. Each letter in SPAWN stands for a particular writing prompt. They are as follows:

S—Special Powers: students are given special powers to do things and then write about how they would use these powers.
P—Problem Solving: students write about the solution of a problem and how they would do this.
A—Alternative Viewpoints: students put themselves in the place of someone or something and write about it.
W—What If?: students are asked to write on what if something happened or changed.
N—Next: students are asked to write on what they think will happen next.

After students have had time to write their explanations, allow students time to work in pairs to share their writing by exchanging papers and providing feedback to one another. Pick a few students from the class to share what they wrote and give feedback as a class.

Activity 2: Solving More Complex Equations (GLEs: 8, 11; CCSS: A-REI.1; CCSS: WHST.9-10.2d)

Materials List: paper, pencil, math textbook

Discuss the methods of solving multi-step equations that incorporate properties of equality (reflexive, symmetric, transitive, and substitution) to obtain a solution. Require students to solve equations which cannot be done mentally, and have students show, explain, and justify the steps used when solving the equations. Have students solve equations that include integral and rational coefficients. Include real-world problem solving that requires writing and solving algebraic equations (i.e., perimeter applications, area problems, sum of angles in a polygon, distance/time
relationships, percent increase/decrease, and proportions). Use the student math textbook as a resource for these types of problems.

Once students became proficient at the skill of solving more complex equations, utilize professor know-it-all (view literacy strategy descriptions). In this particular use of the strategy, form groups of 3 to 4 students. Have each group go to the board and act as the “professors” and invite the other students at their desks to create complex equations for the “professors” to solve. Allow the “professors” to huddle before sharing and explaining their answer. Insist that the other students hold the “professors” accountable for their responses. Groups should each take a turn being the “professors.” This is a fun way to review a concept and also to see if the students really grasped the material being presented.

**Activity 3: Explain Each Step (CCSS: A-REI.1; CCSS: WHST.9-10.2d)**

**Materials List:** paper, pencil

Students need to be able not only to solve equations but truly understand the processes that allow for the algebraic manipulation going on in solving equations. Therefore, in this activity, present students with the following equation: \( \frac{3}{4}x - \frac{7}{3} = \frac{1}{2} + \frac{3}{8}x \). Have each student write an informal but thorough, well-written explanation for each step they used to solve the equation. Have students utilize a two-column approach where one column shows the steps used, and the other column has the reasoning/explanation of the steps being used to solve the equation. Pick up student papers and provide feedback to students. A sample response is shown below.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4}x - \frac{7}{3} = \frac{1}{2} + \frac{3}{8}x )</td>
<td>Since an equation works like a balanced scale, to keep an equation balanced, you must perform the same operation at all times to both sides. In this first step, I am choosing to multiply by 24 on both sides. I chose 24 because this is the LCD for all fractions in this equation. Doing so will eliminate the “fractions” and will be simpler to solve.</td>
</tr>
<tr>
<td>( 24(\frac{3}{4}x - \frac{7}{3}) = (\frac{1}{2} + \frac{3}{8}x)24 )</td>
<td></td>
</tr>
<tr>
<td>( 18x - 56 = 12 + 9x )</td>
<td>When multiplying by 24, I distributed the multiplication over all of the terms in the original equation. This is the result of that multiplication/distribution process.</td>
</tr>
<tr>
<td>( \frac{18x - 56}{-9x} - \frac{9x}{9x} )</td>
<td>Again, to keep an equation balanced, you must perform the same operation at all times to both sides. In this step, I am subtracting 9x from both sides in order to</td>
</tr>
<tr>
<td>( 9x - 56 = 12 )</td>
<td></td>
</tr>
</tbody>
</table>
get the “x” terms on the same side. I subtracted because I want to “undo” the “addition” so that the +9x on the right side is eliminated. The goal is to solve for the “unknown” value by isolating the variable term on one side. This accomplishes that goal.

\[
9x - 56 = 12 + 56 \\
9x = 68
\]

I added 56 to both sides of the equation to get the constant terms on the same side. Since the variable term is on the left, I want the constant terms on the right so that the variable term is isolated. Since I had to “undo” the negative 56, I had to use the inverse operation, thus “add 56” to both sides.

\[
\frac{9x}{9} = \frac{68}{9} \text{ or } 7\frac{5}{9}
\]

In this last step, in order to isolate the variable and solve for “x,” the x is attached to the 9 by multiplication. In order to “undo” the 9 from the x, I must use the inverse operation, division by 9. This results in the final solution as shown.

Activity 4: Independent vs. Dependent Variable (GLE: 37; CCSS: RST.9-10.4)

Materials List: paper, pencil, math textbook

Discuss the concept of independent and dependent variables using real-world examples. For example:
- The area of a square depends upon its side length
- The distance a person travels in a car depends upon the car’s speed and the length of time it travels
- The cost of renting a canoe at a rental shop depends on the number of hours it is rented
- The number of degrees in a polygon depends on the number of sides the polygon has
- The circumference of a circle depends upon the length of its diameter
- The price of oil depends upon supply and demand
- The fuel efficiency of a car depends upon the speed traveled
- The temperature of a particular planet depends on its distance away from the sun

Present students with data from ten real-world contexts, and have the students work in groups to determine which is the dependent and the independent variable. Discuss each situation as a class. Explain that a two-dimensional graph results from the plotting of one variable against another. For instance, a medical researcher might plot the concentration in a person’s bloodstream of a particular drug in comparison with the time the drug has been in the body. One of these variables is the dependent and the other the independent variable. The independent variable is the time lapsed since the drug was taken, while the dependent variable is the drug concentration. Explain
to students that conventionally the *independent* variable is plotted on the *horizontal axis* (also known as the *abscissa* or *x-axis*) and the *dependent* variable on the *vertical axis* (the *ordinate* or *y-axis*). Relate this all pictorially with graphs.

Next, have students create *vocabulary cards* ([view literacy strategy descriptions](#)) for the terms *independent* and *dependent* variable. When students create vocabulary cards for terms that are related to what they are learning, students should see connections between words and critical attributes associated with the word. In the middle of a single card, the vocabulary word is written; and in the four corners of the card would be the definition, characteristics, examples, and illustrations.

An example of a vocabulary card for *dependent* variable is shown below:

<table>
<thead>
<tr>
<th><strong>Definition</strong></th>
<th><strong>Characteristics</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The variable that is measured to determine how it is affected by some related quantity</td>
<td>In an xy coordinate grid, the dependent variable is located on the vertical axis</td>
</tr>
</tbody>
</table>

**Dependent Variable**

- **Examples**
  - The area of a square is dependent on its side length
  - The distance a car travels is dependent upon its speed and the time traveled

- **Illustrations**

Have students create *vocabulary cards* throughout the course when new terms come up and keep them to review individually and with partners in preparation for quizzes, tests, and class assignments.

**Activity 5: Graphing from a Table of Values (GLEs: 4, 15, 24)**

**Materials List:** paper, pencil, graph paper, math textbook, graphing calculator

Provide students with a table containing data that will form a line and have them construct a scatter plot on graph paper. For example, the table which follows displays the amount of oil being pumped from a well in relation to the number of days the well is operated. Let students first determine the dependent and independent variables, and have them use this information to appropriately graph the data (dependent variable on vertical axis and independent variable on horizontal axis). Afterwards, discuss what pattern students see in the data (i.e., it appears to form a linear function).

<table>
<thead>
<tr>
<th>Days the Well is Operated</th>
<th>Amount of Oil Pumped</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
</tr>
</tbody>
</table>

Have students answer questions based on the graph. For example, have students determine the amount of oil that was pumped after 4 days based upon the results of the graph. Ask them to
determine if they think this answer is an exact value or an approximate value and why they think so. Next, have students draw a line through the data points and let them see that the line extends through the origin. Talk about the initial value and intercepts in real-world terms. For this example, the line intercepts the graph at (0,0) which means that the number of barrels pumped is 0 barrels after 0 days.

<table>
<thead>
<tr>
<th># of Days Pump is On</th>
<th>2</th>
<th>5</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Barrels Pumped</td>
<td>12</td>
<td>30</td>
<td>54</td>
<td>60</td>
</tr>
</tbody>
</table>

Connect the paper and pencil work associated with this problem to using a graphing calculator to do the work. Demonstrate for students how to input data into lists, how to plot this data in a scatter plot, and how to determine a line of best fit for the inputted data. Provide students with additional work on this topic using their math textbook as a resource.

**Activity 6: Direct and Inverse Relationships (GLE: 37; CCSS: WHST.9-10.10)**

Materials List: paper, pencil

Discuss with students what is meant by the terms Directly Related and Inversely Related in the context of real-life situations at an elementary level. If two variables have a direct relationship, as one variable increases, the other will also increase in value. Likewise, as one variable decreases, the other also decreases in value. An example where a direct relationship exists is the cost to feed a family—as the number of members in the family increases, the cost to feed the family also increases.

In contrast to the direct relationship, in an inverse relationship, as one variable increases, the other variable decreases, and vice versa. An example of an inverse relationship is the relationship that exists between the number of workers it takes to do a job and the time it takes to finish the job. For instance, suppose it takes 6 workers 1 day to paint a house. If the number of workers decreased, the time it takes to do the job would increase (an inverse relationship). Discuss several real-life examples with students. Have students write a math learning log entry about a real-life situation that involves a direct relationship and an inverse relationship, and then discuss these as a class.

**Activity 7: Graphs and Direct and Inverse Relationships (GLEs: 37; CCSS: WHST.9-10.2d)**

Materials List: paper, pencil, graphs from science and other books, math textbook

Provide students with several line graphs relating different quantities (taken from science books, or their math textbook). Have students work in groups to obtain the following information for each graph:
1. What is the independent variable? What is the dependent variable?
2. Does the graph portray a direct or inverse relationship between the variables? Explain your reasoning.
3. Is the graph linear?

Afterwards, talk with students about increasing and decreasing functions and how they are related to direct and inverse relationships. For example, in an increasing function, a direct relationship exists. In contrast, for a decreasing function, an inverse relationship exists. Relate all of this information graphically. The goal here is to begin getting students to be able to analyze real-life relationships that are presented graphically and to connect this later with the equations which model these graphs.

**Activity 8: Going on Vacation! (GLEs: 15, 24, 37)**

Materials List: paper, pencil, graph paper, graphing calculators, Going on Vacation BLM

Have students work with a partner on the Going on Vacation BLM. The problem requires students to identify the independent and dependent variable, create a table of values relating the two variables, use the table to graph the relationship, write an equation to fit the graph, and then analyze the graph. This will be the first time in this course that students find an equation to fit the table/graph, but students should have been required to find an equation in earlier grades to fit an input/output table; therefore, it should not be totally new to them. However, keep in mind that this particular part of the activity may need more direction for some students. After students create charts and graphs using pencil and paper, demonstrate for students how to use a graphing calculator to do the assigned work by inputting the table of values into lists, how to create a scatter plot of the data, and how to find an equation for the line of best fit. Compare the equation that the calculator produces with the equations that the students have written.

**Activity 9: Analyzing Distance/Time Graphs (GLEs: 25, 37)**

Materials List: paper, pencil, math textbook, Analyzing Distance/Time Graphs BLM

Have students work in groups on Analyzing Distance/Time Graphs BLM. In this activity, each graph presented displays the distance three different cars traveled over a certain time period. Have students analyze the graphs, and discuss the questions related to the graphs. Be sure to discuss the difference between the dependent and independent variables, and relate the slope of a distance time graph with the speed or “rate of change” of the moving object.

After a full discussion of the activity has taken place, provide additional problems in which the student has to find the rate of change using the slope in real-life situations. Use the math textbook as a resource for additional problems.
Activity 10: Slope—What does it Tell Us About a Graph? (GLEs: 15, 24, 25, 37)

Materials List: paper, pencil, graph paper, What Does Slope Tell Us About a Graph? BLM

In this activity, students will interpret the meaning of slope as a rate as it applies to a real-life situation. Provide students with a copy of What Does Slope Tell us About a Graph? BLM. Let students work individually first, then allow students to pair with a partner to discuss their solutions. Afterwards discuss the results as a class. Students should understand that in a real-life situation the slope tells the rate at which the values change. Students should also realize that a slope in a real-life situation has units associated with it. In this particular problem, the units associated with the graph are kilograms per year. The slope tells us how a person’s weight changes over time.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will review magazines, newspapers, or journals (or recall personal experiences) for real-world relationships that can be modeled by linear functions (include function and graph).
- The student will compile a portfolio of work for Unit 3 to be handed in for a grade.
- The student will draw numberless graphs that relate to a situation in real-life, explaining the graph in words and relating it to the motion or situation depicted.
- The student will take paper and pencil tests related to the concepts that were taught in this unit.

Activity-Specific Assessments

- **Activity 6:** The student will write a short paragraph explaining what is meant by direct and indirect relationships and give examples.
- **Activity 7:** The student will write the relationship that exists between the variables for each graph provided him/her in the activity.
- **Activity 10:** The student will make an oral presentation of his/her findings and explain the processes that led to his/her conclusions.
Algebra I–Part 1
Unit 4: Linear Equations and Graphing

Time Frame: Approximately four weeks

Unit Description

This unit focuses on developing an understanding of graphing linear equations in the coordinate plane.

Student Understandings

Students recognize linear relationships, simplify linear expressions, and graph linear equations in two variables. They use a variety of techniques when graphing linear equations including input-output tables, two points, and slope and one point. They graph manually and use a graphing calculator.

Guiding Questions

1. Can students graph data from input-output tables on a coordinate graph?
2. Can students recognize linear relationships in graphs of input-output relationships?
3. Can students perform simple algebraic manipulations of collecting like terms and simplifying expressions?
4. Can students determine the slope of a line given a graph or two points?
5. Can students determine how the values of “m” and “b” in a linear equation in the form \( y = mx + b \) affect the graph?

Unit 4 Grade-Level Expectations (GLEs) and Common Core State Standards

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)</td>
</tr>
</tbody>
</table>
11. Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)

13. Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)

15. Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)

**Geometry**

24. Graph a line when the slope and a point or when two points are known (G-3-H)

25. Explain slope as a representation of “rate of change” (G-3-H) (A-1-H)

**Data Analysis, Probability, and Discrete Math**

29. Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)

**Patterns, Relations, and Functions**

37. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

38. Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)

39. Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

**ELA CCSS**

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>RST.9-10.4</td>
<td>Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.</td>
</tr>
<tr>
<td>WHST.9-10.2d</td>
<td>Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.</td>
</tr>
<tr>
<td>WHST.9-10.10</td>
<td>Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.</td>
</tr>
</tbody>
</table>
Sample Activities

Activity 1: Lines on a (GLEs: 13, 15, 24, 38; CCSS: WHST.9-10.2d)

Materials List: paper, pencil, graph paper, graphing calculators

Have students create a table of values to graph the following two equations: \( y = 2x \) and \( y = -2x \). Have students graph the two equations on two different coordinate planes using paper/pencil and graph paper.

Once students have graphed the two equations on graph paper, have each student write a two-column chart comparing the two graphs. In one column, students should explain the similarities of the two graphs, and in the other column, students should list the differences for the two graphs. An example of such a chart is shown below.

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Both Graphs Linear</td>
<td>• Slopes are different (one positive</td>
</tr>
<tr>
<td>• Both Pass through Origin</td>
<td>and one negative)</td>
</tr>
<tr>
<td></td>
<td>• One graph increasing; one graph</td>
</tr>
<tr>
<td></td>
<td>decreasing</td>
</tr>
</tbody>
</table>

Afterwards, have students exchange each other’s chart. Students are then to analyze the work of their peer and provide feedback. This analysis of one another’s work is a form of questioning the content view literacy strategy descriptions). In this particular use of the strategy, as students read their fellow classmates’ work, ask them to determine the following:

- Has the student identified that one of the similarities is that both equations form a line, thus they are both linear equations?
- Has the student identified that one of the similarities is that both lines pass through the origin?
- Has the student observed that one of the differences is that in the case of the first graph, \( y = 2x \), the graph is increasing and in the second graph, \( y = -2x \), the graph is decreasing?
- Did the student use correct mathematical language?
- Is there anything that the student wrote that was unclear to you as the reader? If so, what is it that you need clarification on?

Students are then to provide feedback to their peers based on these guiding questions. Afterwards, students will get back their original charts and make modifications based on the feedback given.

Finally, when going over this activity as a class, discuss what the terms increasing and decreasing mean mathematically and graphically (in an increasing graph, as the \( x \)-values increase, the \( y \)-values increase; in a decreasing graph, as the \( x \)-values increase, the \( y \)-values decrease). Finally, have students graph the two equations using a graphing calculator and discuss setting up a window to view graphs, as well as learning how to trace along the graphs using the trace feature.
Activity 2: How Does “m” Affect the Graph of an Equation in the Form of $y = mx$ (GLEs: 13, 37, 38, 39)


This lesson builds on the previous activity and students’ prior work with slope in Unit 3. Begin the lesson by making copies of the Anticipation Guide: What about “m”? BLM and provide each student with a copy. Anticipation guides (view literacy strategy descriptions) are a way to activate prior knowledge of content and helps give students a purpose for what they are about to learn by having them respond to statements before and after the lesson being presented. In this particular use of the strategy, have students respond to the statements presented on the BLM. After students have had the opportunity to give their responses, allow them the opportunity to get in small groups to discuss their responses with their peers. At this point, the students are more prepared to gain information from the activity that is about to be presented. When the activity is completed, have students revisit the anticipation guide and modify their thinking if necessary, including having students explain what caused them to modify their thinking.

Provide students with graphing calculators and have them work in pairs during this activity. Discuss the formula $d=rt$ which relates the distance traveled by an object as determined by the rate at which the object travels and the time it travels at this rate. Explain to students that instead of using the letters $d=rt$ to model the formula, the form $y=mx$ will be used by the graphing calculator to graph this relationship. In place of the variable $r$ (the rate at which the object is moving) the variable $m$ will be used to represent that value, while the variables $y$ and $x$ will represent the values $d$ and $t$. Have students determine what the graph of $y = 5x$ looks like on their graphing calculators. Relate this graph and equation to moving at a rate of speed of 5 miles per hour, where the $x$-values represent the time in hours and the $y$-values represent the distance traveled in miles. Show students how the $x$ and $y$ values (which show up at the bottom of the calculator screen as the line is traced) indicate the time and distance values, respectively. Use the table function of the calculator to show students that the calculator also lists these values in a table with increments that can be changed by the user. Also, note that the graph passes through the origin.

Next, have students investigate how the graph of $y = mx$ would change if the value of $m$ is changed to different positive integers (use 1, 3, 10, and 50). Students should see that the changes in $m$ create changes in the graph’s steepness, but that the graph is still increasing as $x$ gets greater. Have students use negative integer values for $m$ (use -1, -3, -10, and -50) in the equation $y = mx$, and have students investigate what this change does to the original graph. Students should see that the graph is now a decreasing graph, but that the graph is still linear and still passes through the origin. Discuss the findings with students, and help students to realize that in the equation $y = mx$, the number $m$ tells about the “steepness” or the “slope” of the line. Students should see that positive values for $m$ result in an increasing linear function, while negative values for $m$ result in a decreasing linear function. Students should also see that the closer the value of $m$ is to zero (have students put in a value of 0 for $m$ to verify this) the more the line becomes horizontal, and that making the value of $m$ really large results in a more vertical graph. This will
be important later when discussing horizontal and vertical lines. Another important point is that
the steepness of a graph with an “m” value of 1 and -1 is the same—the only difference being
one graph increases while the other graph decreases. The “steepness” is the same but the actual
slope value is different. Once all of these ideas have been fully discussed as a class, have
students revisit the anticipation guide BLM to modify their thinking as necessary. Discuss the
changes that were made and the reasons for the changes as a class.

Activity 3: Developing the Slope Formula (GLEs: 13, 25)

Materials List: paper, pencil, graph paper, math textbook

In this activity, begin helping students understand how to determine slope of a line using
\( \frac{\text{rise}}{\text{run}} \) and develop the slope formula when given two points. In this
teacher-led activity, explain to students that when building a house,
the pitch of a roof is usually given as a ratio between the rise of the
roof and the run of the roof.

Provide students with graph paper and write the following roof
pitches on the board. Have students create scale drawings of the roof
pitches on coordinate graph paper:
- Roof 1—rise 3 feet to a run of 1 foot;
- Roof 2—rise of 8 feet to a run of 2 feet;
- Roof 3—rise of 1 foot to a run of 1 foot;
- Roof 4—rise of 4 feet to a run of 1 foot.

After students make sketches of each roof, discuss the steepness of each roof. Have the students
compare which is the steepest and which is the least steep, and have them notice the fact that
Roof 2 and Roof 4 have the same steepness. Relate this activity to the fact that the steepness of a
line segment (or roof) is the slope of the line and is the ratio of \( \frac{\text{rise}}{\text{run}} \). Discuss the fact that Roof 2
has a ratio of \( \frac{8}{2} \) or 4 and Roof 4 has a ratio of \( \frac{4}{1} \) or 4, hence the slope of both graphs is the same.
Explain that the larger the ratio is numerically, the steeper the slope of the line segment (or roof)
will be.

Next, provide students with problems where they have to determine the slopes of several lines on
a coordinate grid by finding their \( \frac{\text{rise}}{\text{run}} \). After students have successfully found the slope using
the rise/run method, use the same lines and show students how the slope can be calculated with
the “slope formula” using two points which are on the line. Lead students to understand how the
slope of a line can be calculated using the formula, \( m = \frac{y_2 - y_1}{x_2 - x_1} \) (relate this formula to rise/run and
“change in vertical” in relation to its “change in horizontal” values). Explain that this formula
can be used to determine slope anytime they are given two points on a line without actually
having to plot the points on a coordinate grid. Provide additional practice for students on the skill of finding the slope given two points, using the math textbook as a resource.

**Activity 4: Slope as a Rate of Change (GLEs: 9, 25, 37; CCSS: RST.9-10.4)**

Materials List: paper, pencil, graph paper, math textbook, Slope as a Rate of Change BLM

Provide students with graph paper and copies of the Slope as a Rate of Change BLM. Have students work in groups on the BLM which involves questions related to the concepts of slope and linear equations. Discuss the results as a class. After discussing the answers with students, guide them to understand the fact that the slope of a line is constant. In the activity, no matter what two points were chosen to determine slope, the slope has the same value. This is a BIG IDEA! Also, when determining slope in real-life contexts, it is important that students understand that there is not only a number associated with the slope, but there are also units associated with this slope. The slope represents a rate which has real-life meaning attached to it. Provide additional work having students determine the slope as a rate of change in real-life contexts. Use the math textbook as a resource for additional problems of this type.

Have students create a vocabulary card (view literacy strategy descriptions) for the term “slope” and have students exchange cards and provide feedback to one another. Discuss some of the cards that were created as a class. When students create vocabulary cards for terms that are related to what they are learning, students should see connections between words and critical attributes associated with the word. In the middle of a single card, the vocabulary word is written, and in the four corners of the card are the definition, characteristics, examples, and illustrations. An example of a vocabulary card for slope is shown below.

<table>
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<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>A measure of the steepness of a line</td>
<td>A line can have no slope (vertical), 0 slope (horizontal), positive slope (increasing) or negative slope (decreasing).</td>
</tr>
</tbody>
</table>

**SLOPE**

Examples

\[ m = \frac{(y_2-y_1)}{(x_2-x_1)} \]

represents the slope formula to be used when finding the slope given two points.

The slope of the line \( y = 5x - 3 \) is 5 since this is in slope intercept form \( y = mx + b \).

Illustration

Once students make their vocabulary cards, allow time for them to review the words and information individually and with a partner. This can serve as helpful review for other class activities and in preparation for quizzes and tests.
Activity 5: Finding the Intercepts of a Linear Graph (GLEs: 13, 24; CCSS: WHST.9-10.10)

Materials List: paper, pencil, graphing calculators, graph paper

In this activity, the goal is to help students understand what intercepts are and how to determine the x- and y-intercepts for an equation and its graph.

Have students use a graphing calculator to graph the line \( y = 2x + 6 \). Discuss with students the definition of an intercept—the point where the graph intersects an axis. Ask students what they think the terms x-intercept and y-intercept mean, and clarify should there be misinterpretation of the terms.

Show students how to find the intercepts using both calculator technology and algebraically using paper and pencil methods. First, have students trace the graph and find the x-intercept. Ask what the y-value is when the graph intersects the x-axis. They should respond that the y-value is 0. Have students substitute 0 for \( y \) in the equation \( y = 2x + 6 \), and then solve for \( x \) (using paper/pencil). Thus the point (-3, 0) is the x-intercept for the graph of \( y = 2x + 6 \). Next, have students trace the calculator graph to the y-intercept and ask students to indicate to find the x-value. Have students substitute 0 for \( x \) (using paper/pencil) in the same equation and solve for \( y \) to determine that the point (0, 6) is the y-intercept for the graph. Have students plot the intercepts on graph paper and draw a line through them. Ask students to compare this graph with the one drawn on the graphing calculator. It is important to remember that substituting \( x = 0 \) to find the y-intercept (and \( y = 0 \) to find the x-intercept) is counter-intuitive to many students.

Next, have students determine the intercepts for the equation \( y = -2x - 4 \), and then have them use the intercepts to create a graph for the equation. Have students work individually and then form pairs to exchange answers and discuss how they determined the solutions. Discuss the results as a class.

Give students the equations \( y = 3x - 6 \) and \( y = -3x + 6 \). Have students write a math learning log (view literacy strategy descriptions) entry about the similarities and differences they should see in the graphs of these equations. Allow time for students to exchange their entries to check each other for accuracy and logic. Students should revise their entries based on peer feedback.

Activity 6: Is the Data Linear? (GLEs: 15, 29, 39; CCSS: WHST.9-10.10)

Materials List: paper, pencil, graphing calculator, math textbook

In this activity, provide students with data from the input/output tables shown below, and have students predict, without actually graphing, if the data is linear. Discuss the predictions and have students explain why they made their decisions.
Next, have students enter the data into their graphing calculators and graph the data in a scatter plot to determine if their prediction is correct. Ask students to examine the data for patterns in the tables that might help determine the answer to the question of whether a set of data is going to be linear or non-linear without the need to actually graph the data. It may be necessary to give students a hint by asking them to look for patterns within the $x$-values and then within the $y$-values of each chart. They should also compare the $x$ values in one table to the $x$ values in the second table, and they should do the same with the $y$-values. Students should see that the difference between any two $x$ values in both tables is 1, so the data sets are alike in that respect. They will probably explain that the difference in any two $y$ values in Table 1 is 4. Some may see the $y$-values in the second chart as being doubled each time; others may explain that the difference in any two $y$ values is not constant or the same (the differences are doubled). For this example, the difference lies in the pattern of the $y$-values. Ask which table shows a linear graph. Guide students to understand that if data is linear, there are two things to determine – change in the $x$-values and the change in the $y$-values. Explain that linear relationships result when the change in $y$-values divided by the change in $x$-values for any two points is constant. Relate this to the slope formula used in Activity 4. For example, the data in the Table 1 is linear because there is a constant ratio (or slope) of 4 between any two points, while the data in Table 2 does not have this characteristic.

Provide other examples of data for students to analyze to become proficient at this skill using the math textbook as a resource. In addition, provide opportunities for students to actually collect data themselves, and then determine if the data is linear or non-linear. For example, students could investigate the relationship between the side length of a square and its perimeter where the side length is the independent variable and the perimeter is the dependent variable. Students could likewise investigate the relationship between the side length of a square and its area where the side length is the independent variable and the area of the square is the dependent variable. In these two cases, students should see that there is a linear relationship between side length and perimeter, but a non-linear relationship between side length and area.

Have students write a math learning log (view literacy strategy descriptions) entry relating the process they could use to determine whether a set of data is linear or non-linear without actually graphing the data. Let students exchange their logs and provide feedback to one another, then have students make any adjustments to their initial logs and turn them into the teacher.
Activity 7: Slope-Intercept Form of a Line (GLEs: 13, 38, 39)

Materials List: paper, pencil, graphing calculators, Match the Equation with the Graph BLM

In Activity 2, students learned that any equation in the form \( y = mx \) forms a line. This activity extends to equations in slope-intercept form, \( y = mx + b \). Students will investigate the effect of \( b \) on the graph of the line.

Students should already understand how the value of \( m \) affects the slope of the line based upon what they learned in Activity 2. Using the graphing calculator, let students discover what happens to a line when the value of \( m \) is held constant and the value of \( b \) is changed. Starting with the equation \( y = 2x \), have students discover what happens to the graph when \( b \) is changed using equations such as \( y = 2x +1 \); \( y = 2x +2 \); and \( y = 2x-1 \). Make sure that students use both positive and negative values for \( b \). Students should see that the effect of changing \( b \) translates the line \( y = mx \) up or down the \( y \)-axis by \( b \) units. Students should also recognize that \( b \) is the \( y \)-intercept for the line. Once students have investigated thoroughly the effects of changing the values of \( m \) and \( b \) on a linear graph, provide students with copies of Match the Equation with the Graph BLM. Allow students the opportunity to work on this alone and then pair up with another student to compare their answers before going over the results as a class.

Activity 8: Graphing Using a Point and the Slope (GLEs: 13, 24)

Materials List: paper, pencil, graph paper, math textbook

This activity is designed to use what has been learned about slope in order to graph lines on a coordinate plane. Have students plot the point \((0, 4)\) on a coordinate grid. Have students draw some lines that pass through the point \((0, 4)\). Students should realize that an infinite number of lines actually run through the point \((0, 4)\).

Next, using a new coordinate grid, tell students to find the line that passes through \((0, 4)\) but has a slope of \(\frac{3}{2}\). Let the students work in groups to find other points on the line graph using the slope of \(\frac{3}{2}\). (Remind students of the relationship between slope and the rise/run). Have students list some other points and ask them how many lines run through the point \((0,4)\) and have a slope of \(\frac{3}{2}\). Students should come to the realization that there is really only one distinct line that has those characteristics. They should also realize that there are an infinite number of points that could be produced in order to draw the line with these characteristics. Finally, have students graph other lines using a point and a slope with which to make their graphs. Use the math textbook as a resource for additional problems of this type.
Activity 9: Equivalent Forms of Equations (GLEs: 8, 11, 15)

Materials List: paper, pencil, graph paper, math textbook

Provide students with the following pair of linear equations in two variables: \( y = -2x + 4 \) and \( 4x + 2y = 8 \). Have students make a table of \( xy \) values for the two equations. After the table of values is found, have students graph the two lines using graph paper. Students should see that the two equations produce the same line. Provide students with additional pairs of equations, some which produce the same line and some which do not. Have students decide which equations and graphs are the same and which are different, and discuss what they did to figure this out. Refer to the equations which were used initially: \( y = -2x + 4 \) and \( 4x + 2y = 8 \). Remind students that both equations produced the same line. Explain that these two equations are equivalent forms of a linear equation. One of the equations is written in slope-intercept form, while the other is written in standard form and multiplied by a factor of 2. Discuss with students the two different forms of writing linear equations and how to translate a linear equation from standard form \([Ax + By = C]\) to slope-intercept form \([y = mx + b]\) using algebraic manipulation. Explain that the benefit of having an equation in \( y = mx + b \) form is the ease with which one can determine the slope and \( y \)-intercept and then use that information to sketch the graph. The benefit of writing an equation in standard form is that there are no fractions, and one can easily see what the \( x \)- and \( y \)-intercepts are for the graph mentally. Provide students with the practice necessary to develop the skill of transforming an equation from standard form into slope-intercept form, and vice versa. Also, have students identify the \( x \)- and \( y \)-intercept and slope for each equation and graph the line using paper and pencil. Use the math textbook as a resource for additional work on this concept.

Activity 10: Modeling Real-life Situations (GLEs: 9, 15, 24, 25, 37)

Materials List: paper, pencil, graph paper

This activity is intended to be a teacher-led activity. Introduce the following situation to students:

A family is going on a trip. They travel \( h \) hours in a day, averaging 50 mph. Write an equation to represent the distance traveled, \( d \), in miles after traveling \( h \) hours.

Allow students time to write an equation that matches the situation. Discuss this as a class. Have students explain how they named any variables they used to make their models.

Next, have students construct a graph using paper and pencil to display the equation graphically. In the process, ask students to determine which of their variables represents the independent variable and which represents the dependent variable. In this case, the relationship used to model this situation is the distance traveled per day (\( d \)) equals the speed (50 mph) times the number of hours driven (\( h \) or \( t \)). So the equation they could use to model the situation should be of the form: \( d = 50h \). The \( x \)-axis (independent) is the number of hours driven (scale from 0 to 24 hrs) and the \( y \)-axis (dependent) is the number of miles driven (scale from 0 to 1,200 miles). The graph would consist of a straight line starting at the origin (0,0) and ending at the point (24, 1200). Have the students answer questions concerning the graph. Some possible questions might be:
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How far does the family travel in 3 hours? Answer = 150 miles. How many hours will it take to cover 200 miles? Answer = 4 hours. What does the slope of this graph represent in real life terms? Answer: The rate of speed which is 50 miles per hour. Help students to appreciate the value of representing an equation graphically and the ease with which information from the situation can be obtained.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will write a short paragraph explaining the connection between slope and rate of change.
- Project the following equations \( y = x - 2 \) and \( y = -x + 2 \) and have students determine slopes, \( x \)- and \( y \)-intercepts, and explain what is the same and what is different about the two graphs.
- The student will take paper and pencil tests on all concepts studied during this unit.

Activity-Specific Assessments

- **Activity 4**: The student will write about a type of rate that occurs in real-life that has not been discussed in class and explain what this rate means in real-life terms. For example, if a person grew 9 inches over a four year time span, that means that the person grew at an average rate of 2.25 or 2 ¼ inches per year over that four years.

- **Activity 5**: The student will find the \( x \)- and \( y \)-intercepts for the linear equation \( 3x + 5y = 12 \) and use them to graph the equation on grid paper.

- **Activity 6**: The student will create two data tables—one that will produce a linear function and one that is not linear.

- **Activities 9**: The student will write two equivalent forms of a linear equation. For example, the equation \( 3x + 5y = 10 \) can also be written as \( 5y = -3x + 10 \) or \( y = -\frac{3}{5}x + 2 \).
unit: Algebra I–Part 1

Unit 5: Graphing and Writing Equations of Lines

**Time Frame:** Approximately three weeks

**Unit Description**

In this unit, the emphasis is on writing and graphing linear equations in both real-life and abstract situations. Interpretation of the real-life meaning of equations and the relationship between the values of coefficients in the linear equation and their effect on graphical features is reinforced.

**Student Understandings**

Students understand the meanings of slope and y-intercept and their relationship to the nature of the graph of a linear equation. They write and interpret equations of lines using the slope-intercept, point-slope, and standard form for the equation of a line. Given equations, students can graph and interpret information from the graphs.

**Guiding Questions**

1. Can students write the equation of a linear function given appropriate information about the slope and intercept?
2. Can students use the basic methods for writing the equation of a line (i.e., two-point, slope-intercept, point-slope, and standard form) and translate from one form to another?
3. Can students perform the algebraic manipulations on the symbols involved in a linear equation to find its solution and relate its meaning graphically?
4. Can students discuss the meanings of slope and intercepts in the context of an application problem?
5. Can students interpret and analyze the results from a linear equation which models real-life situations and apply meaning to this equation in terms of its slope and y-intercept?

**Unit 5 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)**

<table>
<thead>
<tr>
<th>Grade-Level Expectations</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLE #</td>
<td>Algebra</td>
</tr>
<tr>
<td>11.</td>
<td>Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)</td>
</tr>
</tbody>
</table>
13. Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)

15. Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)

Geometry

24. Graph a line when the slope and a point or when two points are known (G-3-H)

25. Explain slope as a representation of “rate of change” (G-2-H)

Patterns, Relations, and Functions

37. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-ID.7</td>
<td>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td>
</tr>
</tbody>
</table>

ELA CCSS

<table>
<thead>
<tr>
<th>CCSS #</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>WHST.9-10.2d</td>
<td>Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.</td>
</tr>
</tbody>
</table>

Sample Activities

Activity 1: Writing a Linear Equation Given Slope and Y-intercept (GLEs: 11, 13, 24)

Materials List: paper, pencil, graph paper, math textbook

In Unit 4, students were taught to graph a line given its slope and y-intercept. In this activity, students will write the equation of a line given its slope and y-intercept. Discuss with students how to write and graph an equation in slope-intercept form (i.e., $y = mx + b$) given the slope, $m$, and y-intercept, $b$. For example, if a line has a slope of 2 and a y-intercept of -3, the equation would be written as $y = 2x -3$ and its graph would be as shown:
Explain to students that anytime the slope and \( y \)-intercept for a line are known, it is very simple to write its equation using this format. Provide additional examples for students, and make the connection between how to write, as well as how to graph, linear equations when given a slope and a \( y \)-intercept. In addition, lead students through the process for changing an equation from slope-intercept form into standard form \((Ax + By = C)\) by the use of algebraic manipulation. Because students are not solving for a value, this is a very hard process for many students. Provide ample opportunity for students to show proficiency in this skill. Use the textbook as a resource for additional problems. Students should be able to convert equations from one form to the other with ease.

**Activity 2: Writing a Linear Equation Given a Point and a Slope (GLEs: 11, 13, 24)**

**Materials List:** paper, pencil, graph paper, math textbook

Introduce the point-slope form of a linear equation and how it can be used to find the equation of a line. Indicate that this is especially useful when a portion of the graph of the line is given, but the \( y \)-intercept is not included in the portion of the graph shown. The point-slope form of a line is derived from the slope formula: 
\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
\]
By cross multiplication: 
\[
(y_2 - y_1) = m (x_2 - x_1).
\]
Assuming \((x_1, y_1)\) is the known point, \((y_2 - y_1) = m (x_2 - x_1)\) becomes \((y - y_1) = m (x - x_1)\). Demonstrate to students how this formula can be used to write an equation. For example, if the slope is 5 \( (m = 5) \) and the line contains the point \((3, 4)\), the point-slope equation would be given as follows: 
\[
(y - 4) = 5 (x - 3).
\]
Emphasize how the point and slope are replaced in their respective places in the formula.

Have students work to rearrange or transform the equations into either the slope-intercept form \((y = 5x - 11)\) or standard form \((5x - y = 11)\). Continue to have students draw the graph of an equation. Knowing the point and slope, students can plot the point and count off the slope to graph. Emphasize the connection between the point-slope and slope-intercept forms of the equation for graphing. Both provide a point (the intercept or another point) and both provide a slope; therefore, plotting and counting slope to find a second point can be used to graph each of these. Note that the standard form of the equation might be most useful in determining the \( x \) and \( y \) intercepts for the graph since one of the terms in the equation is eliminated when 0 is substituted for either \( x \) or \( y \).

Provide additional examples and practice using the math textbook as a resource for students to achieve proficiency in writing equations of lines given a point and a slope.
Activity 3: Determine an Equation for a Line Given Two Points (GLEs: 11, 13, 15; CCSS: WHST.9-10.2d)


Make copies of the What’s the Equation of the Line? BLM. Allow students to work in groups of three to four on this BLM. In this activity, students must use what they learned in Activity 2 (determining the equation of a line given its slope and a point on the line) and extend that knowledge to be able to determine the equation of a line given two points on the line. The intent of the activity is to develop the understanding that to find the equation for a line, two things must be known about the line in question—the slope and a point that lies on the line. Students should first realize that the two points must be used to determine the slope of the line that contains them. Then, using this slope and one of the two data points given, students use the procedures previously discussed to determine the equation of the line in all three forms—point-slope, slope-intercept, and standard form.

Have students respond to a SPAWN writing prompt (view literacy strategy descriptions) (using the letter P--Problem Solving). The prompt is: Explain how you found the solution to problem 4 on the BLM. In their responses, students should explain how they determined the equation of the line containing the points (-3, 5) and (2, -1) and put the answer in slope-intercept form. Have students turn in their SPAWN writing and provide feedback as part of a formative assessment activity. For students who were not successful or needed modifications, have students rewrite their SPAWN writing and turn in for further feedback from the teacher.

Finally, provide students with additional problems of the types shown on the BLM using the math textbook as a resource.

Activity 4: Equations of Vertical and Horizontal Lines (GLE: 13)

Materials List: paper, pencil, math textbook

In this activity, writing equations of lines is extended to include equations of vertical and horizontal lines. In a teacher-led discussion, explain to students why the slope of a horizontal line is 0, then use the point-slope formula to help students see that the equation for a horizontal line is \( y = a \), where \( a \) is actually the \( y \)-coordinate for any point on the line. Repeat the same process using points on vertical lines. Lead students to see that since division by 0 is undefined, the slope of a vertical line is undefined. Ask students to write the equation of a vertical line based on what they know about the equations of horizontal lines. Lead students to see that the equation of a vertical line is \( x = a \), where \( a \) is the \( x \)-coordinate for any point on the line. Provide students additional examples and problems using the math textbook as a resource.

At the end of this activity, have students participate in a version of professor know-it-all (view literacy strategy descriptions). In this particular use of the strategy, form groups of 3 students to come up to the board and act as the professor. Call on groups randomly. Students should quiz the professors on how to determine the equation for a line given a slope and a point, two points, or a vertical or horizontal line (based upon problems created by the questioners). Students ask the
questions, the professors answer the questions and aren’t allowed to sit until the class feels its question has been answered satisfactorily. Each “professor” in the group should be required to answer some part of the question. Professor know-it-all is a fun way to review a concept and also to see if the students really grasp the material being covered.

Activity 5: Fahrenheit and Celsius—how are they related? (GLEs: 9, 13, 15, 25, 37; CCSS: S-ID.7)

Materials List: paper, pencil, Fahrenheit and Celsius—How Are They Related? BLM

*Note: This activity has not changed because it already incorporates the CCSS listed.*

In this activity, students will work in groups on the Fahrenheit and Celsius—How Are They Related? BLM. In order to accomplish this task, students will have to extend what they have learned in determining equations of lines and apply that knowledge to solving a real-life problem. The relationship between temperatures measured in degrees Fahrenheit and Celsius is linear and can be obtained by knowing that water freezes at 32°F and 0°C while water boils at 212°F and 100°C. Discuss these important benchmarks with students, and have students work in groups of three to perform tasks associated with the BLM. In this activity, students must also interpret the meaning of slope and the x- and y-intercepts in a real-life setting. After students have had an opportunity to work on this task, discuss the results as a class.

Activity 6: Writing an Equation from a Table of Values (GLEs: 9, 10, 12, 13, 15, 37; CCSS: S.ID.7)

Materials List: paper, pencil, graph paper, From Tables to Equations BLM

*This activity has not changed because it already incorporates the CCSS listed.*

In this activity, students apply the skill of determining equations of lines for a real-life situation and interpreting its meaning.

As a precursor to this activity, present the following question to students: “What is a polygon?” Utilize the discussion (view literacy strategy descriptions) strategy Think-Pair-Square-Share. In this particular strategy, after giving students the question, ask students to think alone about the problem for a short period of time, then have each student pair up with another student to share their thoughts. After they have had the opportunity for discussion, have pairs of students share with other pairs, forming small groups of four. Once this process has taken place, gather oral responses to the question for a full class discussion of the question presented. The goal of discussion is to provide a deeper processing of content and rehearsal of newly learned content. Students should be led to understand that a polygon is a closed geometric figure with straight sides joined at the endpoints of those segments.
Next, ask students if they remember what the sum of the interior angles of a triangle is, and review with students the fact that the sum of the three angles of any triangle is 180°. This should have been learned in previous math classes.

Provide copies of the From Tables to Equations BLM. The table on the BLM shows the relationship between the number of sides of an n-sided polygon and the sum, S, of its interior angles. Have students use the data table provided to answer the questions presented on the BLM. Let students work in pairs to perform the indicated tasks. After students have had an opportunity to perform the tasks, discuss the results as a class. Again, students are required to find the equation and to interpret the real-life meanings of the slope and x- and y-intercepts.

**Activity 7: Wages vs. Hours Worked (GLEs: 9, 13, 15, 25, 37; CCSS: S-ID.7)**

Materials List: paper, pencil, graph paper, Wages vs. Hours Worked BLM

*Note: This activity has not changed because it already incorporates the CCSS listed.*

Provide students with copies of the Wages vs. Hours Worked BLM. Allow students to work in groups on this BLM. This activity is another opportunity to have students analyze a real-life situation, make a table of values that match the situation, and use the graph the data generated in the table. Students then create an equation to match the data and interpret information from the graph. Afterwards, students use the equation to help them answer other questions based on the situation. After students have had the opportunity to answer the questions from the BLM, fully discuss the problems that were presented as a class.

**Activity 8: The Stock Is Falling! (GLEs: 9, 13, 15, 25, 37; CCSS: S-ID.7)**

Materials List: paper, pencil, graph paper, graphing calculator, The Stock is Falling! BLM

*Note: This activity has not changed because it already incorporates the CCSS listed.*

Provide students copies of The Stock is Falling! BLM. Allow students to work in groups on the BLM. In this particular activity, students make the mathematical connection between a verbal situation, a table relating the situation and its graph, and the algebraic representation of the situation. Fully discuss the questions presented as a class. Follow up the activity by showing students how to input the equation they found into a graphing calculator and how to use it to determine the x- and y-intercepts and the value of the stock at 10 weeks to integrate technology into this activity.
Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will write a learning log (view literacy strategy descriptions) entry that explains how to determine slope, x-intercept, and y-intercept, given an equation.
- The student will write a letter to an absent classmate explaining the connection between slope and rate of change.
- The student will create portfolios containing samples of their activities.

Activity-Specific Assessments

- **Activity 1**: Provide the student with a graph of a line. The student will write the equation in slope-intercept form and then translate the equation into standard form.
- **Activities 2 and 3**: The student will write the equations and sketch the graphs when given a point and the slope of the line or when given two points on a line.
- **Activity 8**: The student will create a problem for a real-world context in which a decreasing linear situation exists. The student will make a graph and write an equation to represent the situation.
**Algebra I–Part 1**

**Unit 6: Inequalities and Absolute Values in One Variable**

**Time Frame:** Approximately three weeks

**Unit Description**

In this unit, an examination is made of the nature of solving linear inequalities in one variable and graphing their solutions on a number line. The unit also includes an introduction to absolute value equations and inequalities.

**Student Understandings**

Students recognize and distinguish between strict inequality (< and >) statements and relaxed inequality/equality (≤ and ≥) statements. Students solve linear inequalities in one variable and graph their solutions on the number line. Students graph simple absolute value inequality relationships on the number line.

**Guiding Questions**

1. Can students perform the symbolic manipulations needed to solve linear inequalities in a single variable and graph their solutions on the number line?
2. Can students interpret and graph simple absolute value equalities on the number line?
3. Can students relate absolute value inequalities in one variable to real-world settings (i.e., measurement, absolute value distances) and graph their solutions on the number line?

**Unit 6 Grade-Level Expectations (GLE) and Common Core State Standards (CCSS)**

<table>
<thead>
<tr>
<th>Grade-Level Expectation</th>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11.</td>
<td>Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)</td>
</tr>
<tr>
<td></td>
<td>14.</td>
<td>Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)</td>
</tr>
</tbody>
</table>
Sample Activities

Activity 1: Solving Inequalities with a Single Variable (GLEs: 11, 14; CCSS: WHST.9-10.2d)

Materials List: paper, pencil, math textbook

Students should have been introduced to solving inequalities and graphing the solutions in 7th grade. Use this activity to review the basic steps in solving an inequality and expressing the resulting graphs for the solution sets. Relate the solving of inequalities to its counterpart—solving equations in one variable. To visually represent the concept of an inequality (in one variable), on the board or overhead projector, draw an out-of-balance scale. Make the left side the lower side and write $4 - x$ on this side. On the other side, write $3$. Tell the students that the picture represents the inequality some number minus four is greater than negative three and write $x - 4 > 3$ above the scales. Relate how the solution to the inequality becomes $x > 1$ by adding four to both sides of the inequality (just as would be done if it were an equation being solved). Discuss how to graph such an inequality on a number line. Provide additional examples for students to model with the classroom including problems which involve flipping the inequality symbol when multiplying or dividing both sides by a negative. Use each of the inequality symbols ($<$, $>$, $\ge$, and $\le$) in the problems, and explain how each affects the way in which the graphical solution is expressed using parentheses, brackets, dots, and open holes, as appropriate. When students understand how to solve and graph inequalities with a single variable, provide additional practice using the math textbook as a resource. Pick one of the inequality problems that is assigned and have students respond to a SPAWN writing prompt (using the P or Problem Solving category) (view literacy strategy descriptions). The prompt is: Explain how you solved one of the inequality problems you solved from the textbook and
provide your reasoning for the steps you used to come to your solution. Use the SPAWN writing as a way to check student understanding of the concept as part of a formative assessment activity. Allow time for students to check their responses with a partner, then share their responses with the class. Provide overall clarification and feedback to students as needed to ensure understanding.

Activity 2: Real-life Inequalities (GLEs: 11, 14; CCSS: A-CED.1)

Materials List: paper, pencil, math textbook, Real-life Inequalities BLM

Note: This activity has not changed because it already incorporates the CCSS listed.

Present the following situation to students:
Problem: The math club raised $6800 to go on a trip to the math competition in Chicago. A travel agent charges a $300 fee to organize the trip and an additional charge of $600 per person for each student attending the competition.
Let students work in groups of three to find the following information:
- Write an inequality showing the relationship between the money that was raised and the cost for sending $x$ people on the trip, assuming the amount raised has to be more than the amount needed to go on the trip.
  Solution: $6800 > 300 + 600x$
- Solve the inequality to determine the maximum number of people that can go on the trip.
  Solution: $x < 10 5/6$; the maximum number of people that can go on the trip is $10$—more money would be needed to send $11$ people.

Allow students the opportunity to work in their groups on the problem, and then discuss the solution as a class. After doing this problem and discussing it thoroughly, provide students a copy of the Real-life Inequalities BLM. Allow students to work on these problems in their groups, and then go over the BLM as a class. Once students are comfortable with real-life inequality problems, provide additional work on this topic using the math textbook as a resource.

Activity 3: Absolute Value as Distance (GLE: 9)

Materials List: paper, pencil

To begin this activity, have students utilize the Think-Pair-Square-Share discussion (view literacy strategy descriptions) for the following question: “Why do you think the absolute value of a number is always positive?” After giving students the question, ask students to think alone about the problem for a short period of time, then have each student pair up with another student to share their thoughts. After they have had the opportunity for discussion, have pairs of students share with other pairs, forming small groups of four. Once this process has taken place, gather oral responses to the question for a full class discussion. The goal of discussion is to provide a deeper processing of content and rehearsal of newly learned content. It also creates the
opportunity to clarify misconceptions students may have about absolute value. Students should be led to understand that the absolute value of a number is really the distance the number is from zero on the number line. For example, write \( |x| = 3 \) on the board. Explain to students how to interpret the equation. In this particular example, the number or numbers, \( x \), which are a distance of 3 units from zero are those that should be marked on the number line. Have students find numbers on the number line that meet these criteria. Connect this solution mathematically with the following statements: \( |-3| = 3 \) and \( |3| = 3 \). When students see the absolute value symbol, they should immediately think of distance. Since distance is associated with a positive value, the absolute value of a number is positive. Provide additional examples in which students practice this concept. This concept of absolute value as distance is laying the groundwork to actually have students solve more complex absolute value equations and inequalities in the next few activities.

**Activity 4: Solving Absolute Value Equations (GLEs: 11)**

Materials List: paper, pencil, math textbook

The previous activity should have laid the foundation for what is next—solving an absolute value equation. Using what they learned about absolute value as a distance, relate this to solving the following absolute value equation: \( |x - 3| = 7 \). Conceptually, lead students to understand that there are actually two situations that need to be considered. One is when \( x - 3 = 7 \) (when the expression inside the absolute value is positive) and the other is when \( x - 3 = -7 \) (when the expression inside the absolute value is negative). Since both situations will result in solutions that provide a distance of 7, students should see that \( |x - 3| = 7 \) has two solutions for \( x \). In this particular problem, the two values of \( x \) that make the equation true are 10 and -4. These solutions are derived from solving each individual equation (i.e., \( x - 3 = 7 \) and \( x - 3 = -7 \)). Provide students with additional examples and practice from their math textbook, have students explain in words what each equation means geometrically in terms of distance on a number line, and solve the equations for the unknown variable.

**Activity 5: Absolute Value Inequalities I (GLE: 9, 11, 14; CCSS: WHST.9-10.10)**

Materials List: paper, pencil, math textbook

Lead the students through the following activity using whole-group instruction.

Begin this lesson by presenting the following problem: “What is the solution to the absolute value inequality \(|x| < 2|\)?” Next, write the following statement/prompt on the board: “The solution to this problem is all the numbers on the number line less than 2.” Utilize SQPL (view literacy strategy descriptions) to guide the lesson. SQPL (student questions for purposeful learning) is a strategy designed to focus students on the content that is about to be taught by first presenting students with a thought-provoking prompt and then allowing students to ask and answer their own questions about the content being presented. In this particular case, after presenting the problem/prompt, have students individually decide if the statement is true or false and why they agree or disagree with the statement. Next, have students pair up and brainstorm questions based
on the prompt, including reasons why they think the prompt is true or false. Elicit student questions and write them on the board. Prepare students for the presentation of the information by telling them to answer as many of the questions as they can in their pair-groups. Students should be directed to utilize the class discussion and teacher-led instruction for each SQPL question to answer these questions. Go down the list of student SQPL questions asked by students, then use these to guide the lesson. Some sample questions from students may be:

“What effect does an inequality have on the solution to an absolute value problem?”
“How is this problem related to distance on a number line and how is this different than a problem with an equal sign?”
“Why would only the numbers less than 2 be the solution? Isn’t this the same solution as if the problem were written as $x < 2$?”
“How is the problem $x < 2$ different from the problem $|x| < 2$?”

Next, take a poll of the students to see who agrees and who disagrees with the statement/solution for the prompt that was presented. Form one group of students who agree and another group of those who disagree. Ask students to discuss in their groups what they think this particular problem means in terms of distance, and then come up with a graphical solution to the problem through debating their thinking on the problem. When the groups are ready to talk about the problem as a class, have representatives of each group debate the issue. Use the classroom discussion and student questions/answers to drive the lesson. The teacher should act as a mediator for the discussion/debate and lead them toward the ultimate goal which is how to solve these types of inequality problems. For example, in this particular problem, students are asked to locate all points on the number line that are less than two units from zero. That is what $|x| < 2$ means. Students should ultimately realize that the solution presented is not correct. The true solution is all the numbers between -2 and +2. When written as an inequality, the solution is written as $-2 < x < 2$ or $(x > -2$ and $x < 2)$, and then graph the inequality that results on a number line. Writing an inequality this way may be new for students since it is a compound inequality. Provide additional problems of this type for students to solve. Include absolute value inequalities with greater than, less than or equal, and greater than or equal. When working with a distance greater than some amount, students should understand that “visually” this becomes a compound inequality. For example, to show all of the points which are more than 2 units from 0 on the number line this would be expressed algebraically as $|x| > 2$. Relating this graphically on a number line, students should see that to express this information, two inequality statements would have to be written: $x > 2$ or $x < -2$. This is critical to students’ understanding and solving absolute value inequalities of this type. In closing, ask students to look over their original SQPL questions from the beginning of the lesson and go through the answers. Check to be sure students have answered their SQPL questions accurately.

Use the math textbook as a resource for additional work on this topic.
Activity 6: Absolute Value Inequalities II (GLEs: 9, 11, 14)

Materials List: paper, pencil, math textbook

Extend student understanding of absolute value inequalities by considering expressions such as $|3x + 2| < 7$. Lead students to understand how this problem, like the absolute value equations solved earlier, has two cases to consider—when $3x + 2$ is less than positive 7 and, when it is greater than -7. Show this graphically on a number line. Students should recognize that it can be written in the same fashion as the earlier inequality (i.e., $-7 < 3x + 2 < 7$). Ask students to use previously learned rules to solve the complex inequality. Provide additional examples as are necessary for students to fully understand the skill of solving absolute value inequalities. Use the math textbook for additional work on this topic.

Activity 7: Real-life Absolute Values (GLEs: 11; CCSS: A-CED.1)

Materials List: paper, pencil, math textbook, Real-life Absolute Values BLM

Note: This activity has not changed because it already incorporates the CCSS listed.

Provide students with copies of Real-life Absolute Values BLM. Allow students to work in small groups on the BLM. Discuss the results as a class. If possible, provide additional work on this topic using the math textbook as a resource.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will use sample work from the activities to place in a portfolio that would showcase knowledge of inequalities.
- The student will describe the difference between an equation and an inequality in words and give an example of each using real-world examples.
- The student will write a letter to a classmate explaining what an inequality is and how to solve inequalities and graph them.
- The student will create absolute value inequality statements and share with the class on a math bulletin board.
Activity-Specific Assessments

- **Activity 1**: The student will solve and graph inequalities in one variable.

- **Activity 3**: The student will explain in words why there is no solution to the absolute value equation $|x| = -3$.
  
  *Answer: Since absolute value represents a distance, there are no numbers, $x$, that are a distance of -3 units from zero, since distance is always positive. Hence, there is no solution to this problem.*

- **Activity 5**: The student will explain (using distance on a number line) what the inequality $|x| \geq 4$ means and find its solution.
  
  *Answer: This inequality represents the values that are a distance greater than or equal to 4 from zero on the number line. The solution is $x \leq -4$ or $x \geq 4$.  

Algebra I–Part 1
Unit 7: Systems of Equations

**Time Frame:** Approximately five weeks

**Unit Description**

This unit examines the nature and mathematical procedures used to find and interpret solutions for real-life and abstract problems involving systems of equations.

**Student Understandings**

Students graph and interpret the solution of a system of two linear equations. Students relate the existence or non-existence of the solution of a system of equations to the slope of the two lines. They develop algorithmic ways of determining the solutions to a system of linear equations.

**Guiding Questions**

1. Can students explain the meaning of a solution to a system of two linear equations?
2. Can students determine the solution to a system of two linear equations by graphing, substitution, or elimination?
3. Can students relate the solution, or lack of solution, for a system of equations to the slopes of the lines in the system?
4. Can students identify coincident lines by their slopes and y-intercepts and relate this to the possibility of an infinite number of solutions?

**Unit 7 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)**

<table>
<thead>
<tr>
<th>Grade-Level Expectations</th>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
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<tbody>
<tr>
<td><strong>Number and Number Relations</strong></td>
<td>4.</td>
<td>Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)</td>
</tr>
<tr>
<td></td>
<td>11.</td>
<td>Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)</td>
</tr>
<tr>
<td></td>
<td>15.</td>
<td>Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)</td>
</tr>
</tbody>
</table>
16. Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)

**Geometry**

25. Explain slope as a representation of “rate of change” (G-3-H) (A-1-H)

**Patterns, Relations, and Functions**

37. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

38. Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)

39. Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

**CCSS for Mathematical Content**

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-CED.2</td>
<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
</tr>
</tbody>
</table>

**ELA CCSS**

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
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</thead>
<tbody>
<tr>
<td>WHST.9-10.10</td>
<td>Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.</td>
</tr>
</tbody>
</table>

**Sample Activities**

**Activity 1: When Will They Meet! (GLEs: 15, 16, 25, 37)**

Materials List: paper, pencil, graph paper, colored pencils, When Will They Meet? BLM

Using the When Will They Meet? BLM, students will interpret the graph of a distance/time relationship and answer questions based upon the analysis of the graph. They will also use the point of intersection for two lines to answer a real-world problem.
Activity 2:  Is There a Point of Intersection? (GLEs: 11, 16, 38, 39)

Materials List: paper, pencil, graphing calculators, Is There a Point of Intersection? BLM

In this activity, students use graphing calculators to explore whether two lines will intersect. Provide students with copies of Is There a Point of Intersection? BLM and graphing calculators to work in small groups on the BLM investigation. For this activity, students are only to determine whether or not there is a point of intersection, not to actually determine the point of intersection which will be done in the next activity. The purpose is to get students to analyze equations in slope-intercept form and determine which linear equations will intersect based upon their slopes and y-intercepts. Through the investigation, students should realize that there are three possibilities when two linear equations are graphed on the same coordinate grid:
1. Exactly 1 point of intersection if the slopes are different
2. No point of intersection if the slopes are the same but different y-intercept
3. Infinitely many points of intersection if the slope and y-intercept are identical

Activity 3:  Finding a Point of Intersection by Graphing Method (GLEs: 4, 16, 39)

Materials List: graph paper, pencil (two colors), graphing calculators, math textbook, ruler

In this activity students determine the actual point of intersection for the graphs of two lines using the graphing method. Before graphing, ask students to decide whether or not there should be a point of intersection based upon what they learned in Activity 1 by rewriting the equations in slope-intercept form and comparing slopes. For example, present the following two equations:
\[ x + y = 6 \] and \[ 3x - 4y = 4. \] By writing the two equations in slope-intercept form, students should see that the two equations have different slopes so there should be exactly one point of intersection.

Next, have students graph the two lines on the same coordinate graph using two different colored pencils and graph paper. Have the students determine “visually” where the point of intersection appears to be on the graph. Ask students if they are absolutely certain that this is the point where the two lines intersect, or if this is an approximate value. Point out the fact that using graphing to determine where two lines intersect has limitations as the actual point of intersection for the two lines is approximated.

Discuss with students that a point of intersection for two linear equations is a common solution to both equations (the point makes both equations true). To verify that a point is a solution to both equations, the \( x \) and \( y \) values for the coordinates can be substituted into both equations and should make both equations true.

Provide additional examples for students including examples in which there are no solutions and some with infinitely many solutions. After the examples are done by hand, demonstrate for students how to use the graphing calculators to determine the point of intersection by using the “trace,” “zoom,” and “intersect” features.

Provide additional work on this topic using the math textbook as a resource.
Activity 4: Using Substitution to Solve a System of Equations (GLE: 16)

Materials List: paper, pencil, math textbook

In this activity, extend the idea of finding a point of intersection to the mathematical language of solving a system of equations. Discuss this new terminology with students. Using the previous example in Activity 2, (e.g., \(x + y = 6\) and \(3x - 4y = 4\)), remind students that the point of intersection (4, 2) was found graphically. The drawback to using a graphical approach is that the exact point of intersection is not always certain, especially if the coordinates are not whole numbers. If this is the point of intersection, the \(x\)- and \(y\)-coordinate in each of the two equations can be replaced with \(x = 4\) and \(y = 2\), and they should make both equations true, which in this case they do. Therefore, the point (4, 2) is referred to as the “solution to the system.” No other values for \(x\) and \(y\) will make both equations true at the same time because this is the only point that the two equations have in common.

At the point of intersection for the lines, the \(x\) in one equation is equal to the \(x\) in the other equation; likewise for the \(y\) coordinates. This idea is the basis for the technique of solving systems by “substitution.” In the example problem, the equations in slope-intercept form are as follows: \(y = -x + 6\) and \(y = \frac{3}{4}x - 1\). Since the \(y\) values are equal at the point of intersection, by substitution the resulting equation is: 

\[-x + 6 = \frac{3}{4}x - 1\]

This can then be solved for the variable \(x\), giving the value of \(x = 4\), which is the \(x\) value for the point of intersection. The \(y\) value for the point of intersection, \(y = 2\), can be found by substituting the \(x\) value found into either of the original equations.

Provide additional examples for students to become proficient at the substitution method of solving systems of equations. Use the math textbook for additional work on this topic.

Activity 5: Solving Systems by Elimination (GLE: 16)

Materials List: paper, pencil, graphing calculators, math textbook

Demonstrate how to solve systems by using the elimination method. Point out that the elimination method is used when the two equations are put in standard form. In the elimination method, the goal is to eliminate one of the variables and solve for the variable that remains. Include examples where there is no solution (no point of intersection) and examples where there are infinitely many points of intersection. In these two special cases when elimination is used, both the \(x\) and \(y\) terms are eliminated concurrently. When this occurs, one of two things will happen. In one case, there will be an equation that makes sense, such as \(0 = 0\), which would indicate the two equations are exactly the same and thus have infinitely many points of intersection. In the second case, an equation will remain that does not make sense, such as \(0 = 8\). This would indicate the two equations have the same slope and different \(y\) intercepts and are parallel to one another thus having no point of intersection. This can be related to what was done in Activity 1 by rewriting the equations in slope-intercept form and comparing the slopes and \(y\)-intercepts for the two equations.
Provide additional systems for students to solve using the elimination method. Have students check their solutions by graphing the equations using graphing calculators. Provide additional practice using the math textbook as a resource in which students must determine which method, elimination or substitution, best fits the problem.

To complete work with this activity, have students participate in a version of professor know-it-all (view literacy strategy descriptions). In this particular use of the strategy, form groups of 3 students to come up to the board and act as the professor. Call on groups randomly to come to the front of the class to answer questions from other students. Students at their desks should ask questions related to the topics they have learned with solving systems of equations. The professors take turns answering the questions and aren’t allowed to sit until the class feels their question has been answered satisfactorily. Have different groups act as the “professors” to review content while the other members of the class ask questions. The teacher should oversee this and coach students when necessary to fill in any gaps or misunderstandings.

Activity 6: Break-even Point! (GLEs: 11, 16, 25, 37; CCSS: A-CED.2)

Materials List: paper, pencil, Starting a Business BLM, graph paper, ruler

Note: This activity has not changed because it already incorporates the CCSS listed.

Provide students with copies of Starting a Business BLM and allow students to work on the problem in small groups. Students may have problems when working through some of the questions, so assist groups if they need additional help. When completed, discuss the work as a class.

Activity 7: Pizza Parlor (GLEs: 16, 25, 37; CCSS: A-CED.2; WHST.9-10.10)

Materials List: paper, pencil, Pizza Parlor BLM, graph paper, ruler, colored pencils

Note: This activity has not changed because it already incorporates the CCSS listed.

In this activity, another opportunity is provided for students to explore the real-world meaning of a point of intersection for two lines for a given situation. Have students work on the Pizza Parlor BLM with their group members. As groups complete the BLM, have students exchange their work with another group. Utilizing the literacy strategy of questioning the content (QtC) (view literacy strategy descriptions), have students review the work that was done by that group. As they read through the work, they should ask the following questions and make comments and provide feedback to groups.

- Is the work on writing cost equations correct? If not, what recommendations can you make to correct what is wrong?
- Is the explanation clear and concise regarding the price that Mr. Moreau should charge? If is unclear, write a note to explain what is not clear.
When creating the graph, does the group use proper graphing techniques, labels, and appropriate scales? If not, what is incorrect?

Does the group understand what the point of intersection means in real-life terms and is the explanation clear? If not, what is incorrect?

QtC is a great way to let students evaluate one another’s work. After this process, groups should get their original BLMs and make any modifications needed to their answers or explanations. Discuss the results as a class.

During this activity, students should discover that the point of intersection for this particular graph tells that the cost and revenue are the same. At that point, the cost to make the pizzas is equal to the revenue that was made selling the pizzas. Before this point, the revenue is less than the cost. After this point, the cost is less than the revenue and a profit is made. Students need to understand the value of making a graph. In addition to being a visual representation of a problem, the graph provides useful information. This activity also uses many of the concepts learned thus far with graphing and writing equations.

Activity 8: Which is the Better Offer? (GLEs: 9, 16, 25, 37; CCSS: A-CED.2; WHST.9-10.10)

Materials List: pencil, graphing calculator, Which is the Better Offer? BLM

This activity has not changed because it already incorporates the CCSS listed.

Provide copies of Which is the Better Offer? BLM to students and have them work in pairs on the activity. The activity deals with renting a van and involves a real-life application of systems of equations. Discuss the BLM when students have completed the activity.

Next, have students get into groups of four. Have each group of students create a real-life problem whose solution would require solving a system of equations. Students should come up with a situation and several questions to accompany their problem. Each group should then create an answer key for its problem. Once each group has created a problem and the accompanying solution key, have each group exchange problems with one another. Each group then should work the other group’s problem that was created and provide feedback on the problem and questions that were created.

This activity is a modified form of the literacy strategy known as text chain (view literacy strategy descriptions). Text chain is one of the strategies used to reinforce content and gives students an opportunity to demonstrate their understanding of newly learned material. It is especially useful in promoting application of content through writing. In this case, students write text chains in the form of story problems on concepts being learned and then solve the problem. Typically, what is done is in a group of four, one student writes the first sentence of the story problem; the second student writes the second sentence of the story problem; the third student writes a question based on the first two sentences; and finally, the fourth student solves the problem and the other three check the work. In this particular use of the strategy, have each group create the problem and the questions to accompany the problem. Each group should work together to come up with the answers and then exchange the problems they created with another group.
group, whereby each group solves the problem presented by their peer group. Once completed, groups should return their work to the group that created the problem, where work is checked and feedback is provided. During this process, the teacher should monitor work and at the end of the lesson should pick the best questions and solutions and have groups present questions/solutions to class. An example of a text chain that a group might create is shown below:

Example: Jennifer has two choices for her cell phone plan. In plan A, there is a fixed fee of $25 and each additional minute costs 35 cents. In plan B, there is a fixed fee of $35 and each additional minute costs 15 cents. Determine two equations with two unknowns that would allow you to solve the system to determine the number of minutes Jennifer would have to talk in order for both plans to cost the same amount. Determine both the number of minutes as well as the cost associated with both plans.

Solution: $c = 25 + .35x$ and $c = 35 + .15x$ where $x$ is the number of minutes. Jennifer would have to talk 50 minutes and the cost would be $42.50

Activity 9: Extension Problem with Solving Systems (GLEs: 11, 16)

Materials List: pencil, paper

Present the following problem to students and have them work in small groups to come up with a solution. The problem is intended to extend the lesson on solving systems by providing a novel question for students to solve which utilizes the content that has been learned in this unit.

Problem: Suppose $3x + y = 19$ and $x + 3y = 1$. Find the value of $2x + 2y$.

Monitor student groups and guide student groups as needed. Have student groups share their solutions as well as the approaches they took to solve the problem. Look for students who may have solved the problem in alternate ways. One possible solution is to use elimination method and determine the values of $x$ and $y$ first ($x = 7$ and $y = -2$) and then use the values to determine the value of $2x + 2y$ (which would be 10).

Another way to solve the problem might be to add the two equations as shown below:

\[
\begin{align*}
3x + y &= 19 \\
-x + 3y &= 1 \\
4x + 4y &= 20
\end{align*}
\]

Dividing both sides of the equation by 3, results in $2x + 2y = 10$ (the same solution found using the other approach).

Once all approaches have been thoroughly discussed as a class, clear up any remaining misconceptions that students may have about the content.
Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The students will take paper and pencil tests on the skills associated with this unit.
- The student will write a short paragraph explaining his/her algorithm for determining the number of solutions, given the equations for two lines.
- The student will explain, in writing and using a graph, what it means for a certain point to be a solution to a set of equations.

Activity-Specific Assessments

- **Activity 2**: Provide students with pairs of equations. Without the aid of graphing calculators, students will determine whether there will be one, none, or infinitely many points of intersection based upon the slope and intercept of each equation.

- **Activity 3**: The student will graph two linear equations using paper and pencil and use the graph to approximate the point of intersection.

- **Activity 4**: The student will determine the exact point of intersection for two linear equations using the elimination method.

- **Activity 5**: The student will create three problems and solve them using elimination which show one solution, no solution, and an infinite number of points of intersection.

- **Activity 6**: The student will create cost and revenue equations for as real-world problem, and use the equations to determine the break-even point for the particular situation.
Algebra I–Part 1

Unit 8: Solving Systems Using Matrices and Systems of Inequalities

Time Frame: Approximately four weeks

Unit Description

In this unit, the solving of systems of equations is extended to include the use of matrices. The unit also presents matrices and shows how they can be utilized in real-life applications, and includes solving systems of inequalities.

Student Understandings

Students develop the concept of a matrix and matrix operations of addition and multiplication and the relationship between solving matrix equations $ax = b$ and $Ax = B$ as $x = a^{-1}b$ and $x = A^{-1}B$ respectively. They apply matrices to the solution and interpretation of a system of two or three linear equations. They also graph and interpret systems of inequalities.

Guiding Questions

1. Can students explain what a matrix is and how it is applied in real-life situations?
2. Can students perform operations with matrices (addition, subtraction, multiplication)?
3. Can students determine the solution to a system of two or three linear equations with two or three variables by matrix methods?
4. Can students use matrices and matrix methods by hand and calculator to solve systems of equations $Ax = B$ as $x = A^{-1}B$?
5. Can students explain the meaning of a solution to a system of inequalities (in both abstract and in real-life problems)?
6. Can students solve, graph and interpret linear inequalities in two variables?

Unit 8 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
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<th>Grade-Level Expectations</th>
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<td><strong>Algebra</strong></td>
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### Grade-Level Expectations

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<td>14.</td>
<td>Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)</td>
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<td>16.</td>
<td>Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)</td>
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### CCSS for Mathematical Content

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<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
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### ELA CCSS

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<td>Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.</td>
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### Sample Activities

**Activity 1: Matrices—an Introduction (GLEs: 5, 16; CCSS: RST.9-10.4)**

Materials List: paper, pencil, graphing calculator, Vocabulary Self-Awareness for Matrices BLM, Movie Cinema Matrix BLM

Begin the activity by administering the Vocabulary Self-Awareness for Matrices BLM whereby students analyze what they know about matrices. The goal of *vocabulary self-awareness* ([view literacy strategy descriptions](#)) is to help bring all students to a comfortable level with the vocabulary being used. In this particular use of the strategy, terms and processes involved with matrices are introduced at the beginning of a unit, in this case using the BLM. This is a variation of the typical use of the *vocabulary self-awareness* strategy whereby only vocabulary is assessed. In this modification of the strategy, both vocabulary and processes are being assessed to determine the students’ comfort level with the material that will be taught in the activity.

Have students complete the Vocabulary Self-Awareness for Matrices BLM and use the results to target the vocabulary which students feel least comfortable with to help guide instruction throughout this entire unit. After students have completed the BLM, continue with the activities.
presented. At the end of the unit, have students go back through the Vocabulary Self-Awareness for Matrices BLM and have them assess their comfort level with the topics again. The goal is to have all students replace any O or – marks with + marks to indicate their understanding of what was taught on matrices.

Discuss that a matrix is a rectangular array of numbers. Provide students with copies of the Movie Cinema Matrix BLM which displays the items sold at different times at a movie cinema on a Monday. Discuss with students how the chart can be written as a rectangular array and enclosed with brackets or parentheses. The enclosed array is called a matrix. The advantage of writing the numbers as a matrix is that the entire array can be treated as a single mathematical entity. A matrix can be named with a single capital letter as shown.

The numbers that make up a matrix are called the entries, or elements, of the matrix. The entries of matrix M are all numbers, but the matrix itself is not a number, just as a multiplication table is not a number. Various operations can be used on matrices (addition, subtraction, multiplication) and will be discussed in the other activities.

A matrix is often classified by its order or dimension; that is, by the number of rows and columns that it contains. For example, matrix M has 4 rows (rows run across horizontally) and 3 columns (columns run up and down vertically). This means that matrix M is a $4 \times 3$ matrix. When a matrix has the same number of rows as columns, it is called a square matrix. Relate the rows and columns of the chart with the matrix. Introduce students to the matrix function on the calculator, and show them how to enter the data in this BLM into a matrix.

**Activity 2: Adding Matrices (GLEs: 5, 16)**

Materials List: paper, pencil, graphing calculator

Provide students with several pairs of matrices that can be combined by addition. An example is provided below. Using matrix M, from the previous activity which shows the sales on a Monday at a movie cinema, and matrix T, which displays the same items sold on a Tuesday, have students find $M + T$ and describe what this new matrix describes.

$$M = \begin{pmatrix} 4 & 20 & 25 \\ 8 & 24 & 18 \\ 10 & 34 & 28 \\ 34 & 38 & 55 \end{pmatrix} \quad \quad T = \begin{pmatrix} 6 & 15 & 24 \\ 5 & 8 & 22 \\ 8 & 25 & 15 \\ 13 & 22 & 16 \end{pmatrix}$$
Students should see that the new matrix, \( M + T \), really describes the total sales of items at the concession stand at the movie cinema for Monday and Tuesday. To add two matrices such as this, the only things that can be added are “like terms.” In this case, recall that snacks are in the first column, and the movie times are given in each row. Because the two matrices contain the same types of information at the intersection of each row and column, the corresponding values in matrix \( M \) and matrix \( T \) are alike and can be added. Adding the numbers from row 1, column 1 in each matrix gives \( 6 + 4 = 10 \), which means that the total value of the snacks sold at the 1:00 show on Monday and Tuesday is $10. The complete new matrix that results from the addition of \( M + T \) is shown. Discuss with students the results of the new matrix and how to interpret its meaning.

Have students find \( M - T \) and have them explain in real-world terms what this new matrix represents (the difference between Monday and Tuesday sales). Follow this by showing students how this can be done using the graphing calculator. Provide additional problems of this type for students to become proficient at this skill.

**Activity 3: Multiplying a Matrix by a Scalar (GLEs: 5, 16)**

Materials List: paper, pencil, graphing calculator

In this activity, the work with matrices is extended to include multiplying a matrix by a scalar. Continuing the work from the movie theater example, present the following situation to students:

> How can a matrix be used to determine \( \frac{1}{2} \) the sales of the concession stand on Monday at the cinema?

Discuss with students how each sale would then be “halved.” This would result in a new matrix to display the results. Have students get into small groups to brainstorm (view literacy strategy descriptions) how to find \( \frac{1}{2} M \) and determine what the results would mean in real-world terms. 

**Brainstorming** involves students working together to generate ideas quickly without stopping to judge their worth. In brainstorming, students in pairs or groups freely exchange ideas in response to an open-ended question, statement, problem, or other prompt. Students try to generate as many ideas as possible, often building on a comment or idea from another participant. This supports creativity and leads to expanded possibilities. The process activates the students’ relevant prior knowledge, allows them to benefit from the knowledge and experience of others, and creates an anticipatory mental set for new learning. Once students have had the opportunity to brainstorm...
their ideas about the problem presented, have a class discussion on the ideas students came up with. Compare their ideas with the process which follows. The teacher should act as a guide/facilitator to ultimately lead students to understand the main objective of this activity which is how to multiply a matrix by a scalar.

Therefore, after determining students’ thoughts and possible solutions to the problem presented, guide students to see this problem in terms of operations with matrices. Explain that determining “what is \( \frac{1}{2} \) the sales of the concession on Monday” requires multiplying a matrix by a scalar. In this case, all elements in the original matrix, M, are multiplied by the scale factor of \( \frac{1}{2} \). This creates a new matrix as follows.

\[
\begin{array}{ccc}
4 & 20 & 25 \\
8 & 24 & 18 \\
10 & 34 & 28 \\
34 & 38 & 55 \\
\end{array}
\]

\[
\begin{array}{ccc}
2 & 10 & 12.50 \\
4 & 12 & 9 \\
5 & 17 & 14 \\
17 & 19 & 27.50 \\
\end{array}
\]

Be sure to connect this idea with the students’ original ideas which were found in the brainstorming process. Show students how to multiply a matrix by a scalar using the graphing calculator, and then provide additional practice for students on this skill.

**Activity 4: Multiplying Two Matrices (GLEs: 5, 16)**

**Materials List: paper, pencil, graphing calculator**

Multiplication of two matrices is not defined so straightforwardly as addition, subtraction, and multiplication of matrices by a scalar. The process is much more complex.

Write on the board the two matrices, P and Q as shown below. Both are 2 by 2 matrices. Demonstrate how to do the matrix operation of multiplication to find the product. Explain that matrix multiplication is a “row times column” process where the entry of a row in the first matrix multiplies the corresponding entry in the column of the second matrix. The sums of the products are found until the entries of a row are all used. The process is repeated for the next row until all elements are accounted for.

For example, look at matrix P and Q shown below.

\[
P = \begin{pmatrix}
10 & 15 \\
15 & 20
\end{pmatrix}, \quad Q = \begin{pmatrix}
8 & 6 \\
10 & 5
\end{pmatrix}, \quad PQ = \begin{pmatrix}
230 & 135 \\
320 & 190
\end{pmatrix}
\]
The process begins by multiplying each entry in the first row of P by the corresponding entry in the column of Q. The two products are then added together to form the first element in the new matrix called PQ. In this example, in row 1 of matrix P multiply the number 10 by the first entry in column 1 matrix Q, which is 8. This product is 80. Add to this the product of the second element in row 1 of matrix P, which is 15, by the second entry in column 1 of Q, which is 10, giving a product of 150. When the two individual products are added, the result is $80 + 150$ for a sum of 230. Thus, 230 is the first entry in the new matrix PQ. The complete new matrix PQ is shown. Discuss with students how the entire matrix was formed.

Repeat this activity reversing the matrices to show students that matrix multiplication is not a commutative operation. Instead of P times Q, multiply Q times P, and prove to students that the resulting matrix, QP, is different than PQ. Matrix QP is shown below.

$$QP = \begin{pmatrix} 170 & 240 \\ 175 & 250 \end{pmatrix}$$

Provide additional opportunities for students to gain experience at this skill. Then show students how this can be done easily using graphing calculator technology.

**Activity 5: The Inverse Matrix (GLEs: 5, 11, 16)**

Materials List: paper, pencil, graphing calculators

In Activity 4, students were introduced to the concept of multiplying one matrix with another. Students discovered that the order in which matrices are multiplied results in different answers (unlike multiplication of real numbers). In this activity, the idea of an inverse matrix is introduced.

The inverse matrix of matrix A, symbolized by $A^{-1}$, is the matrix that will produce an identity matrix when multiplied by A. In other words: $AA^{-1} = I$ and $A^{-1}A = I$. The identity matrix, symbolized by I, is one of a set of matrices that do not alter or transform the elements of any matrix A under multiplication, such that $AI = A$ and $IA = A$. Show students that the identity matrix for any 2 x 2 matrix is as follows:

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Demonstrate, using paper and pencil method as well as the graphing calculator, that any 2 x 2 matrix, when multiplied by this identity matrix, results in the original matrix. Connect this idea with that of multiplying any real number by 1 which will result in the original number.

Next, show students how solving a matrix equation is related to solving linear equations. For example, if $ax = b$, to solve for $x$ we can multiply both sides of the equation by $\frac{1}{a}$, which is the
multiplicative inverse of \( a \). \( \frac{1}{a} \) can also be written as \( a^{-1} \). Show students the inverse key on a calculator, and relate finding the multiplicative inverse of 5 as \( \frac{1}{5} \) or 0.2. Use several examples to demonstrate this to students. To solve the equation \( ax = b \), use \( ax = ba^{-1} \), and since \( aa^{-1} = 1 \), \( x = ba^{-1} \).

Just as normal equations can be solved using this approach, so too can matrix equations. For example, suppose \( AX = B \), where \( A \), \( B \), and \( X \) are all matrices. To find \( X \) when \( A \) and \( B \) are known, multiply both sides of the matrix equation by the inverse of \( A \), or \( A^{-1} \). Thus: \( A^{-1}AX = A^{-1}B \) which when solved for \( X \) gives us \( X = A^{-1}B \).

Provide the following example to students.

\[
A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}
\]

Show students how to use a graphing calculator to find \( A^{-1} \).

\[
A^{-1} = \begin{pmatrix} 0.8 & -0.6 \\ -0.2 & 0.4 \end{pmatrix}
\]

Have students prove that \( AA^{-1} = I \) (Identity Matrix) by multiplying each matrix by the other using paper and pencil as well as a calculator. Provide additional 2x2 matrices for students to determine their inverse using the graphing calculator. This skill will be utilized when solving systems using matrices in the next two activities.

**Activity 6: Solution of Two Equations: Found Five Ways (GLEs: 5, 11, 16)**

Materials List: graph paper, pencil, graphing calculators, ruler, paper

Write the following pairs of equations on the board: \( 2x + 3y = 5 \) and \( 2x + 4y = 9 \). Have students solve the system of equations first by graphing using graph paper, then by graphing on a graphing calculator using the intersect function. Next, have students solve the problems using the substitution and elimination methods.

Finally, show students how the two equations can be solved using matrices. Discuss how to write the two equations into matrices as follows:

\[
\text{Let } A = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 9 \end{pmatrix}
\]
Point out to students that matrix A comes from the coefficients associated with the \(x\) and \(y\) terms from the two equations. Matrix X contains the values that need to be determined. Matrix B is created from the constants (5 and 9) from the two linear equations.

The idea here is to find \(x\) and \(y\), which are the values for matrix X. Since \(AX = B\), to solve for \(X\) take the inverse matrix of \(A\) or \(A^{-1}\) of both sides; therefore, \(X = A^{-1}B\). Show students how this can be done using the calculator. Verify that the point of intersection is the same regardless of the method. The point of intersection for the two lines is \((-3.5, 4)\). Provide additional examples for students to become proficient in, using matrices to solve such equations. Repeat this activity using systems that have no solutions, one solution, or an infinite number of solutions.

**Activity 7: Solving Three Equations with Three Unknowns Using Matrices (GLEs: 5, 11, 16; CCSS: A-CED.2; RST.9-10.4)**

Materials List: paper, pencil, graphing calculator, math textbook, Solving Systems Using Matrices BLM

This activity has not changed because it already incorporates the CCSS listed.

Provide students with copies of the Solving Systems Using Matrices BLM. Have students work in groups on the activity. When students have had an opportunity to work through the activity, discuss fully as a class. Since the goal of the activity is to have students extend their knowledge of matrices, this activity may challenge the students. Another option would be to do the BLM with the class as a teacher-led activity. Discuss with students how matrices can be used to solve systems with 2, 3, or even more unknowns utilizing the procedures they have learned. For additional practice on this skill, find problems which have three unknowns for students to solve using matrices. Use the math textbook as a resource.

At the end of this activity, remember to have students re-assess their comfort level using the Vocabulary Self-Awareness for Matrices BLM from Activity 1. Discuss what the students have learned during the course of this unit and what they still don’t feel comfortable with. Then use the results from the vocabulary self-awareness (view literacy strategy descriptions) activity to guide any additional instruction.

**Activity 8: Linear Inequalities (GLE: 14; CCSS: WHST.9-10.10)**

Materials List: graph paper, pencil, math textbook, graphing calculators (optional), ruler

In this activity, students compare graphing a linear equation with a linear inequality. In a linear equation, points that form the actual line are solutions to the linear equation. In a linear inequality, an area associated with all the coordinate points that make the inequality true is shaded and the line itself is a boundary for the region shaded.
Begin the activity by having students graph the equation $3x + 4y = 12$. Next, discuss the difference and the similarity between this linear equation and the following linear inequalities:

- $3x + 4y > 12$
- $3x + 4y \geq 12$
- $3x + 4y < 12$
- $3x + 4y \leq 12$

Discuss how each graph differs—whether the line is dashed (if $>$ or $<$) or solid (if $\geq$ or $\leq$), and where the shading is located (to the right or to the left of the line). To aid in helping students understand which side of the line gets shaded, have students pick points and replace their coordinates in the inequality to see if they make the inequality true. If the particular points make the inequality true, then that is the side of the line that gets shaded.

Provide additional examples, including real-life problems that utilize this skill, using the math textbook for a resource to help students become proficient at this skill. Include vertical and horizontal lines and inequalities such as $x > 2$ or $y < 5$. Also show students how a graphing calculator can be used to graph inequalities at this time.

Afterwards, present the following inequality for students to consider: $2x + 3y \geq 500$. Have students get in small groups and come up with a text chain (view literacy strategy descriptions). In this particular activity, a modified form of the text chain strategy will be employed. Typically, what is done is in a group of four, one student writes the first sentence of the story problem; the second student writes the second sentence of the story problem; the third student writes a question based on the first two sentences; and finally, the fourth student solves the problem and the other three check the work. In this particular use of the strategy, have each group create a real-life problem situation that could be modeled by this inequality statement, along with the question and graph to accompany the problem. Once completed, have each group share their real-life scenario to the class along with the question and graph they created. The other students should critique whether the problem is a feasible one that fits the inequality as well as whether the graph that was created as the solution is correct. Guide the discussion as well as monitor the accuracy of the student responses to the problems presented.

An example of a possible real-life scenario to the problem generated by a text chain group would be as follows:

- **Student 1**: Jan is selling hats and earrings she made in a craft class to make money to go on a trip. She charges $2 per hat and $3 for each pair of earrings. **Student 2**: She needs to make at least $500 to have enough money for her trip. **Student 3**: What are all the solutions (hats and earrings) that would allow Jan to meet these requirements? **Graph the solution.**
- **Student 4**: Solves the problem and graphs the solution.
Activity 9: System of Inequalities (GLE: 14)

Materials List: graph paper, two different colored markers, math textbook, ruler

Discuss with students how to solve a system of inequalities by graphing both inequalities on a single coordinate grid and looking at the intersection of areas as the solution to the system. For example, have students graph the two inequalities shown below:

\[
y > 2x - 5 \\
3x + 4y < 12
\]

Using two different color markers, have students shade in both inequalities and discuss the intersection of the two areas as being the solution to the system of inequalities. Point out that in order to be a solution, the coordinates must satisfy both inequalities. Provide students with additional practice on this skill using the math textbook as a resource.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will research and write a one-page report on the history of matrices.
- The student will show proficiency in operations with matrices using paper and pencil tests.
- The student will use matrices to solve systems of equations with two and three unknowns.

Activity-Specific Assessments

- **Activity 2**: Provide the student with real-life data similar to the data shown in the activity, and then have the student create two matrices for the data. Once the matrices have been created, the student will find the sum of the two matrices and interpret its meaning.

- **Activity 3**: Provide the student with two 2 x 2 matrices to multiply by hand. Afterwards, the student will use technology to check to see if the answers are correct.
• **Activity 5:** Provide the student with pairs of matrices to determine if they are inverse matrices of one another using paper and pencil as well as using graphing calculator technology.

• **Activity 6:** Have students find the solution to a system of equations in two variables using five different methods.