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2012 Louisiana Transitional Comprehensive Curriculum

Course Introduction

The Louisiana Department of Education issued the first version of the Comprehensive Curriculum in 2005. The 2012 Louisiana Transitional Comprehensive Curriculum is aligned with Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS) as outlined in the 2012-13 and 2013-14 Curriculum and Assessment Summaries posted at http://www.louisianaschools.net/topics/gle.html. The Louisiana Transitional Comprehensive Curriculum is designed to assist with the transition from using GLEs to full implementation of the CCSS beginning the school year 2014-15.

Organizational Structure

The curriculum is organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. Unless otherwise indicated, activities in the curriculum are to be taught in 2012-13 and continued through 2013-14. Activities labeled as 2013-14 align with new CCSS content that are to be implemented in 2013-14 and may be skipped in 2012-13 without interrupting the flow or sequence of the activities within a unit. New CCSS to be implemented in 2014-15 are not included in activities in this document.

Implementation of Activities in the Classroom

Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Transitional Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the CCSS associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

Features

Content Area Literacy Strategies are an integral part of approximately one-third of the activities. Strategy names are italicized. The link (view literacy strategy descriptions) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at http://www.louisianaschools.net/lde/uploads/11056.doc.

Underlined standard numbers on the title line of an activity indicate that the content of the standards is a focus in the activity. Other standards listed are included, but not the primary content emphasis.

A Materials List is provided for each activity and Blackline Masters (BLMs) are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for the course.

The Access Guide to the Comprehensive Curriculum is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities. This guide is currently being updated to align with the CCSS. Click on the Access Guide icon found on the first page of each unit or access the guide directly at http://sda.doe.louisiana.gov/AccessGuide.
Grade 7 Mathematics
Unit 1: Rational Number Relationships

Time Frame: Approximately 4 weeks

Unit Description

The focus of this unit is connecting and extending the relationships of fractions, decimals, integers and percents to enable deeper understanding and flexibility in thinking. Conceptual understanding of proportionality is developed.

Student Understandings

Students demonstrate their grasp of fraction, decimal, percent, and integer representations and operational understandings by comparing, ordering, contrasting, and connecting these numbers to real-life settings and solving problems. They demonstrate an understanding of reasonableness of answers by comparing them to estimates. Students can distinguish between unit rates and ratios and recognize quantities that are related proportionally.

Guiding Questions

1. Can students represent in equivalent forms and evaluate fractions, ratios, percents, decimals and integers?
2. Can students connect fractions, ratios, decimals, and integers to real-life applications?
3. Can students use proportional relationships to solve multistep ratio and percent problems in the context of real-life applications?
4. Can students demonstrate that the decimal form of a rational number terminates in 0s or eventually repeats?
5. Can students demonstrate the equality of ratios in a proportion?
6. Can students illustrate the reasonableness of answers to such problems?

Unit 1 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>Grade-Level Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Number Relations</strong></td>
</tr>
<tr>
<td><strong>GLE #</strong></td>
</tr>
<tr>
<td>1.</td>
</tr>
</tbody>
</table>
2. Compare positive fractions, decimals, percents, and integers using symbols (i.e., <, ≤, =, ≥, >) and position on a number line (N-2-M)

7. Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)

8. Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)

10. Determine and apply rates and ratios (N-8-M)

11. Use proportions involving whole numbers to solve real-life problems. (N-8-M)

CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.NS.2</td>
<td>Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.</td>
</tr>
</tbody>
</table>

Sample Activities

**Activity 1: Decimal Comparisons - Where’s the Best Place? (GLE: 2; CCSS: 7.NS.2)**

Materials List: *Where’s the Best Place BLM, Numbers BLM, learning log*

Students will convert a rational number to a decimal and write inequalities with them.

Students play a game called Where’s the Best Place? Students have four chances to create a fraction by taking turns drawing cards from a pile. Review division of fractions to create a decimal. Also, review symbols used to compare numbers (>, <, ≥, ≤, = ). Place the students in groups of four.

Rules for the game:

- Give each player a copy of the *Where’s the Best Place BLM* to play the game.
- Have students shuffle ten cards numbered 0 through 9 and place them face down in a pile. (Use the Numbers BLM to make the cards or use the cards 2-9 and an ace from a deck of playing cards.)
- One player draws a digit card from the pile. Each player must decide privately whether he/she wants to use that digit as the numerator, denominator or to discard it. The object is to try to create the largest number. **A player can discard only two cards.**
- After the player writes a digit on the game card, he/she cannot erase it and place it elsewhere. Once a digit is drawn, it cannot be used again in that game.
Next, the second player draws another digit card from the pile. Each player must use this card to fill the place for either the numerator or denominator. A player may also choose to discard. Note: If all players have used the cards drawn by the first two players, the game proceeds to the next step. Otherwise, players 3 and 4 continue to draw until all places are filled.

The game is over when all places on each game card are filled. Each student will use division to convert their fraction to a decimal. The teacher should look for decimal conversions that terminate in 0s or eventually repeat and discuss with students [CCSS #7.NS.2 (d)] The player with the greatest number wins.

Write an inequality using the rational numbers generated by the group. Depending on the needs of students, the teacher may choose whether the students will write the inequality with the fractions or the decimals.

Example for one player:
Draw #1—“4” is placed as the numerator.
Draw #2—“7” is discarded.
Draw #3—“6” is discarded (Two discards have been used, so the player is forced to play the next card.)
Draw #4—“3” is placed as the denominator.

Student divides \( \frac{4}{3} \) and gets 1.3. He/she compares to the other players to determine who has the largest number.

Students should respond to the following prompt in their learning logs (view literacy strategy descriptions). This learning log should be a small notebook used primarily for recording math understanding. Explain to the students that their learning logs will be used all year to record new learning and write questions that they want answered through math class. Have students copy the prompt. For longer prompts, the teacher should copy the prompt and have students tape, glue, or staple into the learning log.

Prompt 1:
Some of the digits in the following numbers are hidden.

A. 3.\_\_\_\_\_\_  B. 3.\_\_\_\_\_\_

Give an example when each situation is true. Using mathematics, justify your answers.
1. the value of A is larger than the value of B
2. the value of B is larger than the value of A
3. the value of A is equal to the value of B
Activity 2: Fraction Comparisons (GLEs: 1, 2)

Materials List: several pieces of chart paper for every pair of students, Fraction Comparisons BLM for each pair of students

Students will write equivalent and nonequivalent fractions as well as inequalities to compare them.

To check the depth of understanding students have in dealing with equivalent fractions, have the students complete the Fraction Comparisons BLM while working with a partner. Circulate around the room and ask questions to find what strategies the students are using to find equivalent fractions.

A copy of the information provided on the BLM is reprinted below.

1. Using chart paper, complete the following situation. Be prepared to share your work in 20 minutes.
   a. Write two fractions that are equivalent. Explain how you know that they are equivalent.
   b. Look at the fractions you wrote in Part A. Write two other fractions, one that is equivalent to your first fraction and one that is equivalent to the second fraction.
   c. Are the four fractions you have written equivalent to each other? Why or why not?

2. Using chart paper, complete the following situation. Be prepared to share your work in 20 minutes.
   a. Write two fractions that are not equivalent. Tell which is larger, and explain how you know.
   b. Look at the fraction you wrote in Part A. Write two other fractions, one that is not equivalent to your first fraction and another one that is not equivalent to your second fraction.
   c. Order the four fractions you have written from smallest to largest, and explain how you know the order is correct.
   d. Write a mathematical statement using the symbols <, ≤, =, ≥, > and your fractions.

Activity 3: Number Line Placement (GLEs: 1, 2)

Materials List: Velcro® strip, masking tape, or string for number line; a set of rational number index cards

Use this activity as a pre-assessment activity to get an idea of the students’ level of understanding of number sense.
Use a Velcro® strip, masking tape, or string taped along the board to represent a number line. Place zero and one on the number line. Have students compare and determine the placement of rational numbers. Have numbers written on index cards for the students to use. (Examples of numbers: 1, \( \frac{1}{2} \), 100%, .08, \( \frac{1}{3} \), 75%, 0) Make sure to include several numbers which are equivalent—fractions, decimals and percents. Do not use negative numbers at this time. Give a card to a student. Have him/her place the card where he/she thinks it belongs on the number line using masking tape or a Velcro® strip. Have a discussion about the placement of this number (e.g., Must it go there? Could it be placed elsewhere?). Give another card for placement to another student. Continue until all numbers have been placed along the number line. There are many questions that can be asked with the placement of each number creating in-depth class discussions. Students may need to move some numbers on the number line once one or two numbers have been placed. Have students make observations about the number line and write 5 inequalities from the number line using the symbols <, ≤, =, ≥, >.

**Activity 4: Representation of Equivalent Fractions, Decimals, and Percents (GLE: 1)**

Materials List: Fraction Pieces BLMs (eight BLMs) for each student, scissors

Give each student a copy of each of the eight Fraction Pieces BLMs. Each BLM should be copied on a different color of paper. Each sheet will have a rectangle divided into equal portions by parallel lines. The rectangle on Fraction Pieces 2 BLM is divided into halves. The rectangle on Fraction Pieces 3 BLM is divided into fourths, etc. Model labeling and cutting strips from the paper using colored overhead sheets (i.e., cut along the parallel lines and then at the marking for \( \frac{1}{2} \)). Show students how to represent each fraction, decimal, and percent with a different colored paper. A red strip of paper is cut into 2 pieces, and each piece is labeled \( \frac{1}{2} \), 0.50, and 50%. A blue strip of paper is cut into 4 pieces, and each piece is labeled \( \frac{1}{4} \), 0.25, and 25%, etc. On the overhead, show the placement of equivalent fractions of two different colors (e.g., 1 red piece is equivalent to 2 blue pieces, shown side by side on the overhead). Lead a discussion which includes equivalencies using decimals and percents. Have students work in groups of 4, cut their papers into pieces as modeled, and develop a presentation showing the maximum number of equivalent fractions.

**Teacher Notes:**
1. Caution, if one graphic is resized, all graphics will need to be resized proportionally. Always copy the original. Copy and compare the sizes of each strip before duplicating in mass to give to students. Many photocopiers do not make exact duplicates of an image and the more copies that are made, the more variance there could be. The heat expands the paper. The last page copied may be more distorted than the first if the machine is hot.
2. Fraction pieces are also used in Unit 1, Activity 5 and Unit 2, Activity 1. Have each group store the pieces in a gallon baggie to for future use.

Activity 5: Compare Fractions, Decimals, and Percents (GLE: 2)

Materials List: fraction strips from Activity 4, 6 index cards for each student (two fractions, two decimals, and 2 percents); Greater Than, Less Than, or Equal To BLM for each student or pair of students.

Review the concepts of equal, greater than, and less than with students who will work in groups of 2. Using the colored overhead strips from Activity 4, demonstrate the concepts of comparing numbers using the terms greater than, less than, greater than or equal to, less than or equal to, and equal.

Give 6 index cards that include two fractions, two decimals, and two percents to each student. Have students form pairs and instruct each student to randomly select an index card. The pair should write an equality using the numbers on the cards drawn and be able to defend their reasoning to others.

Have each pair of students complete the modified math word grid (view literacy strategy descriptions) found on the Greater Than, Less Than, or Equal To BLM to show their understanding of greater than, less than, greater than or equal to, less than or equal to, and equal. A word grid is a visual technique for helping students learn important related terms and concepts. It provides students with an organized framework for learning mathematical concepts by analyzing the similarities and differences of key ideas. (Students should use the values given to them earlier to fill in the left column. Students may also create new fractions by rolling a number cube.) Once the grid is complete, instruct groups to switch inequalities with another group and check each other’s work. While monitoring each group, make anecdotal notes regarding students requiring additional practice.

<table>
<thead>
<tr>
<th></th>
<th>Greater Than, Less Than, or Equal To</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; ½</td>
</tr>
<tr>
<td>¼</td>
<td>☑</td>
</tr>
<tr>
<td>50%</td>
<td>☑</td>
</tr>
</tbody>
</table>

Place a check in any cell to indicate which statements are true when the number in the first column is combined with the information in the top row.
Activity 6: Equivalent Fractions, Decimals, and Percents (GLEs: 1, 2)

Materials List: at least 30 index cards per 4 students

Have groups of four students create a deck of cards using index cards. Cards should represent common fractions such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{3}, \frac{2}{5}$, etc. and their decimal and percent equivalencies. Example: one card will have 0.5, a second card will have $\frac{1}{2}$, and the third card 50%. The three equivalent cards represent a set. Each deck of cards should contain 10 complete sets. A game is played in which five cards are dealt to each player and the rest are laid down for a draw. Use the rules for a Go Fish game. When a student draws a card, he/she asks, “Do you have anything equal to ____?” (e.g., Do you have anything equal to $\frac{1}{2}$? Do you have anything equal to 20%). The students lay down cards when they have all three cards which comprise a set. The first student to use all of his/her cards wins.

Using the cards created, reinforce the concept of greater than, less than, and equal to, greater than or equal to and less than or equal to. Create cards for each of these symbols. Divide students into teams to play a spelling bee type game in which two cards are drawn from the deck of fractions, decimals, and percents. The team drawing the cards has one minute to choose which symbol is appropriate and explain why they chose the inequality symbol. If they cannot, the other team is given a chance. Scoring is one point per correct answer.

Activity 7: Is it Reasonable? (GLEs: 7, 8)

Materials List: teacher-made set of real-life problems involving positive fractions and decimals, paper, pencil, math learning log

Provide students with a list of real-life situations involving positive fractions and decimals. Individually, have the students estimate each answer. As a class, discuss their estimates and methods used for estimating. Example problem: 24% of the 7th graders at West Middle School are helping tutor 4th graders at West Elementary School. If there are 322 seventh graders at West Middle School, estimate how many seventh grade students are tutoring the 4th graders.

Give the students a list of the correct answers, and have them select the appropriate exact answer from the list. Discuss the operations needed to solve the problems. Ask the students to compare their estimations to the exact answers. Were any estimations way off? Have a discussion of how far away from the correct answer is too far. Be sure to point out there is no “set limit”; it depends on the information. Give students examples such as this: when estimating the number of students in a classroom, ten students make a big difference, but if you are talking about estimating the number of people at a concert, ten people would not make a difference. Discuss what makes one estimate better than another?
Students should respond to the following prompt in their math learning log (view literacy strategy descriptions).

Prompt:

Pam’s class is asked to estimate 5.3% of 41.9. Pam estimates 8. Keith estimates 20, and Seth estimates 2. Who has the best estimate? Justify your answer using words and mathematical symbols.

Activity 8: Simple Percent Problems (GLE: 8)

Materials List: newspaper advertisements or B’s Shoe Boutique BLM (at least one example for each group of 4 students), a half or quarter sheet of poster board per group, glue, scissors, markers/colored pencils

Divide students into groups of four. Introduce the idea of shopping when a store is having a sale. Using the store sale advertisements from the newspaper or B’s Shoe Boutique BLM, have the student groups figure 10%, 20%, 30%, 50%, 75%, \( \frac{1}{3} \) and \( \frac{2}{3} \) off the cost of items in the advertisements, or figure the sale price using the percent that is given in the ad. Many times items are advertised as \( \frac{1}{3} \) or \( \frac{2}{3} \) off the original price. Have students work with a partner to prove how they know that \( \frac{1}{3} = \frac{33}{3} \) and that \( \frac{2}{3} = 66\frac{2}{3} \% \).

Have the students check to see if their answers are reasonable. Have students practice estimating 10%, 20%, 30%, 50%, 75%, \( \frac{1}{3} \) and \( \frac{2}{3} \) off the items in the ads, and then compare these answers to the answers they originally figured.

Give each group a budget and assign different discounts. Have students choose items from the sale papers, estimate the percents to determine if they have enough money to make the purchases they want, and then calculate the exact prices. On a quarter/half sheet of poster board, have the students create a display indicating their choices, the method used to calculate each price, and the total cost of their purchases. Allow students to cut and paste pictures of the items, and require them to show their work. As an extension, have students add the local sales tax or create a grocery shopping scenario.

Activity 9: Tipping at a Restaurant (GLE: 8)

Materials List: Tipping at a Restaurant BLM for each student or group, pencil

Discuss with the class the tip customers leave at restaurants, noting that customers pay their server a tip for providing good service. A typical tip is 15% to 20% of the cost of the meal. Indicate to students that they need to use estimation skill to figure a tip that will be left for the server because the check will seldom be a whole number. Discuss with the
students how to round in reasonable ways. Discuss mental math strategies when finding the tip at a restaurant.

Present the following situation to the class. Your bill at Logan’s Restaurant is $19.45. What is a 10% tip on this bill? Instruct students to round off the amount to something they can reasonably work with. Some may say $19.50, but ask if this is reasonable for the situation. They may then say $1.95. Would it be more reasonable to leave $1.95 or $2.00? So a better process might be to round $19.45 up to $20.00 and then calculate a 10% tip for $20.00. Then have the students calculate 10% of $19.45. Have students compare their estimate with the calculation and check for reasonableness.

Practice several different amounts where students will need to use estimation and rounding to get a 10%, 15%, and 20% tip. Stress techniques that apply the distributive property: 15% is a 10% tip plus half that amount, 20% is double a 10% tip. Additional scenarios may be found on Tipping at a Restaurant BLM.

Activity 10: Rates (GLE: 10)

Materials List: sale papers/grocery items that can be used to figure unit cost and/or a copy of Grocery Shopping BLM, pencil, paper

Provide the students with a list of items they can purchase along with the prices. These items can be 6-packs of soft drinks, ounces of potato chips, pounds of peanuts, and so on. Be sure to use items that can be used to figure unit cost. Discuss with students why unit cost does not necessarily mean single units of an item and about how a “unit” changes with a situation. For example, in a grocery store you wouldn’t buy a peanut or a potato chip, so you would use some other unit such as weight. Also, you usually don’t buy one soft drink, so you say “units” are six-packs. Ask students to consider the following “what if’s?” to extend the conversation about unit cost:

- What if you want to compare the price of a six-pack to the price of a 20-pack?
- What if you are talking about buying your soft drinks from a vending machine?

Have the students calculate the unit price of each item. Also, extend this to include rates such as $45.00 for 8 hours of work, driving 297 miles in 5 hours, reading 36 pages in 2 hours, and so on. Have the students figure unit rates for these types of problems also.

Put students in small groups, and give the students 5-10 minutes to review the information from the activity and to respond to one or both of the following situations. They should also write at least 3-5 questions they anticipate being asked by their peers and 2-5 questions to ask other experts. When time is up, the teacher will randomly select groups to assume the role of professor know-it-all (view literacy strategy descriptions) and provide their answers and reasoning for the situation. Professor Know-it-All is an effective review strategy because it positions students as “experts” on newly learned information and concepts to inform their peers and be challenged and held accountable by them. They will also have to provide “expert” answers to questions from their peers about their reasoning. After the activity, students will reflect in their math learning log.
Situation 1: Lucy and CJ are in charge of buying chips for a class party. They plan to purchase 1.5 to 2 oz of chips for each of the 24 students. Use the information below to help them make the best purchase.

Big Al’s Grocery

1 – 1.75oz can for $0.75
12 – 1.75oz cans for $8.95
1 – 6oz can for $2.52

Solution:

<table>
<thead>
<tr>
<th>Size</th>
<th>Total Ounces</th>
<th>Total Cost</th>
<th>Cost per Ounce</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 1.75 oz can</td>
<td>1.75</td>
<td>$0.75</td>
<td>0.428</td>
</tr>
<tr>
<td>12 – 1.75 oz cans</td>
<td>21</td>
<td>$8.95</td>
<td>0.426</td>
</tr>
<tr>
<td>1 – 6 oz can</td>
<td>6</td>
<td>$2.55</td>
<td>0.425</td>
</tr>
</tbody>
</table>

Situation 2: Kenneth and Jena are in charge of buying sodas for a class party. They plan to purchase 6 oz of soda for each of the 26 students. Use the information in the table to help them make the best purchase.

PJ’s Grocery

<table>
<thead>
<tr>
<th>Container Size</th>
<th>Capacity in ounces</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Liter</td>
<td>33.8 oz</td>
<td>$1.09</td>
</tr>
<tr>
<td>2 Liter</td>
<td>67.6 oz</td>
<td>$1.29</td>
</tr>
<tr>
<td>3 Liter</td>
<td>101.4 oz</td>
<td>$1.99</td>
</tr>
</tbody>
</table>

Solution: This table shows the different combinations of containers the students may use to get their target of 156 ounces. Be sure students are able to defend their choices; they may not choose the overall lowest unit cost which requires them to purchase additional soda. They will need to purchase a minimum of 156 ounces of soda.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Size</th>
<th>Total Ounces</th>
<th>Total Cost</th>
<th>Cost per Ounce</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1-L</td>
<td>169 oz</td>
<td>$5.45</td>
<td>$0.03</td>
</tr>
<tr>
<td>2</td>
<td>2-L</td>
<td>169 oz</td>
<td>$3.67</td>
<td>$0.021</td>
</tr>
<tr>
<td>3</td>
<td>2-L</td>
<td>202.8 oz</td>
<td>$3.87</td>
<td>$0.0190</td>
</tr>
<tr>
<td>1</td>
<td>3-L</td>
<td>202.8 oz</td>
<td>$3.28</td>
<td>$0.0194</td>
</tr>
<tr>
<td>2</td>
<td>3-L</td>
<td>202.8 oz</td>
<td>$3.98</td>
<td>$0.0196</td>
</tr>
<tr>
<td>1</td>
<td>3-L</td>
<td>169 oz</td>
<td>$4.17</td>
<td>$0.024</td>
</tr>
</tbody>
</table>
Activity 11: Ratio Patterns (GLEs: 10, 11)

Materials List: pattern blocks or pieces of paper in 5 colors with squares, rectangles and triangles, scissors, pictures of quilts, patterns, repeating patterns

Use the following website to show the class pictures of quilts and patterns that have a repeating pattern such as an AB, ABA, or ABC pattern: http://www.quilt.com/QuiltBlocksPage.html.

Distribute five different colors of paper (pattern blocks, if available) marked with varying shapes including squares, rectangles, and triangles about 2 inches in size. Divide students into groups, and have them cut out the shapes. Show students two shapes - an equal number of red squares and blue triangles. Discuss the ratio of red pieces to blue pieces or squares to triangles. Group different color pieces and shapes to create designs. Discuss how repeated patterns are pleasing to the eye. Ask volunteers to come forward to create a pattern with pieces. Have the volunteers give the ratio of the colors or shapes. Divide students into groups to create their own patterns using different color ratios. Next, give each group a different ratio of reds to greens and blues to yellow, etc. (e.g., the ratio of 3 blue to every 4 green or 2 red for every 5 yellow) and have students create a pattern and demonstrate how their ratio was used to create the pattern. Introduce the concept of proportion for the patterns the students have created (e.g., 4 green for every 2 red is the same as 8 green for every 4 red). Demonstrate how to set up a proportion: \( \frac{4}{2} = \frac{8}{4} \). Help students realize the two fractions are equivalent; the numerator and denominator of the second have only increased by a common factor of 2. Cross multiply to create an equation that shows the cross products are equal: \( 4 \times 4 = 2 \times 8 \).

Give the following situation to students and instruct students how to set up and solve the proportion:

Grandma’s quilt was made with the ratio of yellow squares to red squares as 3 to 4. If she has 15 red squares she needs to use, how many yellow squares will she need if she keeps the same ratio of yellow to red? Ask students which units are being compared? Red squares to yellow squares. Ask students to write a ratio comparing the squares (including the units):

\[
\frac{3 \text{ red squares}}{4 \text{ yellow squares}}
\]

Next, ask students to use the ratio to write a proportion that will help us to determine how many yellow squares will be needed.

\[
\frac{3 \text{ red squares}}{4 \text{ yellow squares}} = \frac{15 \text{ red squares}}{x}
\]
While working with setting up proportions, it is important that students write the proportions with the units described in the situation. This will enable them to make sense of the problem contextually rather than just working with “naked numbers.” At this point, discuss other possible ways to set up the proportion keeping in mind that the two ratios must be set up in the same manner. A conversation regarding the use of the variable may also be necessary. Ask students to work with a partner to determine the different mathematical ways that can be used to find the number of yellow squares needed. *Students should see that:* 1) the second ratio has increased by a common factor of 5 so the number of yellow squares needed is 20 (4 x 5); 2) that cross multiplication can be used to find the yellow squares: 3x = 60, so x = 20

Challenge the students to create their own quilt patterns and to determine the ratio/proportion of the colors they used. Have the students change the ratio of the colors used to a percent of colors used.

**Activity 12: What’s the Recipe? (GLEs: 7, 11)**

Materials List: a different recipe for each pair of students, What’s the Recipe BLM

Discuss situations where a recipe may need to be reduced or increased. Discuss the fact that all ingredients must be increased or decreased proportionally in order for the recipe to turn out correctly. (For example, use a recipe for making chocolate chip cookies that makes 24 cookies, but the recipe needs to be increased so that everyone in a class of 36 gets a cookie.) If the given recipe produces 2 dozen cookies, what would the recipe be for producing 1 dozen cookies? 4 dozen? 6 dozen? 7 dozen? ½ dozen? Give each pair of students a different recipe or the What’s the Recipe BLM, and have them reduce or increase the recipe proportionally by two given amounts. Also, have students create new recipes based on a given percent of the original recipe.

A hot chocolate recipe is a good choice; after calculating how much of each ingredient is needed, the students could make hot chocolate for the class. Water could be heated in a coffee pot.

**Activity 13: What’s the Situation? (GLE: 10, 11)**

Materials List: Chart paper and markers for each group of students; What’s the Situation Group Cards BLM (cut out each problem to be distributed to each group), calculators

Use a modified math SQPL (view literacy strategy descriptions) strategy to prompt students to ask and answer their own questions. In this modified version, students will be prompted to ask questions about a mathematical diagram. Once the diagram is presented, allow students to pair up and brainstorm questions that can be answered from the diagram. Elicit students’ questions and write them on the board, overhead or computer.
Students work in groups of four to answer questions about the mathematical diagram. At the end of the activity, be sure to check that students have found answers to their questions and that their answers are accurate.

In this activity, students will describe in words the percent situation illustrated in a mathematical model and then pose questions whose answers can be determined by reasoning from the model.

Arrange students in groups of four and display Situation 1 from the What’s the Situation BLM using a document camera or overhead. Ask groups to describe in words the first situation illustrated, using no other numbers in their descriptions than those shown on the model. If students are familiar with the game Pictionary®, the teacher may tell them this is similar. A drawing is presented, labeled with minimal words and numbers, to represent a situation. Their job is to tell what situation is represented, using only the visual clues given in the drawing.

**SITUATION 1:** Ashleigh’s new bicycle.

80%  

$136 \quad \text{Original price}$

After agreement is reached on the description of the situation, have each group write questions on their chart paper whose answers can be determined by reasoning from the model. Students should estimate either before solving or to check the answers for reasonableness. Discuss, exploring students’ methods of answering the questions they pose (calculators may be needed to carry out the computations suggested by what students see in the model). Ask students to describe how they know whether their answer is reasonable. Here is the situation represented by the drawing: Ashleigh bought a new bicycle. She paid $136, which is 80% of the original price of the bike. *The point here is to interpret what is given before drawing conclusions.*

Questions that could be asked include:
- What is the value of 10%? (Since 80% is $136, 10% is $136 ÷ 8 = $17. To help students see the proportional relationships, label the model in increments of $17)
• What was the original price of the bicycle? (100% is 10 x $17 = $170 or \(\frac{80}{100} = \frac{136}{x}\) which is $170, the original price)

• What is the amount of the discount? (Since 10% is $17, the discount was 20% or 2 x $17 = $34)

• What is the difference between the original and the price paid? (Subtract the price paid from the original--$170 - $136 = $34, the same thing as the discount).

To check the depth of understanding students have in solving percent problems, distribute the remaining problems from What’s the Situation Group Cards BLM to each group of four. Circulate around the room and ask questions to find what strategies the students are using when thinking about percent problems. Each group will present their questions and solutions to the class. The answer key includes additional questions to extend student thinking.

Activity 14: Proportional or Not? (GLE: 11)

Materials List: Pencil, grid paper, Proportional or Not BLM

In this activity, students will compare proportional and nonproportional situations involving the rates of two taxicab companies.

Background: Being able to solve problems that reflect proportional situations involves more than solving traditional missing-value problems by using the standard cross-product algorithm. By comparing proportional contexts with nonproportional contexts, students can strengthen their understanding of the multiplicative relationship behind proportions.

Before beginning this activity, lead a class discussion on the nonnumeric proportion problem below that involves two girls biking on a path. Because students tend to “misuse” numbers by applying them to some algorithm, they should focus on underlying proportional relationships and create an environment that nurtures proportion sense. One way is to strip problems of numbers that force students to examine the relationship between variables directly.

Ask students to consider the following problem:

Catherine and Rachel like to ride their bicycles along the bike path in Forever Green Park. Today, they both started riding at the beginning of the trail; each rode continuously at a constant speed, making no stops, to the end of the trail. Rachel took more time than Catherine to reach the end of the path. Which girl was biking faster? Why? Explain your answer. Students should easily see that Catherine rode faster because the bikers made no stops, rode the same distance, and they took different amounts of time to reach the end. Additional discussion should
include: Catherine could have taken a shortcut decreasing her distance and affecting her time; although both started at the beginning, the text did not say they started at the same time. This problem not only focuses on the relationship between time and speed but also allows students to analyze other factors such as distance and starting time that might affect whether the relationship is truly proportional.

Pair students and distribute Proportional or Not BLM. Students will decide whether the quantities in each taxicab situation are in a proportional relationship by testing for equivalent ratios in a table, graphing on a coordinate plane, and observing whether the graph is a straight line through the origin. In the situation that is proportional, students should be able to identify the constant of proportionality (unit rate) in the table, graph and verbal description of the relationship. After the table and graph have been constructed, students will use a graphic organizer (view literacy strategy descriptions) to compare and contrast the two situations. Graphic organizers are visual displays that enable students to assimilate new information by organizing in a visual and logical way. The graphic organizer used in this activity is a Venn diagram, which will help students make sense of the table and graph and then decide which situation is proportional. Monitor students’ completion of the diagram to ensure understanding. Once the Venn diagram is completed, students will use it as a study aid when determining the proportionality of any situation.

Activity 15: Number Line with Integers (GLE: 2)

Materials List: Velcro® strip or string and tape for number line (from Activity 3), a set of index cards labeled with positive and negative integers

Using a Velcro® strip or string number line from Activity 3, have students compare rational number and integers and determine the placement of the numbers on the number line. Make index cards for the students to use. (Examples of numbers: 1, -1, \(\frac{1}{2}\), 100%, -4, \(-\frac{1}{3}\), 50%, 0). Make sure to use positive and negative fractions and decimals, whole numbers, and percents to place on the number line. Give a card to a student and ask him/her to place the card where it belongs on the number line. Have a discussion about the placement of this number. Must it go in that exact location? Could it be placed elsewhere? Give out another card for placement. Continue until all numbers have been placed along the number line. Ask many questions about the placement of each number creating in-depth class discussions. Some numbers may need to be moved on the number line once one or two numbers have been placed on the number line.
Sample Assessments

General Assessments:

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage students to create their own questions.
- Ask students to create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- Observe the student’s presentations.
- Have students complete math learning log entries by responding to prompts such as:
  - Explain the meaning of 10%, 20%, 25%, 33 1/3%, 50%, 66 2/3%, 75%, and 100% and write their fractional and decimal equivalents. Give examples of their use in real-life situations.
  - The 4-H advisor is in charge of buying drinks for the club’s landscaping day. She conducted a survey to determine if students liked Dr. Pepper® or Coca Cola®. Here are her results:

<table>
<thead>
<tr>
<th></th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Pepper®</td>
<td>80</td>
<td>75</td>
<td>85</td>
</tr>
<tr>
<td>Coca Cola®</td>
<td>70</td>
<td>90</td>
<td>80</td>
</tr>
</tbody>
</table>

Tell whether the statements 1-4 are accurate based on the information in the table. Explain your answer for each item.

1. 15 more seventh graders prefer Dr. Pepper® to Coca Cola®.
2. The ratio of seventh graders who prefer Dr. Pepper® to Coca Cola® is 5 to 6.
3. 50% of the students surveyed prefer Dr. Pepper®.
4. 7/8 of the sixth graders prefer Coca Cola®.

Tell whether an exact answer or an estimate is needed to determine the grade in which 52% preferred Dr. Pepper®. Explain your answer.

The Coca Cola® for the party would cost $168 and the Dr. Pepper® would cost $180. Will the students pay the same price for Coca Cola® and Dr. Pepper®? Justify your answer using the cost per student.
Teacher may want to copy longer prompts and have students tape, glue, or staple into math learning logs.

- **Assign the following project: Collect several flyers from local restaurants advertising their specials and menu items. In groups of four, students will plan a dinner party at a restaurant for their group with a set budget and prepare a presentation on poster board. The poster will show each person’s order, tax and tip on the total bill, and the final cost.**

### Activity-Specific Assessments:

- **Activity 3:** Given a list of 8 different representation of numbers (fractions, decimals, percents) and a blank number line, the student will place the numbers in the correct position on the number line and write three inequalities using the given numbers and the symbols $<, \leq, \geq, >$.

- **Activity 7:** Present the following scenario to the student, and evaluate the student’s ability to answer the questions asked orally.

  *Latoya is at a grocery store near her house. She has $10.00 but no calculator or paper or pencil. At the right is a list of the items she would like to buy. Use mental calculations and estimation to answer the following questions.*

  1. Latoya believes she can purchase all of the items she wants. Is this reasonable? Justify your answer.
  2. What different items could she buy to come as close as possible to spending $5.00?
  3. Approximately what percent of the $10.00 did Latoya spend on eggs?

  **Solutions:**
  1. No, she cannot buy all the items. By estimating to the nearest half dollar, she will need at least $10.50. She must have $11.13 before tax.
  2. Solutions may vary: Sample solutions: milk, avocado, and cheese or eggs, cheese, honey and avocado
  3. $1.09 out of $10.00 is about 10%.

- **Activity 8:** On a sheet of unlined paper, the student will create an ad for the newspaper. The ad must include the item (a drawn picture) with a description, the regular price of the item, the percent of discount, and sale price. The student will show how he/she arrived at the sale price on the back of the ad.

- **Activity 12:** The student will work the following problem correctly: A certain recipe for brownies calls for 2 teaspoons of vanilla and 6 teaspoons

---

**Item** | **Price**
---|---
Milk | $2.47
Eggs | $1.09
Cheese | $1.95
Bread | $0.68
Honey | $1.19
Cereal | $3.25
Avocado | $0.50
of oil. If you want to make a large batch of brownies for your class using 10 teaspoons of oil, how much vanilla would you need?

Solution: \(3\frac{1}{2}\) teaspoons of vanilla

<table>
<thead>
<tr>
<th>oil</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>vanilla</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3\frac{1}{3}</td>
<td>3\frac{2}{3}</td>
<td>4</td>
</tr>
</tbody>
</table>

- **Activity 13:** Given a situation, students will draw and label a model showing the mathematical relationships in the situation. Students will estimate a solution before solving and then justify why their estimation is reasonable.

*Teacher may choose one or more of the following situations depending on student need:*

a) Jana paid $24.50 for a dress which was on sale for 65% off the regular price. What was the original cost of the dress?

b) A telethon for a local charity raised $45,000. This was 125% of the goal. What was the goal?

c) A length of string that is 180 cm long is cut into 3 pieces. The second piece is 25% longer than the first, and the third piece is 25% shorter than the first. How long is each piece?

*Solutions:*

a) If the dress were 65% off, then she paid 35%, so \( \frac{35}{100} = \frac{24.50}{x} \), or \$70.00\n
(b) 25% of the goal: \$45,000 ÷ 5 = \$9,000, so the goal is 4 x \$9,000 = \$36,000
c) Each section: $180 \text{ cm} \div 12 = 15 \text{ cm}$
Length of 1st string: $4 \times 15 \text{ cm} = 60 \text{ cm}$
Length of 2nd string: $5 \times 15 \text{ cm} = 75 \text{ cm}$
Length of 3rd string: $3 \times 15 \text{ cm} = 45 \text{ cm}$
Grade 7 Mathematics
Unit 2: Situations with Rational Numbers

Time Frame: Approximately four weeks

Unit Description
This unit extends the work of the previous unit to include the operational understandings of multiplication and division of fractions and decimals and their connections to real-life situations including using ratios and rates.

Student Understandings
Students develop an understanding of multiplication and division of fractions and decimals using concrete models and representations. At the same time, they become proficient in computations that involve positive fractions, mixed numbers, decimals, and positive and negative integers using the order of operations. Students also develop an overall grasp for solving proportions involving whole numbers. Students should distinguish between rates and ratios, and set-up, analyze, and explain methods for solving proportions.

Guiding Questions
1. Can students multiply and divide fractions and decimals with understanding of the operations and accompanying representations?
2. Can students add, subtract, multiply, and divide negative integers?
3. Can students set up and solve proportions involving whole number solutions?
4. Can students interpret the results of operations and their representations, for example, between ratios and rates?
5. Can students tell if answers to operations are reasonable?

Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Number Relations</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Solve order of operations problems involving grouping symbols and multiple operations (N-4-M)</td>
</tr>
<tr>
<td>5.</td>
<td>Multiply and divide positive fractions and decimals (N-5-M)</td>
</tr>
<tr>
<td>7.</td>
<td>Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)</td>
</tr>
</tbody>
</table>
8. Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)

10. Determine and apply rates and ratios (N-8-M)

11. Use proportions involving whole numbers to solve real-life problems (N-8-M)

CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Number System</strong></td>
<td></td>
</tr>
<tr>
<td>7.NS.1</td>
<td>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</td>
</tr>
<tr>
<td>7.NS.3</td>
<td>Solve real-world and mathematical problems involving the four operations with rational numbers.</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>7.G.1</td>
<td>Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td>
</tr>
</tbody>
</table>

Sample Activities

**Activity 1: The Meaning of Multiplication of Fractions**  (GLEs: 5, 8)

Materials List: pencils, paper, a piece of newsprint or similar paper for each pair of students, markers

Ensure that students get a real sense of multiplying fractions and making the connection to the meaning of multiplication.

Ask the students to illustrate the meaning of $3 \times 4$ using a picture and/or words. The students should write in words and model three groups of four and/or four groups of three. Make sure the students understand they are adding 3 groups of 4 or 4 groups of 3. This is a good place to review the commutative property. Have a class discussion about a real-life meaning of this problem (e.g., Sam has three groups of candy bars with four candy bars in each group). Extend this concept to include multiplication of a fraction and a whole number (e.g., $3 \times \frac{1}{2}$ add three groups of one half). Discuss how to first estimate an answer. This will provide something to compare to the product so students can make sure their answers are reasonable. Ask, “If you multiply a positive number by a positive fraction less than one, will the product be greater than, less than or equal to the first factor?” Write each problem on the board, and ask a student to model it for the class.

(e.g., add three groups of $\frac{1}{2}$ and/or find $\frac{1}{2}$ group of 3 and drawing the groups). Remind students to check for reasonable answers/models.

After doing several of these types of problems, ask the students to create a rule for multiplying whole numbers and fractions. Continue practicing and modeling various
situations - fractions times fractions, then fractions and mixed numerals. When discussing a whole number times a mixed number, introduce the concept of the distributive property. $2 \times 4 \frac{1}{2}$ means add two groups of four and a half but also could be written as add two groups of four; add two groups of one half, and then add the two sums. Each time the students model and explain their answers, have them check to see if the answers are reasonable.

Using professor know-it-all (view literacy strategy descriptions), have the students work in pairs to create a word problem that involves multiplication of fractions, whole numbers, and mixed numbers. Each group should create/illustrate a model of the problem, write a mathematical sentence that illustrates the situation, and solve their problem. They should also write at least 3-5 questions they anticipate will be asked by their peers and 2-5 questions to ask other experts. Remind students they must be ready to defend the reasonableness of their problems, thought processes, and solutions to the class. After students have been given time to complete their problems, choose groups at random to assume the role of professor know-it-all by asking them to come to the front of the room and answer questions from their classmates. Make sure the professors are held accountable for their responses to other students’ questions about their word problems.

Information about and examples of the commutative property and distributive property can be found at Purple Math.com, [http://www.purplemath.com/modules/numbprop.htm](http://www.purplemath.com/modules/numbprop.htm).

**Activity 2: Multiplication of Fraction Using Arrays (GLEs: 5, 8)**

Materials List: grid paper, pencils, colored pencils, or markers, math learning log, Multiplying Fractions BLM for each student

Have a discussion of the meaning of multiplication of whole numbers (e.g., $3 \times 4$) using arrays. Give students grid paper and have them create an array that could be used to solve the problem $3 \times 4$.

Check to see that students make these drawings and have these understandings.

I have an array of 4 columns with 3 rows in each column. or I have an array of 3 columns with 4 rows in each column.

<table>
<thead>
<tr>
<th>4 columns</th>
<th>3 columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 rows</td>
<td>4 rows</td>
</tr>
</tbody>
</table>

Grade 7 Mathematics Unit 2 Situations with Rational Numbers 2-3
Either way the array is arranged, there are still 12 boxes in the array. The students may use the commutative property to illustrate their arrays if it seems easier for them.

Ask students if they can use arrays in the same way to model multiplication of fractions. In groups, have the students use grid paper to model the situation: Jacque wants $\frac{4}{5}$ of $\frac{1}{2}$ of Nick’s candy bar. How much of the whole candy bar does Jacque get? A student, a group of students, or the teacher should model the problem on the board or overhead after the groups have been given a chance to complete the work.

An example might be as follows:
A candy bar is cut in half, and half is given to Nick. Jacque gets $\frac{4}{5}$ of Nick’s half.

If you divide each half into 5 parts, there would be 10 sections formed. Jacque gets $\frac{4}{10}$ of the whole candy bar. If you rearrange the $\frac{4}{10}$, then the students can see this is the same as $\frac{2}{5}$ of the candy bar.

Allow students to use the grid paper to illustrate and solve the following problems and then create the rule of multiplication for each. Remind students to determine if the product they calculate is a reasonable answer. After students have an opportunity to complete the problem set, randomly select students to share their answers and reasoning. Help students understand that each factor must first be written in fraction form. Next, multiply the two numerators to get the product’s numerator. Then, multiply the two denominators to get the product’s denominator. Last, simplify as needed.

$\frac{3}{6} \times \frac{5}{8} = \frac{2 \times 7}{2 \times 4} = \frac{1}{2} \times \frac{1}{4} = \frac{2}{3} \times \frac{1}{2} = \frac{3}{2} \times \frac{1}{2}$

Some students may choose to use the commutative property because the problem is easier to model. Make sure students can create real-life situations that will describe each of the problems.

Students should respond to the following in their math learning log (view literacy strategy descriptions):
When you multiply two nonzero whole numbers, the product is equal to or larger than the factors. Is the product of two fractions larger than the fraction factors? Explain your reasoning.

After students have responded in their math learning log, they will share what they wrote with a partner to compare reasoning.

Once students have an understanding of multiplying fractions with visual aids, they need to move to multiplying fractions using a set of rules or an algorithm. Have students work in pairs to explain in words and mathematical symbols how to multiply fractions. Remind them to include instructions that explain how to deal with whole numbers and mixed
numbers. After students have time to work, have them share their versions of the rules. Ask probing questions where there are mistakes in student understanding to allow students to discover their mistakes.

Students should be able to describe the following steps:
1. Change any whole numbers to a fraction by writing the whole number as the numerator and 1 as the denominator.
2. Change any mixed numbers to improper fractions by multiplying the denominator by the whole number and adding the product to the numerator to get the numerator of the improper fraction; the denominator will remain the same.
3. Multiply the numerator of the first fraction by the numerator of the second fraction. This is the numerator of the product. Then multiply the denominator of the first fraction by the denominator of the second fraction. This is the denominator of the product.
4. Simplify, if possible, and change improper fractions to mixed numbers.

The Multiplying Fractions BLM contains additional problems for student practice.

**Activity 3: The Meaning of Division of Fractions (GLEs: 5, 8)**

**Materials List:** pencil, paper, Dividing Fractions BLM for each student

In multiplication, most students understand that 4 groups of 2 objects give a total of 8 objects. They need to relate division of fractions to their understanding of the division problem, \(8 \div 4\). Students have difficulty in stating the meaning of division -- take a total of 8 candy bars and divide the bars among groups of 4 students, or 8 separated equally into 4 groups, which means that each group of 4 students gets 2 candy bars. Write a problem on the board. Have students write a situation for the problem, and then solve. Repeat the process several times.

Extend student understanding to include division with a fraction: \(8 \div \frac{1}{2}\) might mean 8 candy bars divided or separated into half pieces. The quotient would indicate how many half pieces there would be after the division. Have students predict if the answer will be more than or less than 8, and then let one student model the problem for the class using a picture and words. The picture helps students see division – that 8 candy bars broken in half would result in 16 pieces. Instruct students to return to the predictions they made. Allow several students to share their prediction, and indicate whether it was reasonable or not. Repeat the process using several examples. Record all problems on the board with the intent that one or more of the students will see a pattern which can be written as a rule after doing a series of problems. (Multiply by the reciprocal or multiplicative inverse.) Remind students of the rules they wrote for multiplying fractions. Have them write a rule for dividing fractions. Students should be able to describe the following steps:
1. Change any whole numbers or mixed numbers to fractions.
2. Leave the first fraction alone.
3. Replace the division sign with a multiplication sign.
4. Write the multiplicative inverse or reciprocal of the second fraction.
5. Multiply the two numerators, and then multiply the two denominators.
6. Simplify the quotient as needed.

Take time to discuss students’ methods before moving on to dividing fractions by fractions and dividing fractions by mixed numbers. Writing word problems is always difficult, especially with fractions! Just make sure the students attach labels to the fractions so that the problems make sense.

Once students have created this “new” rule for dividing fractions, ask them to demonstrate their understanding of dividing fractions by completing a RAFT writing assignment. This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, RAFT writing is used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. It’s the kind of writing that when crafted appropriately should be creative and informative.

Ask students to work in pairs to write the following RAFT:

- **R** – Role (role of the writer—Mr./Mrs. Multiplicative Inverse)
- **A** – Audience (to whom or what the RAFT is being written—6th grade or 7th grade students who do not know how to divide fractions)
- **F** – Form (the form the writing will take, as in letter, song, etc.—Job Description or Descriptive Jingle)
- **T** – Topic (the subject/focus of the writing—explain the role of the multiplicative inverse when dividing fractions)

When finished, allow time for students to share their RAFTs with other pairs or the whole class. Students should listen for accuracy and logic.

The Dividing Fractions BLM contains additional problems for student practice.

**Activity 4: Decimal Positioning**  (GLEs: 5, 8)

Materials List: pencils, chart paper, scissors, glue or tape, math learning log

The decimal position of the two factors in a multiplication problem affects the product of two numbers. The following situation will help to build a deeper understanding of this concept.

Give each group of 4 students chart paper, scissors, and glue or tape. Instruct the students to give an example to each of the following (1-4); students should also cut out and paste a model showing each situation on chart/paper.

1. Give an example of a situation that has a product of 56.
2. Give an example of a situation that has a product of 5.6.
3. Give an example of a situation that has a product of 0.56.
4. Give an example of a situation that has a product of 0.056.
5. Explain how answers were derived. Be prepared to present methods used to the class.

As the students present their methods, ask questions to develop the meaning of multiplication of decimals, not just placement of decimal points. For example, students should be able to use the knowledge that 7 groups of 8 pizzas is 56 total pizzas while 0.7 groups of 8 pizzas means a little over half of eight pizzas or 5.6 pizzas. Present more situations like the one above for the students to internalize the rules they will use for multiplication of decimals.

Include a discussion about estimating and reasonable answers. Ask when it is better to use an estimate vs. an exact answer.

Students should respond to the following in their math learning log without working the problem.

Explain whether the exact product of (1.4)(0.999) will be greater than or less than the estimate (1.4)(1). How can you tell without multiplying 1.4 and 0.999?

After students have responded in their math learning log, they will share their response with a partner to compare explanations.

Teacher note: Ask questions to make sure the students relate the estimate to the multiplicative identity.

Activity 5: Decimal Division (GLEs: 5, 8)

Materials List: pencils, Decimal Division BLM for each student

Ensure that students develop a conceptual understanding of division of decimals, not just to move the decimal so many places, then divide.

In Activity 4, students wrote situations in order to understand the concept of dividing fractions. The problem $\frac{24}{6}$ can be written as 24 cookies divided among 6 people. How many cookies does each person get, or how many sets of 6 cookies are in a package of 24 cookies? Discuss the meaning of this problem.

While students will know the answers to the problems below, the intent is to develop a conceptual understanding of the placement of the decimal in the answer of a division problem. Give each student a copy of the Decimal Division BLM and have him/her work to come up with the patterns they see in the following problems. Remind students to check for reasonable answers.

1. Nikki has $25.
   A. How many 50-cent pieces are in $25? Write this as a division problem and solve it.
   B. How many quarters are in $25? Write this as a division problem and solve it.
C. How many dimes are in $25? Write this as a division problem and solve it.
D. How many nickels are in $25? Write this as a division problem and solve it.
E. How many pennies are in $25? Write this as a division problem and solve it.
Discuss the patterns that students find. Allow students to explain/justify their thought process.

2. Kenneth has $0.50.
   A. How many 50-cent pieces are in $0.50? Write this as a division problem and solve it.
   B. How many quarters are in $0.50? Write this as a division problem and solve it.
   C. How many dimes are in $0.50? Write this as a division problem and solve it.
   D. How many nickels are in $0.50? Write this as a division problem and solve it.
   E. How many pennies are in $0.50? Write this as a division problem and solve it.
Discuss the patterns the students find when fifty cents is used, and pose these questions:
   How many one dollars are in a quarter?
   Does the pattern you found earlier fit this situation? Is it reasonable to have a decimal answer or a whole number answer?
This will cause a bit of concern for the students, because there are no dollars in a quarter; a quarter is a fraction of a dollar. This is where placement of the decimal comes in for division of decimals. More situations like the one above will be needed for the students to get a good understanding of dividing decimals.

Activity 6: Order of Operations—Is It Possible? (GLEs: 3, 5)

Materials List: A number cube or spinner for each student, pencil, Is It Possible? BLM for every student, calculators

Present students with these two sets of problems: \(3 + 4 \times 10\); \((3 + 4) \times 10\) and \(3 + 2^2; (3 + 2)^2\). Have students evaluate each expression. Ask students to compare the two expressions by writing an inequality using <, >, or =. Use student work to lead a class discussion about the use of parentheses and exponents when simplifying expressions. Use a mnemonic such as “Please Excuse My Dear Aunt Sally” to help students remember the order of operations. The first letter of each word corresponds to the first letter in the mathematical operations in the order they are to be performed: Parentheses, Exponents, Multiplication and Division from left to right, then Addition and Subtraction from left to right. Students may have heard other versions from other teachers or created their own version at some point. If students have a difficult time remembering the order of operations, you may want each student to create a mnemonic that is more personal to him/her. Have students work several problems like the two earlier ones before continuing the activity.
Give each student a copy of the Is It Possible? BLM. Instruct students to play Game 1; have students randomly select 4 numbers by either rolling a number cube or spinning a spinner which contains number outcomes. Ask the students to use each of the 4 numbers only once, along with any operations symbols or grouping symbols, to write
mathematical expressions that are equal to each of the numbers 1-9. (Students may be allowed to combine digits to form numbers. Example: I rolled a 3, 6, 2, and 3. I can combine the digits to make \(23 - 3 \times 6 = 5\).) Have students check their answers on a calculator. Instruct students to exchange papers to check one another’s work.

When students have completed Game 1, ask students if it will always be possible to write expressions for each counting number using the 4 numbers? (NO) Have students share with the class examples of what they believe to be impossibilities for creating each of the numbers 1-9 from four numbers generated by the rolls or spins. Challenge students to see if they can form any of the impossible numbers.

Explain to students that in Game 2 they must use a fraction. Example: I rolled a 3, 6, 2, and 3. I then rolled a 5th number to combine with the last 3 to create a fraction. My 5th number was a 3, so I created \((6 - 3 - 2)(\frac{3}{3}) = 1\). Instruct students to play Game 2 and then exchange papers with another student to check their work. Ask the class to discuss which numbers are impossible to form. Repeat the sequence of events for Game 3 which uses a decimal.

2013-14
Activity 7: Let’s Figure It! (CCSS: 7.NS.3)

Materials list: Let’s Figure It! BLM for each student, pencil, math learning log

In this activity, students will solve real-world problems involving the four operations with rational numbers. Students need to be taught that they can, and should ask themselves questions when solving problems. Provide the students with the problems to practice operations with rational numbers by distributing Let’s Figure It! BLM. Model questioning the content (QtC) (view literacy strategy descriptions) with the first problem to help students construct meaning of the processes and operations needed to solve the problem. The goal of QtC is to teach students to use a questioning process to construct meaning of content and to think at higher levels about content from which they are expected to learn. The questions posed in this activity can be used for ANY problem solving process.

Begin the modeling process by posting the following questions in a handout, projected on a board or made into a poster and attached to the classroom wall:
1. How would you describe the problem in your own words?
2. What facts do you have?
3. What do you know that is not stated in the problem?
4. What do you think the answer or result will be?
5. Can you describe the method you used to solve the problem?
6. Can you explain why it works?
7. Does your answer seem reasonable? Why or why not?
8. Is there a different way to explain how you could get the same answer?
Next, initiate discussion to help students construct meaning of the example problem on the BLM by asking them how to describe the problem in their own words, what facts do they have to solve it, and what do they know about the problem that is not stated (Questions 1-3). Encourage the students to estimate a solution by asking what they think the answer or result will be (Question 4). Then, students will solve the problem independently. When finished, students will question a partner on the method they used to solve the problem and explain why their method works (Questions 5-6). In addition, they will ask each other if their answer is reasonable and why (Question 7). Finally, students will determine whether there is a different way to explain how to get the same answer (Question 8). Monitor students’ conversation providing additional modeling and clarification to each pair as needed.

Students will complete the same questioning process with a partner to solve the remaining problems. While QtC is an interactive strategy, the goal is to make the questioning process automatic for students so they use it on their own to solve any problem. Ask students to describe how the questioning process helped them to solve the problems in their learning logs. Students will turn in their learning logs as a formative assessment. Make a note of misconceptions in the problem-solving process and address them in any subsequent problem-solving activities.

Activity 8: Using Symbols and Multiple Operations (GLE: 3)

Materials List: Challenge Numbers BLM, Challenge Symbols BLM, two boxes, large paper and markers or student white boards and dry erase markers, math learning log

To prepare for this activity, the teacher needs to copy each BLM and cut out the pieces. The pieces from each BLM should be folded and dropped into separate boxes. Include as many copies as needed. Additional numbers may be used.

Review the order of operations. Have students, working in groups of four, select 5 numbers and 4 symbols (operation and grouping) from separate boxes and create an expression that they will use to challenge other students’ understanding of the order of operations. Instruct students to write their expressions on large paper or on white boards to present to the class.

To play Challenge, each group will present its expression to the class. The other groups will have three minutes to solve each problem. Have groups write their answers on large paper using markers or small white boards using dry erase markers. When time is called, have each group show its answer. The team presenting will earn one point for each group with an incorrect answer. Be sure to have each group present the problem it created. The group with the most points at the end of the game wins.

Students should respond to the following in their math learning logs (view literacy strategy descriptions).
Two students were asked to compute $36 + 12 ÷ 2 × 3 − 4$. Sam started by multiplying 2 by 3. Jared started by dividing 12 by 2. Who used the correct procedure? Explain your choice. **Jared is correct because the order of operations rule is multiply or divide from left to right and division comes first in the expression.**

After students have responded in their math *learning logs*, they will share their response with a partner to compare explanations.

**Note to teacher:** Inexpensive 4’ x 8’ sheets of white tileboard (normally used in bathrooms) are available at home improvement stores and contain the same material used to make more expensive white boards available through school supply stores. Personnel at the store will usually cut the sheets into 2’ x 2’ squares for a nominal fee.

**Activity 9: Problem Solving Triangle Puzzle (GLEs: 5, 7, 10, 11)**

**Materials List:** Triangle Puzzle BLM for each student, scissors, tape, pencil, paper

Provide students with Triangle Puzzle BLM formed of equilateral triangles. Have students cut the triangles apart, and match each problem to the solution. The triangles will form a symmetrical shape when each problem is answered correctly. Students can then tape the pieces together.

**Directions for teacher to make additional puzzles:** Cut out several equilateral triangles and place together to form a symmetrical shape. Write a problem along one side of a triangle. Find the triangle that shares this side and write the solution along the side of this triangle. Continue this process until all triangles have either an answer or a problem written on each side. Include ratio, proportions, order of operations, percents, decimals, fractions and mixed number situations on the puzzle. Make sure that the same answer is not used more than once as this makes the puzzle very difficult to solve.

**Activity 10: Integer Target Part One (GLE: 7)**

**Materials List:** color markers, newsprint or bulletin board paper, yardstick, green markers/counters, Integers BLM, pencil, paper, math *learning logs*

To prepare for this activity, copy the two pages of the Integers BLM on construction paper, cut out the cards, and place them in a baggie. Create number lines (-30 to 30) on the newsprint or bulletin board paper using colored markers and a yardstick. The cards and number lines will be used in Activity 11. This activity should take approximately one 50-minute class period.
Place students in groups of 2, and give each group a sack of four to six cards made using the Integers BLM (some positive and some negative), a number line, and a green marker/counter.

Discuss ways to find the sum of the cards in the stack. Ask, “If you have no cards, what is your total?” Students should respond that the total is zero. Instruct students to place their marker at zero on their number line.

Each group then turns over the first card in their stack. Ask, “What should groups with a positive card do? What about those with a negative card?” Most groups will recognize that those with positive cards should move to the right on the number line, and those with negative cards should move left. Have one student in each group write an equation to describe the group’s reasoning.

Instruct students to leave their first card up and draw the second card. Ask, “How can you move your marker to the place that shows the sum of your two cards?” After some discussion, students should agree to begin where they left off and move the distance and direction shown by their second card – right for positive and left for negative. Instruct groups to turn over the third card and continue in this manner until the class is comfortable with adding integers.

Now that each group has a small collection of cards face-up in front of it, the group is ready to experiment with giving cards away. Each group chooses a positive card to discard and then find the new sum. Have groups write an equation to describe their thinking. If a group holds the cards -2, 4, 3, and -1, and it gives away the 3, it may write “-2 + 4 + -1 = 1,” reflecting the fact that it has discarded the 3 and added the remaining cards. Another group may reason that its original sum of 4 will be reduced by 3 and write “4 – 3 = 1.”

Continue the process, this time discarding a negative card. The students discover that since adding negative cards lowers the total, giving up negative cards must increase the total.

Students should respond to the following prompts in their math learning logs (view literacy strategy descriptions).

1. Describe one or two ways that you can move to the right on the number line by getting or giving away a card. (Sample answer: I can move to the right on the number line by either getting a positive card or giving up a negative card.)
2. Describe one or two ways that you can move to the left on the number line by getting or giving away a card. (Sample answer: I can move to the left by either giving up a positive card or getting a negative card.)

After students have responded in their math learning logs, they will share their response with a partner to compare strategies. Once students understand these basic concepts and become familiar with the cards and number line, they are ready to begin playing “Integer Target.”
Integer Target Part One and Integer Target Part Two were adapted with permission from “Integer Target: Using a Game to Model Integer Addition and Subtraction” by Jerry Burkhart, math specialist and teacher, Mankato Area Public Schools, Mankato, MN. *Teaching Mathematics in the Middle School*, March 2007, Volume 12, Issue 7, page 388.

**Activity 11: Integer Target Part Two (GLE: 7)**

Materials List: Integers BLM and number lines from Activity 10, Integer Target BLM, a red and a green marker/counter for each student, dice

Allow students the maximum amount of play time as the more they play, the more comfortable with the concepts they become.

Students will play the Integer Target game in pairs or groups of four. Each student will need a red and a green marker/counter, a number line (-30 to 30), and a copy of Integer Target BLM. Each group will need a die and a set of cards (one copy of Integers BLM copied on construction paper).

Have students recall their discoveries of how to move to the right and to the left on the number line. If needed, review several “moves” from the previous activity as a group. Explain to the students they will play a game called Integer Target where the moves are similar to the previous activity. Instruct student groups to read the instructions found on Integer Target BLM. Play a mock game as a class, then have student groups play.

Follow Up: The next day, reinforce and extend what has been learned by having students discuss their actions and express them as number sentences. Have students write their number sentences on the board and describe the actions they represent. Be sure to have the students give a solution. (Example: Given the expression 1 – –4, students might say that a player begins with a card sum of 1 and gives away a –4 card, resulting in a new sum of 5.) Make sure that some examples require students to perform this task in reverse, beginning with the concrete action and finding the equation that would describe it. (I have a sum of 5, and I want a sum of 2. What should I do?)

Have the students play the game a second time. Now, during their turn, they must write an equation that describes the action they took. The form of the equation is “beginning total” +/- “action card” = “new total.”

A computer version of Integer Target is available at [http://www.themathroom.org/?page_id=8](http://www.themathroom.org/?page_id=8)
Activity 12: Which Direction? (CCSS: 7.NS.1)

Materials List: Integers (BLM) cards from previous activities, Which Direction? BLM, number cubes

In this activity, students will extend their thinking about integer sums and differences to the number line. The big idea that students should see is that addition can be modeled by facing to the right on the number line and subtraction by facing to the left. Also, the number being subtracted is shown by direction—forward or backward. For example: \(3 - (-2)\) is modeled on the number line by starting at 3, facing left (subtraction) and moving backwards 2. Students should also generalize subtraction of rational numbers as adding the additive inverse, \(p - q = p + (-q)\).

Begin the activity with the literacy strategy student questioning for purposeful learning, SQPL (view literacy strategy descriptions) by writing the statement, “Subtracting 2 from any number is the same as adding -2 to a number.” This strategy is used to encourage the students to generate questions that they would need to answer to verify the statement. Have students form groups of four, and brainstorm different questions that they might need to answer to determine whether this statement is true or false. Have each group of students highlight 2 or 3 questions that their group has come up with for use with whole class discussion. Write these questions on a sheet of newsprint for use as closure when the students have completed this activity. These questions might include the following: Will this work if I subtract from any number? Will this work if I use any number to subtract? Can I prove if this works with a model? If the questions you want them to discover are not on their list, you might take an opportunity to put your own question on the list.

Next, use a virtual manipulative found at http://www.mathsisfun.com/numbers/casey-runner.html to help students see the direction of addition and subtraction on a number line. Play the game as a whole class asking questions to guide students in determining how integer addition and subtraction is modeled on the number line. Note: Play the “Sprint” version of the race first, then if additional modeling is needed, play the “Flag Race.” Remind students to see if any of the questions they posed at the beginning of the activity can be answered during the game.

Students will use a modified version of a math text chain (view literacy strategy descriptions) in groups of 4 to generate a number sentence that includes three numbers and two operations. The text chain strategy gives students the opportunity to demonstrate their understanding of which direction on the number line they must go to model each number sentence and end up with the correct solution. Each student will have a part in modeling the number sentence on the number line and determining if the statement written in the beginning is true or false.

Distribute a set of Integer cards used in the previous activities (Integers BLM), a copy of “Which Direction” BLM to each student in the group, and a pair of number cubes. Model the text chain with the whole class first so that students understand the process before
trying it with their small group. Shuffle the integer cards and place them face down in the center of the group. Student 1 will draw three cards from the pile and roll the number cubes. Looking at each number cube, an even number represents addition and an odd number represents subtraction. Using the cards drawn and the operations rolled, Student 1 will write a number sentence in the appropriate place on the Which Direction BLM and describe the action in words. For example, if the cards 4, -3 and 2 are drawn and two odd numbers (subtraction) are rolled on the number cube, the following number sentences are possible, but students only use one:

\[ 4 - (-3) - 2 \text{ Start at 4, face left and move back 3 spaces, you are at 7 then move up 1 space (move 2 spaces to the left) You will be at 5} \]
\[ 4 - 2 - (-3) \]
\[ -3 - 4 - 2 \]
\[ -3 - 2 - 4 \]
\[ 2 - (-3) - 4 \]
\[ 2 - 4 - (-3) \]

If an odd number and an even number is rolled, then the expression would contain both subtraction and addition, i.e. \( 4 - (-3) + 2 \) rather than \( 4 - (-3) - 2 \).

Student 2 will model the first part of the expression, \( 4 - (-3) \), on the number line given on the BLM, and describe the action in words (Start at 4, face left and move back 3 spaces landing on 7). Student 3 will model the next part of the expression, \( 7 - 2 \) on the same number line and describe the action in words (Start at 7, face left and move back 2 spaces, landing on 5). Student 4 will determine if their number line helps to determine if the SQPL statement is true or false and describe how this action proves or disproves the statement. In this example, students should see that \( 4 - (-3) \) is the same as \( 4 + 3 \) and would help prove that the statement is true.

Number line for this example: \( 4 - (-3) - 2 \)

Students may need to play a few more rounds to prove whether the statement is true or false. This discovery should lead students to the conclusion that subtraction is the same as “adding the opposite.” Help students understand that although the two expressions are equivalent in the sense that they give the same result, they still have different meanings.

Once the activity is completed, have the students reread the list of questions that were generated at the beginning of the activity using the SQPL statement. As a class, discuss whether these questions were answered as they completed the activity. Have the students
write the ways the SQPL strategy helped them to see that the subtraction of rational numbers is the same as adding the additive inverse, \( p - q = p + (-q) \) in their learning logs.

Activity 13: Integers All Around (GLE: 7)

Materials List: paper, pencil

Provide the students with several different real-life problems involving integers. Example situations might include gaining and losing yards in football, temperature change on a thermometer, number cubes with positive and negative numbers, game shows involving making or losing money, or a dart game with positive and negative amounts on the board. Have the students work the problems, and check their answers to see if they are reasonable.

Have students work in pairs to create 2 to 3 of their own problems involving three or more integers. Instruct groups to exchange problems. First, students will write the number sentence they will use to solve their partner’s problem. Then, students will demonstrate their understanding of solving real-life problems with integers by responding to a SPAWN (view literacy strategy descriptions) prompt. SPAWN is an acronym that stands for five categories of writing prompts (Special Powers, Problem Solving, Alternative Viewpoints, What If?, and Next), to stimulate students’ predictive, reflective, and critical thinking about content-area topics. After writing the number sentence, students will describe in words the Next step in solving the problem and then solve. Students will write their final answer in a complete sentence. Remind students to check their answers for reasonableness. Once students give the problems back to the original owner, the writer of the problems will check to see if the description of what to do next matches the actual problem-solving and then correcting, if necessary.

Example:
In yesterday’s football game, John made the following plays: gained 8 yards, lost 2 yards, lost 3 yards, gained 5 yards. What the total amount of yardage that John gained or lost?

\[
8 + (-2) + (-3) + 5
\]

The next step to solve this problem is to combine the negative numbers, then combine the positive numbers, then add to get the total.

\[
(8 + 5) + (-2 + -3)
\]

\[
13 + (-5)
\]

8

John gained a total of 8 yards on four plays.
Activity 14: Cooperative Problem Solving (GLEs: 5, 7, 10, 11)

Materials List: Cooperative Problem Solving BLM, pencil, paper

To prepare for this activity, copy the Cooperative Problem Solving BLM, and cut the pieces apart. Problems may be separated by placing them in sandwich bags to be distributed to the student groups.

Have students work in groups of two or three to solve real-life situations. Each group should be given one sandwich bag that contains the pieces of one word problem. Each student takes at least one card and keeps it in his/her possession. Have students in each group take turns sharing the information on their cards, then work together to find a solution to the situation. This is a good tool to get all students involved in the problem-solving process. Even the weakest students have a part, because they must contribute the information on their cards and read the information to the group in order for the problem to be solved.

Example of a set of cards that one group would solve:

| The seventh graders are planning to sell cups of hot chocolate at the basketball games this winter. | If 6 spoonfuls of mix make a cup of hot chocolate, | How many spoonfuls of mix will be needed to make 42 cups of hot chocolate? |

Allow time for student groups to share their problem and solution. Lead a class discussion about the different methods used to solve the problems. Students should be able to identify the quantities being compared; point out that when two quantities are compared and written as a fraction, that fraction is called a ratio. Have students find equivalent ratios in the problem they worked. Model how to set up and solve proportions using the problem set in this activity. Remind students that each proportion is two equivalent ratios.

Activity 15: Common Ratios (GLEs: 5, 10, 11)

Materials List: a measuring tape for each group, Common Ratios BLM for each student, pencils, calculators

Remind students how they used proportions in the previous activity to solve problems when equivalent ratios could be written. Include examples to reinforce this concept.

Students will compare their heights with other measurements of their body to the nearest millimeter to determine if there is a common ratio. Be careful with the division of groups during this activity. Give each student a copy of the Common Ratios BLM for recording measurements. Have students work in groups of three or four. Ask them to take turns
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with one student measuring, one student recording on the given chart, and the third being the person measured. Have students complete the chart by measuring the distances described in the chart, and then finding each ratio. Have the students compare their findings to the findings of the other students in the group. Then have the students complete the questions as a group.

Have students research the work of Leonardo da Vinci to see if their proportions relate to his ratios. Have students use da Vinci’s ratios to predict their measurements. What is The Golden Ratio? How do artists use these ratios today? A website that provides information on the Golden Ratio include http://en.wikipedia.org/wiki/Golden_ratio.

Activity 16: In Another World! (GLEs: 5, 11)

Materials List: In Another World! BLM for each student, rulers and/or measuring tapes, pencils, calculators

You are a 65-inch tall Earthling who has landed on the world of Gianormas. Immediately upon arrival, you meet Leonardo who is 50 ft tall! As you look around, you notice that everything in this new world is Leonardo’s size. You assume that everything is to the same scale as it is on Earth.

Students will measure items that may be found in the classroom to the nearest quarter-inch. Then they will use proportions to find the measurement of the same items in a world of giants, then in a world of miniatures. Each student will need a ruler or measuring tape, a pencil, and a calculator to complete the In Another World! BLM. Remember to convert inches into feet or vice versa when necessary.

Activity 17: How Big is this Room Anyway? (GLE: 11; CCSS: 7.G.1)

Materials List: tape measure for each group of four, paper, pencil, poster size paper, index cards

Discuss blueprints. Use a vocabulary card (view literacy strategy descriptions) to help students understand how scale drawings relate to proportional reasoning. A vocabulary card is designed to help students learn content-specific terminology and has been shown to increase depth and breadth of word knowledge, resulting in greater comprehension. Begin by writing the term, scale drawing on the board and draw a large, rectangular card-like frame around it so that it’s in the center of the rectangle. In the corners of the card write a definition, characteristics, examples, and an illustration of the term.
Students will complete the parts of the card that they know. As the activity continues, students will add to the card as understanding of the term increases. Divide students into groups of four. Have students measure the length of each wall of the classroom and record the actual lengths to the nearest ½ foot on the board. Have students determine a scale that would represent the actual size of the classroom on a poster size sheet of paper. (Example: ½ inch = 1 foot.)

Determine the scale lengths of the classroom walls using the scale determined. Draw a classroom model using these measurements to the nearest ½ inch on poster size paper. (This is your classroom floor plan and should hang on the wall)

*Ask: What is the actual area of the classroom? What is the scale area of the classroom? Compare the measures of the actual and scale of the room (dimensions and area).*

Divide students into groups of 3 – 5. Assign each group a different object in the classroom (teacher’s desk, bookcase, trash can, etc.) to measure (remember only length and width are needed for the blueprint). Students should convert actual measurements to the scale measurements and draw and cut out these models from an index card. Each student should measure his/her own desktop and create a scale cut out of his/her desktop for the classroom model – each student should write his/her name on the desktop.

*Ask: What is the actual area of your desktop? What is the scale area of your desktop?*

*Ask: What comparisons do you see as you make observations of the areas of your room and desktop? List your observations. Be sure to bring out that the area of the actual is 4 times the area of the scale.*

Each group should present the scale model of their classroom object (not desks at this time) for the blueprint to the class. Discuss methods of determining these measurements and then glue these objects in the correct position on the classroom model. Have students, one group at a time, place their desk scale drawing on the classroom model. (It might be easier to work from the center out.) Post blueprints/scale models on the wall for all classes to compare.
Using the class scale model of the classroom, have students make predictions about distance from various points in the room (i.e., If the distance from the teacher’s desk to the board is 5 inches on the scale model and the scale of 1 inch represents 4 feet, then the actual distance is 20 feet.) Have students measure the actual distance(s) to check for accuracy of the scale model of the classroom.

Students will complete the vocabulary card and use it as a study aid for tests, quizzes or other activities with the word. A completed vocabulary card may look like this:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>a reduction or an enlargement of an actual object</td>
<td>Written like a proportion</td>
</tr>
<tr>
<td>Scale drawing</td>
<td>Ratio of drawing to actual</td>
</tr>
<tr>
<td>Map</td>
<td>Scale = size of drawing</td>
</tr>
<tr>
<td>floorplan</td>
<td>size of actual object</td>
</tr>
<tr>
<td>drawing of a cell</td>
<td>Illustration</td>
</tr>
</tbody>
</table>

**Activity 18: Scale Drawings** (GLE: 11; CCSS: 7.G.1)

Materials list: Scale Drawings BLM, pencil, paper

Provide the students with the problems to practice scale drawing problems by distributing Scale Drawing BLM. Give students time to work through these situations and then divide them into groups of four to discuss the situations. Have students in groups come to a consensus on the solutions to these problems and then have them prepare for a discussion using professor know-it-all (view literacy strategy descriptions). With this strategy, the teacher selects a group to become the “experts” on scale drawing required in the situation that is selected. The group should be able to justify its thinking as it explains its proportions or solution strategies to the class. All groups must prepare to be the “experts” because they are not told prior to the beginning of the strategy which group(s) will be the “experts.” All groups should prepare questions about scale drawings to ask the “experts” should they not be chose as the “expert” group.
Sample Assessments

General Assessments
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions.
- The student will create and demonstrate math problems by acting them out on using manipulatives to provide solutions on the board or overhead.
- Use the website http://www.rubrics.com to create a rubric to assess student work.
- The student will complete journal entries using such prompts as:
  - Explain whether the exact product of (1.40)(0.099) will be greater than or less than the estimate (1.4)(1). How can you tell without multiplying 1.4 and 0.099?
  - Using the numbers -6, $\frac{1}{2}$, -2, and 5 and any three operations, what problems can you write for which the answer fall between 2 and 0?
- The student will create a portfolio containing samples of his/her ability to work problems such as the following:
  - While watching the LSU football game, Jerrica became very thirsty. The snack stand sold drinks in 4 sizes.
    - Kiddie Size: 20 ounces for $0.80
    - Adult Size: 32 ounces for $0.90
    - Super Size: 44 ounces for $0.99
    - Super Duper Size: 64 ounces for $1.25
    For which size would Jerrica get the most drink for her money? Explain how you made your decision. If the snack stand offered a Mega Size drink, how much should Jerrica expect to pay? Explain your thinking.

Activity-Specific Assessments
- **Activity 2**: The student will solve this problem: Monica needed to triple a recipe for cookies. The recipe called for $2 \frac{1}{2}$ cups of flour and $1 \frac{3}{4}$ cups of sugar. How much will she need of each? Prove your answer. Explain how this problem illustrates multiplication of fractions.
  
  **Solution**: To triple a recipe means to multiply by 3. Three groups of $2 \frac{1}{2}$ for flour, and 3 groups of $1 \frac{3}{4}$ for sugar.

- **Activity 3**: The student will solve this problem correctly: A local coffee house donated twelve pounds of fresh-roasted coffee. The students are running a fund-raiser at school and decide to sell the coffee in bags. How many bags can be made if each bag contains $\frac{1}{4}$ pound? $\frac{1}{8}$ pound? Explain how you arrived at your answer.
Solution: Twelve divided into bags that are $\frac{3}{5}$ of a pound. Students can draw a picture of the situation. Twelve divided into bags that are $\frac{1}{8}$ of a pound. Students can draw a picture of the situation.

- **Activity 4:** The student will solve this problem correctly: Carlos placed a bunch of grapes on a scale at a fruit stand. The bunch weighed 2.7 pounds. Grapes are on sale for $1.59 a pound. Suppose Carlos has a $5 bill. Does he have enough money to buy the grapes? Justify your answer.
  
  Solution: $2.7 \times 1.59 = 4.29$ Yes, Carlos has enough money.

- **Activity 8:** The student will respond to the following situation either to turn in or to write in their math learning logs (view literacy strategy descriptions). Three students were asked to evaluate the following expression: $46 - 30 \div 2 \times 3 + 6$

  Which solution do you think is correct?
  
<table>
<thead>
<tr>
<th>Meghan’s solution</th>
<th>Quinton’s solution</th>
<th>Lane’s solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$46 - 30 \div 2 \times 3 + 6$</td>
<td>$46 - 30 \div 2 \times 3 + 6$</td>
<td>$46 - 30 \div 2 \times 3 + 6$</td>
</tr>
<tr>
<td>$16 \div 2 \times 3 + 6$</td>
<td>$46 - 30 \div 6 + 6$</td>
<td>$46 - 15 \times 3 + 6$</td>
</tr>
<tr>
<td>$8 \times 3 + 6$</td>
<td>$46 - 5 + 6$</td>
<td>$46 - 45 + 6$</td>
</tr>
<tr>
<td>$24 + 6$</td>
<td>$41 + 6$</td>
<td>$1 + 6$</td>
</tr>
<tr>
<td>$30$</td>
<td>$47$</td>
<td>$7$</td>
</tr>
</tbody>
</table>

  Explain why you agree with the solution you selected.

- **Activity 15:** Look for completeness of charts with body measurements and ratios along with a written summary of discovery.

- **Activity 18:** Students will make a scale drawing of the following using a scale of 0.5 in. = 2 ft. using correct proportions.

  A rectangular kitchen 12 feet by 16 feet and a rectangular cooking island in the kitchen that is 4 feet by 5 feet.
Grade 7 Mathematics
Unit 3: Expressions and Equations

Time Frame: Approximately six weeks

Unit Description

This unit ties numerical problem solving to algebraic problem solving. Starting with computations using the distributive property, the unit moves into using properties of operations to generate equivalent expressions to solving real-life and mathematical problems using numerical and algebraic expressions, equations, and inequalities.

Student Understandings

In this unit, students gain an understanding of exponents of 2 and 3 and the evaluation of expressions containing these exponents. They should be able to use mental math to match algebraic inequalities with the situations they model, particularly in using inequalities to approximate the values of square roots that are not perfect. Students should be able to apply the distributive property of multiplication over addition to solve problems. Students will also gain an understanding that rewriting an expression in different forms in a problem context can shed light on how the quantities in it are related. They should be able to use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Guiding Questions

1. Can students apply the distributive property?
2. Can students approximate the square and cube roots of numbers using inequalities and perfect squares and cubes?
3. Can students generate equivalent expressions?
4. Can students write equations and inequalities generated from real-world situations and solve fluently?
5. Can students graph points on a coordinate grid?
6. Can students graph the solution set of an inequality and make sense of it in the context of the problem?
7. Can students link algebraic inequalities with their verbal descriptions.
8. Can students solve and extend patterns using exponents.
## Unit 3 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>Grade-Level Expectations</th>
<th>CCSS for Mathematical Content</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GLE #</strong></td>
<td><strong>GLE Text and Benchmarks</strong></td>
</tr>
<tr>
<td><strong>Number and Number Relations</strong></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Solve order of operation problems involving grouping symbols and multiple operations. (N-4-M)</td>
</tr>
<tr>
<td><strong>Algebra</strong></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)</td>
</tr>
<tr>
<td>18.</td>
<td>Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21, …) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)</td>
</tr>
</tbody>
</table>

### Sample Activities

**Activity 1: Developing the Distributive Property (CCSS: 7.EE.2)**

*This activity has not changed because it already incorporates the CCSS.*

Materials List: pencil, paper

Start this activity by having the students solve one of these problems: 34×8 or 157×5. Next, ask students to explain their methods for solving the given problem. Students may show methods such as repeated addition or just regular multiplication. Students may use the distributive property and not know that they have done so. If students do not mention the distributive process or describe the use of the property for solving such a problem, ask the class if it would be possible to work 34×8 by finding 30×8 and adding this answer...
to $4 \times 8$. If they agree that it is possible to do this, have the students try $157 \times 5$ using the same method.

One way to efficiently multiply a two-digit number by a one-digit number is to use the distributive property. Provide several problems which require the use of the distributive property (e.g., $6(24) = 6(20 + 4) = 6(20) + 6(4) = 120 + 24 = 144$). Remind students that rewriting 24 as $20 + 4$ is writing the number in expanded form, a process they learned in grade school.

Do not allow students to use pencil, paper or calculators for the next problem. Put the following problem $(6 \times 84) + (6 \times 16)$ on the board allowing only 5 - 10 seconds for student think time. Students will not be able to find a solution in the time allowed because they are trying to multiply 6 and 84, then multiply 6 and 16, and add the two. Give students a hint that the distributive property could be used to work this problem quickly. Let students see if they can come up with how the distributive property could be used. $(6 \times 84) + (6 \times 16) = 6(84 + 16) = 6 \times 100 = 600$.

After practicing several forms of distributive property problems, ask: “Will the distributive property work when dealing with subtraction?” Have the students prove why by making up problems to show verification. $(7 \times 73) - (7 \times 33) = 7(73 - 33) = 7 \times 40 = 280$

Will the distributive property work with fractions? Try this one: $\frac{1}{2} \times 6 \frac{2}{3}$. Hopefully, this will surface: $(\frac{1}{2} \times 6) + (\frac{1}{2} \times \frac{2}{3}) = 3 + \frac{1}{3} = 3 \frac{1}{3}$. You take half of 6 and half of two-thirds, then just group the answers together. It might not always be this easy, but it will work.

Pair students and have them develop their own examples using whole numbers and fractions with addition and subtraction. Ask them to share the examples with another pair of students to see if they can solve the problems using the distributive property.

**Activity 2: Distribute It With Candy Bars! (CCSS: 7.EE.2)**

This activity has not changed because it already incorporates the CCSS.

Materials List: pencil, Candy Bars BLM for each student

Give each student a copy of Candy Bars BLM. Work through the problems with the students, or have students work in pairs, then have a class discussion about the problems.

The Candy Bar BLM gives the students a scenario that involves selling three types of candy bars. Different types of candy bars are packaged with different numbers of bars in each box. Students must use the distributive property to solve problems about selling the candy bars. Additional problems may be used as needed.
Activity 3: Perimeter of a Corral (CCSS: 7.EE.2)

This activity has not changed because it already incorporates the CCSS.

Materials List: pencil, paper

Have groups of students explore how to determine the perimeter of a rectangular corral that has a width of $x$ feet and a length of $x + 40$ feet.

Have students write the expressions for finding the perimeter different ways, explaining the method each time. Possible examples of expressions include adding all the side lengths, $x + x + (x + 40) + (x + 40)$, showing that there are two lengths and two widths, $2(x) + 2(x + 40)$, and using the distributive property, $2[x + (x + 40)]$. Have a class discussion of the various methods found. Give each group a different value for $x$ and have each student in the group use a different expression to find the perimeter. Ask group members to compare their answers with one another. Did all the expressions give the same perimeter? Why or why not? Repeat this activity using various widths and lengths.

Present the class with a SPAWN (view literacy strategy descriptions) prompt. SPAWN is an acronym for six categories of writing prompts to develop student thinking and get them to articulate in their own words what they’ve learned. Using the W or “What If?” category of SPAWN, present students with the following prompt:

Matt’s dog needs a fenced yard. Matt’s uncle Thomas has 132 feet of chain fence he will give Matt. What if Matt uses the back of his house as one side? He then only has to put fence around three other sides. His house is 32 feet long. Describe two ways Matt could make a fenced in yard for his dog. Include sketches and dimensions in your description.

Students can write their response to this SPAWN prompt in their learning logs (strategy link). Once students have completed their responses, have them share with a partner or the whole class to stimulate discussion about approaches to solving the problem.

Activity 4: Number Line Square Roots (CCSS: 7.EE.3)

This activity has not changed because it already incorporates the CCSS.

Materials List: square tiles, Square Roots BLM or index cards labeled with similar problems, learning logs
Give each student 4 square tiles and instruct them to use the tiles to make a larger square. Have a class discussion in which students explain what they know about the new square. *(Sample Answer: The new square is 2 tiles by 2 tiles. Its area is 4 square tiles.)* Discuss the concepts of perfect squares and square roots. Squaring a number and finding the square root are inverse operations just as addition and subtraction “undo” each other. Indicate that a perfect square number gets its name because you can make a square with that number of given tiles; therefore, 4 is called a perfect square. You cannot make a square with 5 tiles, so 5 is not a perfect square. Another way of knowing if a number is a perfect square is by determining its square root. The square root of a perfect square is a whole number. \( \sqrt{4} = 2; 2^2 = 4 \). Be sure students can distinguish between squares and square roots.

Now, have two students use their combined blocks to make the biggest square possible. Have groups explain their solution and reasoning. The largest square possible measures 2 by 2; the next biggest square would be 3 by 3 or 9 square tiles. The students only have 8 tiles. 8 is not a perfect square; there is no whole number that can be multiplied by itself (squared) to get 8.

Have students use other sums of square tiles to model perfect squares. Instruct students to find the first twenty perfect squares; \(1^2=1, 2^2=4, 3^2=9, 4^2=16, \) and so on. Draw a number line on the board and number it from -20 to 20 (or more) on which students will determine the placement of square roots. Cut apart the problems on the Square Roots BLM or put similar problems on index cards. Examples of problems: What is the approximate value of \( \sqrt{6} \)? What is the value of \( \sqrt{64} \)? Make sure to have several types of problems.

Pass out all cards to the students. Have a student read his/her card aloud, and place a mark on the number line where he/she thinks the answer belongs.

Have a class discussion about the placement of the number on the number line each time. Ask questions regarding the placement of the answers. Example: Is your answer for the \( \sqrt{6} \) closer to 2 or 3? Why? Continue until all problems have been read aloud and the answers placed on the number line.

Have students respond to the following prompt in their math learning logs (view literacy strategy descriptions): Is \( \sqrt{21} \) closer to 4 or 5? Justify your reasoning with words, numbers, and a number line.
Activity 5: Area of a Corral (GLE: 12)

Materials List: paper, pencil

Have students work in groups of 2 or 3 to explore how to determine the area of a rectangular corral that has a width of \(x\) feet and a length of \(x + 30\) feet.

\[
\begin{array}{|c|c|}
\hline
x & 30 \\
\hline
\end{array}
\]

Ask each group of students to choose and substitute a three digit number for \(x\) and find the area of the corral. Instruct students to show more than one method for finding the area. (i.e., finding the areas of the two smaller rectangles and then adding them or finding the area of the large rectangle in one step). Make sure students do not leave out the units. Remind students that area is measured in square units: if \(x = 400\) ft then,

\[
(400\text{ ft})^2 + (400\text{ ft} \times 30\text{ ft}) = 160,000\text{ ft}^2 + 12,000\text{ ft}^2 = 172,000\text{ ft}^2
\]

Once students have completed the problem above, ask them to demonstrate their understanding of area by completing a RAFT writing (view literacy strategy descriptions) assignment. This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, RAFT writing has been used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. It’s the kind of writing that when crafted appropriately should be creative and informative.

Ask student groups to write the following RAFT:

- **R** – Role of the writer—Store Clerk at Sam’s Feed and Seed
- **A** – Audience or to whom or what the RAFT is being written—Mr. Mobley, owner of corral
- **F** – Form, the form the writing will take, as in letter, song, etc.—estimate
- **T** – Topic or the subject focus of the writing—On company letterhead, create an estimate for Mr. Mobley, the owner of the corral. Include a drawing of his corral, the area of the corral, and the projected cost to plant grass seed in his corral. A 5 pound bag of grass seed will cover 2,000 square feet and costs $15.00.

Allow time for student groups to share their RAFTs with the class and discuss their estimate/work with the class. Students should listen for accuracy and logic in their classmates’ RAFTs.
Activity 6: Four in a Row (GLEs: 3, 12, 29)

Materials List: overhead, overhead pens, coordinate grid transparency, pencils, graph paper, cup

Show students how to simplify order of operations problems that contain square roots such as $5x^2 + \sqrt{16} - 2$.

Divide the class into two teams. To start play, put an algebraic expression on the overhead and display a coordinate grid on the board. Have one team give coordinates for a point they wish to capture on the coordinate grid. Circle the point on the coordinate grid. Instruct the team to substitute the coordinates of the point chosen into the algebraic expression. If the team provides the correct answer, award the team that point and fill in the circle with the color chosen for that team. If the team in play cannot provide a correct answer, give the opposing team an opportunity to fill in the circle. For example, give the expression $x + y^2$. Team 1 wants to capture (2, 3), so they substitute the coordinates for the variables in the expression: $2 + 3^2$ and evaluate to get 11. Have teams alternate playing until one team has captured four grid points in a row horizontally, vertically, or diagonally.

Be sure to have several expressions already made up before play begins, and see that the expressions contain exponents (especially 2 and 3) and square roots. Example of expressions: $2x + 3y^2$, $\sqrt{16x - \frac{y}{2}}$, $(x + y)$, $x^2 + 2y$, $3x - y^3$, $2x^2 + \sqrt{25}$

Extension:
After game has been played as a whole class, divide each team into two new teams to play. Before play begins, have each team create 8-10 expressions, cut them apart, and place them in a cup. Provide additional expressions to each game to ensure that all of the skills are being covered. To start play, one team picks an expression out of the cup, chooses the coordinates for the point they wish to capture, then substitutes the coordinates of the point into the expression and evaluates it. The other team must check their work. If a team provides a wrong answer, the other team has a chance to capture the point by providing a correct solution. Teams take turns until one team has captured four points in a row horizontally, vertically, or diagonally.

Activity 7: What’s My Value? (GLE: 12)

Materials List: What’s My Value? BLM, pencils

Before beginning this activity, lead a class discussion on the building of the cubes in Activity 4 in this unit.

The What’s My Value? BLM provides a list of algebraic equations involving exponents of 2 and 3 and square roots. Students are instructed to determine which replacement
values from the second column should be used to make the equation in the first column true.

Examples:  

<table>
<thead>
<tr>
<th>Equation</th>
<th>Replacement Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{x} + 5 = 7 )</td>
<td>( x = 7 )</td>
</tr>
<tr>
<td>( x^2 - \sqrt{3} = 46 )</td>
<td>( x = 4 )</td>
</tr>
<tr>
<td>( x^3 + \sqrt{4} = 29 )</td>
<td>( x = 3 )</td>
</tr>
</tbody>
</table>

After students have completed What’s My Value? BLM, lead a discussion about how students found their answers. Most students will use the guess and check method for most of the problems. Suggest the work backwards strategy if none of the students mentions it. Model the strategy using several of the equations from this activity. Then give students some additional equations similar to these to work using the work backwards strategy.


Materials List: Paper, pencil, What’s My Number Part 1 BLM

In this activity, students will use variables to describe relationships among the steps in a number puzzle. Students’ fluency in describing how to generalize the steps in a think-of-a-number puzzle demonstrates their readiness to move to more formal work with symbols.

Present the following puzzle from the What’s My Number Part 1 BLM to students, having them perform the steps as directed, first choosing whole numbers, and then determine their results. Pair students to compare final results. Each student’s final result will be 13.

Step 1: Choose a two-digit number.
Step 2: Add that number to itself.
Step 3: Add 20
Step 4: Subtract 12
Step 5: Subtract the original number.
Step 6: Add 5
Step 7: Subtract the original number.
Step 8: What is the final result?

Next, students will work with a partner to write an algebraic expression that describes what happens to the original number \( n \) at each step. One way to think about this task is to acknowledge that only one variable is involved—the initial number that is selected. Here is one way this can be written:

Step 1: \( n \)
Step 2: \( n + n = 2n \)
Step 3: \( 2n + 20 \)
Step 4: \( 2n + 20 - 12 = 2n + 8 \)
Step 5: \( 2n + 8 - n = n + 8 \)
Step 6: \( n + 8 + 5 = n + 13 \)
Step 7: \( n + 13 - n = 13 \) (final result)

Once complete, have pairs group with another pair to create a group of 4 and share expressions. Make sure that groups can explain how the expressions are related, especially if the expressions are written differently. For example, why is \( 2n + 8 - n \) equivalent to \( n + 8 \)? Did someone write \( n + n + 8 - n \) instead? How is this expression related?

After formatively assessing student understanding on Puzzle 1, create new groups based on student understanding. Puzzle 2 can be assigned to students that still need practice and Puzzle 3 can be assigned to students that demonstrate fluency with writing equivalent expressions. Students can solve using numbers on the back of the BLM and write the algebraic expressions in the box provided.

Activity 9: What’s My Number? (Part 2) (CCSS: 7.EE.2)

Materials List: What’s My Number Part 2 BLM

In this activity, students will explore a number puzzle to determine if the result will be the same if the initial number is not a whole number. In the previous activity, students applied the properties of operations with whole numbers to generate equivalent expressions until the final result was reached.

Puzzle 2 from previous activity with solution:

<table>
<thead>
<tr>
<th>Step 1: Write down any whole number.</th>
<th>Step 1: ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Add the number that is 1 less than the original number.</td>
<td>Step 2: ( n + n -1 = 2n - 1 )</td>
</tr>
<tr>
<td>Step 3: Add 9 to this result.</td>
<td>Step 3: ( 2n - 1 + 9 = 2n + 8 )</td>
</tr>
<tr>
<td>Step 4: Divide the sum by 2.</td>
<td>Step 4: ( (2n + 8) \div 2 = n + 4 )</td>
</tr>
<tr>
<td>Step 5: Subtract the original number.</td>
<td>Step 5: ( n + 4 - n = 4 ) (final result)</td>
</tr>
<tr>
<td>Step 6: What is the final result?</td>
<td>Step 6: ( 4 )</td>
</tr>
</tbody>
</table>

Ask students if they think the result will be the same for other rational numbers such as fractions, decimals, or integers? Distribute the What’s My Number BLM for students to use to explore the possibilities. Once the grid is complete, pair students to compare the expressions generated and challenge them to prove that the result of the puzzle is the same with a different non-whole number. Students will respond in their learning logs (view literacy strategy descriptions) to the question posed at the beginning of the activity using an example from the word grid to explain their reasoning.
Activity 10: Tiling Tubs (CCSS: 7.EE.2)

Materials List: Tiling Tubs BLM

This activity will help students to see that a variety of equivalent expressions can be represented by one situation. Unlike problems in which students may have looked at a pattern of change as a figure increases in size, in this task, students must be able to state the relationship that is determined by knowing a variable side length, s, and the context of the problem. Encourage students to sketch how they “see” the algebraic expression on the drawing to prove how the expression and drawing are related.

Distribute Tiling Tubs BLM and have students complete the following task independently:

Hot tubs and in-ground swimming pools are sometimes surrounded by borders of tiles.
1. How many 1-foot square tiles will be needed for the border of a square hot tub that has edge length s feet?
2. Express the total number of tiles in as many ways as you can.
3. Be prepared to convince your classmates that the expressions are equivalent.

Next, students can discuss ways to determine whether one expression is equivalent to another. Discuss the use of the commutative, associative, and distributive properties in conjunction with the order of operations. Possible solution strategies: 4(s + 1); s + s + s + s + 4; (s + 1) + (s + 1) + (s + 1) + (s + 1); 2s + 2(s + 2); 4(s + 2) – 4; (s + 2)^2 – s^2

Activity 11: Equation Relationships (CCSS: 7.EE.2)

Materials List: paper, pencil, Equation Relationships BLM

This activity will help students to visualize the equivalence between different expressions using the distributive property, 6 – 3a and 3(2 – a); the commutative property, 6 – 3a and -3a + 6; or selected values of a (6 – 3a = a + 14 when a = -2).

To review the distributive and commutative properties, students will complete a word grid (view literacy strategy descriptions). A word grid provides students with an organized framework for learning words by analyzing the similarities and differences. Learning vocabulary through the use of word grids allows students to contextualize vocabulary knowledge. On a sheet of paper, students will construct the following grid:
### Expressions and Equations

<table>
<thead>
<tr>
<th>Expression</th>
<th>Distributive Property</th>
<th>Commutative Property</th>
<th>Neither</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 9 + 6 = 6 + 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. 5 \times 76 = (5 \times 70) + (5 \times 6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. -2 + 8 = 8 - (-2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. 6(3 \cdot x) = 18 \cdot x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. 3(5) - 3(4) = 3(5 - 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. 5 - 3 = 3 - 5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students will place a check mark in the correct column that the expression illustrates (distributive, commutative or neither). Once the grid is complete, ask students to identify the expression(s) which illustrate the distributive property (B and E) and what they have in common. The result is the same whether you multiply first and then add (or subtract) or if you add (or subtract) first and then multiply. Ask students to identify the expression(s) which illustrate the commutative property (A and C) and what they have in common. The order in which you add or multiply two numbers does not change the sum or product. Next, ask students to identify the expression(s) that were neither distributive nor commutative and why. Students may think D illustrates the distributive property simply because it contains parentheses. These expressions are equivalent simply through multiplication. Students may think F illustrates the commutative property simply because the numbers are reversed without paying attention to the operation of subtraction, which is not commutative. To summarize the grid, ask students to rewrite D to show the distributive property $6(3 + x) = 6(3) + 6(x)$ and F to show the commutative property $5 + 3 = 3 + 5$ or $5(3) = 3(5)$ in the “neither” column.

Distribute the Equation Relationships BLM. Students will work with a partner to substitute the given values for $a$ in the equations and record the solutions in the appropriate place in the table. Next, students will write three observations describing how the quantities are related.

Questions to ask when the table is complete:
- For what value of $a$ is $-6a = a + 14$?
- For what value of $a$ is $a + 14 = 6 - 3a$?

Students should be able to visualize the equivalence between different expressions using the distributive property, $6 - 3a$ and $3(2 - a)$ and the commutative property, $6 - 3a$ and $-3a + 6$. Ask students to add the expressions from the table that illustrate either the distributive $6 - 3a = 3(2 - a)$ or the commutative property $6 - 3a = -3a + 6$. 

Grade 7 Mathematics ◊ Unit 3 ◊ Expressions and Equations
Activity 12: The Mystery Line (CCSS: 7.EE.4)

Materials List: Mystery Line BLM

In this activity, students will use the information given in the diagram below to set up a series of equations to find the length of section \( z \).

\[
\begin{array}{ccccccc}
& & & & Z & \downarrow & y \downarrow \downarrow x \downarrow x \downarrow y \downarrow z \downarrow &
\end{array}
\]

\[
\begin{array}{ccccccc}
& & & & 40 & \downarrow & 58
\end{array}
\]

Solution:
The section that is 40 units in length can be represented as \( y + x + x \) or \( 2x + y = 40 \).
The section that is 58 units in length can be represented by \( x + x + y + z \) or \( 2x + y + z = 58 \).

Next, substitute 40 for \( 2x + y \) in the first equation: 
\( 40 + z = 58 \) so \( z = 18 \)

Activity 13: Situations with Equations (CCSS: 7.EE.4)

This activity has not changed because it already incorporates the CCSS

Materials List: Equations BLM, newsprint or large paper (2-3 sheets per group of four), markers

Lead a class discussion about algebraic equations. Include examples like those found on Equations BLM. Ask students how they would go about solving one of these. Lead students to solve the equations using symbolic steps.

Have students work in groups of four to make a team. Give each team 2-3 equations from the Equations BLM. Have each team work together to solve their equations on newsprint or large paper, using symbolic steps and/or diagrams to justify their work. Ask students to write a situation that would represent the equations given to them.

Inform the students after the teams have completed their work, they will assume the role of professor know-it-all (view literacy strategy descriptions) and present their work and situations with the class. One member of each team should explain the addition/subtraction grouping, one the multiplication/division grouping, one the checking step, and another the situation that represents the equation. Remind students that as professor know-it-all, they must be prepared to defend/prove their reasoning for their work.
Send one group to the front of the room at a time. When students present, be sure they explain their procedures verbally and that they don’t just give written steps. Also, prepare the other students to formulate questions to ask of the professors and to hold them accountable for the accuracy of their answers.

Instead of using newsprint or large paper, give each group blank transparencies and let them use the overhead when presenting.

Add equations to this problem set, if needed.

**Activity 14: Headlines and Stories (CCSS: 7.EE.4)**

Materials List: Paper, pencil, chart paper, markers, post-it notes

In this activity, students will work in groups of four to write a situation (story) that must match a given equation (headline) and solve. Using a text chain (view literacy strategy descriptions), each member of the group will complete a step to reach the final solution. In this case, a text chain will give students the opportunity to demonstrate understanding of how to solve equations by writing out each step in a collaborative context.

Divide students into groups of 4 and distribute a sheet of chart paper and markers to each group. Write the following equation on the board: $2p + 3 = 11$. Have one member from each group copy the equation at the top of the chart paper and explain that the equation is the “headline” to a story they will write. Student 1 will write the story (real-life situation that matches the equation). Student 2 will complete the next step (subtracting 3 from both sides). Student 3 will complete another step (dividing both sides by 2). Student 4 will make sure all the steps are correct and write the ending to the story which includes the answer. If any step is incomplete, the next student must correct the step as well as complete his/her own step.

An example of the story for this headline might look like this:

**HEADLINE:** $2p + 3 = 11$

**STUDENT 1:** The class needed twice as many pizzas for a party than what they had, then they realized they needed three more to give them a total of 11 pizzas to feed everyone. How many pizzas did they have originally?

**STUDENT 2:** $2p + 3 = 11$

$2p + 3 - 3 = 11 - 3$

**STUDENT 3:** $rac{2p}{2} = \frac{8}{2}$

$p = 4$

**STUDENT 4:** The class had 4 pizzas originally.

After Student 4 has checked to make sure all steps are correct, the group must also reach a consensus as to the logic and accuracy of the text chain and make any corrections. Each
group will exchange with another group to evaluate the story for logic and accuracy. Groups will use post-it notes or a colored marker to ask clarifying questions or to make corrections on the chart paper and return to the group that wrote it. Next, each group will present their text chain to the class (since only one other group saw it) and respond to the questions posed by the other groups on their chart paper.

Activity 15: Inequality Bingo (CCSS: 7.EE.4)

This activity has not changed because it already incorporates the CCSS

Materials List: Inequality Chart BLM (cut in two), Inequality Bingo BLM, Verbal Inequalities BLM (cut apart), a bowl or box, Algebraic Inequalities BLM (cut apart), small squares of paper

Before playing the game, students will complete a vocabulary self awareness chart (view literacy strategy descriptions) to assess their understanding of the meaning of inequality phrases and symbolic representation. This awareness is valuable for students because it highlights their understanding of what they know, as well as what they still need to learn in order to fully comprehend inequalities.

Distribute the Inequality Chart BLM and ask students to rate the meaning of each phrase according to their level of familiarity and understanding. A plus sign (+) indicates a high degree of comfort and knowledge, a check mark (✓) indicates uncertainty, and a minus sign (-) indicates the meaning of the phrase is new to them. Also, ask students to try to supply the meaning of the phrase and the symbolic notation. For phrases with check marks or minus signs, students may have to make guesses about the meaning and symbolic notation.

During the course of the game, students can revisit the chart to revise their entries in a different color to demonstrate growing understanding of verbal inequalities and the appropriate symbolic notation.

Version 1. Have students match algebraic inequalities with verbal statements by using a modified Bingo game played with a 3 x 3 grid on each card.

Give each student a copy of Inequality Bingo BLM. Point out there are two games on one sheet. On the first Bingo card, have students fill two spaces with the symbol <, two spaces with the symbol >, two spaces with the symbol ≤, two spaces with the symbol ≥ and one = symbol in the center (free space).

Randomly select numbered, simple word statements from a bowl or box and read them to the class. Have students mark off the symbol that should be used to represent the relationship indicated in the problem. Also, instruct students to mark the number of the problem on each square as they are filled as this will facilitate the verification of a winner. For example, the teacher reads problem #3 – I have less than Mary. The student
is expected to write #3 in one of the boxes that contains a < symbol then cover the symbol. When a student wins the game, let the class verify the winning card. The numbered simple word statements are on the Verbal Inequalities BLM.

Allow students to make up their own word statements that involve greater than, less than, and other comparisons for future games.

Version 2. Extend the activity by having students match algebraic inequalities with verbal statements and vice versa. Give students the list of inequalities found on Algebraic Inequalities BLM, and have them write one in each of the boxes on the second Bingo card. Randomly select word statements that are on the Algebraic Inequalities BLM from a bowl, and read them to the class. Have students use a square of paper to cover the inequality that represents the relationship indicated. For example, if Joe’s age, x, added to twice Morgan’s age, y, is greater than 26 is read, cover or mark the space containing \( x + 2y > 26 \) on his/her Bingo card. When a student wins the game, let the class verify the winning card.

It would be a good idea to display the sentences on the overhead or where students can see them.

For variation, continue playing any of the versions of the game until a black out is achieved.

Students can use the revised inequality chart as a reference sheet when working with inequalities. As students’ learning of inequalities extends to graphing, have students add the graph of the inequality to the chart.

**Activity 16: Real-Life Inequalities (CCSS: 7.EE.4)**

Materials List: paper, pencil, Inequality Situations and Graphs BLM

Have students write an inequality that represents a situation where a person has an allowance of $25 a month and must spend no more than 60% of this amount ($15) on snacks and entertainment. Have students find solutions to the inequality. Sketch a number line on the board and have the students determine a method of plotting their solutions. Lead a discussion about the meaning of the inequality and its solutions. Have the students work in pairs to complete the following inequality.

There is a building code in some states that requires at least twenty square feet of space for each person in the classroom. Suppose a classroom is 28 feet long and 18 feet wide. How many people can be in the classroom? Explain your answer and write an inequality to represent the solution.

\[ 20p \leq 504 \]
\[ p \leq 25.2 \text{ means “no more than 25 people”} \]
Distribute Inequality Situations and Graphs BLM to students. Have the students write the inequality that matches the situation, solve the inequality, sketch a graph to represent the solutions, and be ready to justify their solutions to other groups.

Have students share solutions with other groups and discuss as a class any solutions that they would like to challenge.

**Activity 17: Who Has? (CCSS: 7.EE.4)**

*This activity has not changed because it already incorporates the CCSS*

Materials List: index cards, transparencies cut into strips

Using index cards, make a set of cards showing verbal statements for one-step equations and inequalities – at least one for each student in the class. Example: My mystery number is greater than or equal to 6. Who has the graph and inequality for my mystery number?

Write answers on transparency strips with one strip having the inequality/equation and another strip showing the graph (as points, or open or closed rays on a number line).

Distribute at least one index card and two transparency strips (a graph and an inequality/equation) to each student. Have one student read the verbal statement aloud. Ask the students with the correct inequality and the graph to place them on the overhead. Continue the process until all cards have been used.

Example

*Verbal Statement on index card:* My mom says that I have to save at least $5.00 each time I get paid; who has the graph and inequality for this statement?

*Transparency:* I have the inequality: \( x \geq 5.00 \) (Student puts the inequality on the overhead).

*Transparency:* I have the graph. (Student puts the graph on the overhead)
Activity 18: Patterns to Investigate (GLE: 18)

Materials List: pencil, paper

In this activity, have students describe patterns by making charts, tables or drawing the patterns using a function machine. Ask them to write an expression describing the rule for the numbers in the sequence, and provide the 100th number in the sequence using the rule described.

Start by making a chart or table pairing the term number with each number in the sequence. Indicate that the students are to think of this as a function machine. The object is to try to figure out the rule that the function machine uses.

Table 1

<table>
<thead>
<tr>
<th>Term number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number in the sequence</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>15</td>
<td>55</td>
<td>105</td>
<td>n + 5</td>
</tr>
</tbody>
</table>

Have students make observations and discuss what they notice about the table. Ask them to describe and write the rule that generates the output from the given inputs.

In the table below, fill in the missing values for the following function machine.

Table 2

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td># in the sequence</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
<td>729</td>
<td>$3^{100}$</td>
<td>$3^n$</td>
</tr>
</tbody>
</table>

Have students make observations and discuss what they notice about the table. Ask how is it different from Table 1 and to describe and write the rule that generates the output from the given inputs.

Activity 19: More Patterns! (GLE: 18)

Materials List: toothpicks, pencil, paper

Present the following problem to the students. Have the students work in groups to complete the activity using a function machine.
An engineer designs the skeleton for the walls of a new stadium from equal lengths of steel beams placed in a rectangular pattern shown below. How many steel beams are needed to construct a wall whose length is 57 beams long?

Wall Length 1

Wall Length 2

Wall Length 3

(This is the view from the front.)

The length of the wall is measured by the number of beams along the bottom of the wall.

a. Ask students to model each wall length with toothpicks to represent the beams to a wall length of 6.
b. Then have students make a table, record the total of beams for each wall length, and share their observations with their group members.
c. Instruct students to predict how many beams are needed for walls of lengths 7 and 10.

Example of table with solutions.

<table>
<thead>
<tr>
<th>Wall Length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of beams</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>22</td>
</tr>
</tbody>
</table>

d. Have students write sentences to describe the patterns they see in the table (i.e., write a mathematical statement or rule to describe the rule that relates the length of a wall to the total number of beams). Have each group write its rule on chart paper to share with the class.

Some example or rules the student may see:  
a) The total number of beams is equal to 3 times the number of lengths plus 1.  
b) The total number of beams is equal to 1 less than the length, times 3, plus 4.  
c) The total number of beams is equal to 2 times the number of lengths, plus the number of lengths, plus 1.

e. Have the students describe their rules symbolically using \( l \) to represent the length of the wall and \( b \) to represent the total number of beams needed to construct the wall.

Example equations that go with the above rules:  
a) \( b = 3l + 1 \)  
b) \( b = 4 + 3(l - 1) \)  
c) \( b = 2l + l + 1 \)

f. Have the students determine how many beams are needed for a length of 57 and a length of \( n \).
Sample Assessments

General Assessments

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage the student to create his/her own questions.
- Have the student write equations for real-life situations and vice versa.
- Have the student create and demonstrate math problems by acting them out or by using manipulatives to provide solutions on the board or overhead.
- Use the website http://www.rubrics.com to create a rubric to assess student work.

The student will complete math learning log (view literacy strategy descriptions) entries using such prompts as these:

  - Compare and contrast $x^2$, $2x$, and $x + 2$.
  - Explain the difference between inequality, expression and equation.
  - Explain which represents the larger quantity: $2^3$ or $3^2$?

- The student will create a portfolio containing samples to show understanding of concepts learned in the unit. Example:
  
  Suppose you are developing a pattern. This is the first arrangement in the pattern. When you add squares to the pattern to make it grow, this is the second arrangement in the pattern.

There are several different ways to continue the pattern. Draw what you think the next three arrangements would look like. Make a table of values comparing the arrangement number to the total number of tiles used. Using the table, predict the total number of tiles in the $10^{th}$ arrangement and in the $100^{th}$ arrangement.

Activity-Specific Assessments

- Activity 2: The student will respond correctly to the following situation:

  The seats in a theater are divided into two sections. Section A has 15 rows of seats. Section B has 13 rows. There are 10 seats in each row. How many seats are in the theater?

  Jamal solved the problem using this method. He multiplied to find the seats in each section, and then added to find the total.

  Section A  \[10 \times 15 = 150\]

  Section B  \[10 \times 13 = 130\]
Total Seats \( (10 \times 15) + (10 \times 13) = 150 + 130 = 280 \)
Rosa thinks her method is correct. She added to find the total number of rows, and then multiplied by the number of seats per row.
Rows in A and B \( 15 + 13 \)
Total seats \( 10(15 + 13) = 10 \times 28 = 280 \)
Who is correct? Justify your answer.

- **Activity 4:** The student will work the following problems correctly:
  Between what two consecutive integers is each square root?
  a. \( \sqrt{19} \) b. \( \sqrt{60} \) c. \( \sqrt{102} \) d. \( \sqrt{200} \)

- **Activity 6:** The student will simplify the following expressions correctly:
  a. \( 40 \div 2 + (5 - \sqrt{4}) \)
  b. \( \sqrt{25} + 4 \times 6 \div 3 \)
  c. \( 48 \div \sqrt{64} - \frac{23 - 13}{2} \)
  d. \( 31 + \frac{28}{7 - \sqrt{9}} - 2 \times 10 \)

- **Activity 13:** For the following situations, the student will assign a variable to the unknown quantity, write an equation to find the unknown quantity, and solve the equation to find the unknown quantity:
  a. Rosa, Mel, and Kathy work as a cleaning crew. They shared the money they earned last week equally. Mel spent $26, leaving him with $45. How much money did the crew earn last week?
  b. Peter bought a coat at an end-of-winter half-price sale. Sales tax of $6.24 was added to the sale price, bringing the cost of the coat to $84.24. What was the original price of the coat?
  c. This week, the price of a round-trip train ticket to the city rose by $1.50. Mr. Logan bought 6 tickets and paid $64.50. How much did the round-trip ticket cost before the increase?

- **Activity 16:** The student will graph and write a verbal statement to represent each inequality:
  a) \( n > \$18 \) b) \( c < 20 \text{ gallons} \) c) \( x \leq 5 \text{ books} \)
• **Activity 19:** The student will work the following problem correctly:
  
  A pattern of squares is shown below.
  
  a. Sketch the fourth and fifth figure in this pattern.
  
  b. Create an input-output table comparing the figure number to the number of squares.
  
  c. How many squares would be in the 100\textsuperscript{th} figure?
  
  d. Write an expression for the number of squares in the \( n \)\textsuperscript{th} figure.

<table>
<thead>
<tr>
<th>Figure #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>100</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td># of squares</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>( 100^2 )</td>
<td>( n^2 )</td>
</tr>
</tbody>
</table>

Solutions:

<table>
<thead>
<tr>
<th>figure 1</th>
<th>figure 2</th>
<th>figure 3</th>
</tr>
</thead>
<tbody>
<tr>
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Grade 7
Mathematics
Unit 4: Statistics and Probability

Time Frame: Approximately four weeks

Unit Description

This unit focuses on understanding that statistics can be used to gain information about a population by examining a sample of the population and using data from random samples to draw inferences about the population. Probability will be explored when investigating chance processes and developing, using, and evaluating probability models.

Student Understandings

Students can reason and understand that random sampling tends to produce representative samples and support valid inferences. They use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Further, they are able to generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. Students can use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. Students can approximate the probability of a chance event by collecting data on the chance process that produces it and observing its frequency and predicting the approximate frequency given the probability. In addition, they can develop a probability model by observing frequencies in data generated from a chance process.

Guiding Questions

1. Can students describe a population by taking samples?
2. Can students talk about the relationship between the sample and the population and their meanings?
3. Can students assess the representativeness of their samples?
4. Can students generate simulated samples of the same size to gauge the variation in estimates or predictions?
5. Can students use data to draw inferences about a population?
6. Can students talk about clusters, gaps, and outliers in data and their meanings?
7. Can the students determine probability from experiments and from data displayed in tables and graphs?
8. Can the students compare theoretical and experimental probability in real-life situations?
Unit 4  Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.</td>
<td>Compare theoretical and experimental probability in real-life situations (D-5-M)</td>
</tr>
</tbody>
</table>

CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.SP.1</td>
<td>Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</td>
</tr>
<tr>
<td>7.SP.2</td>
<td>Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.</td>
</tr>
<tr>
<td>7.SP.3</td>
<td>Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.</td>
</tr>
<tr>
<td>7.SP.4</td>
<td>Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.</td>
</tr>
<tr>
<td>7.SP.5</td>
<td>Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability of ( \frac{1}{2} ) indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
</tr>
<tr>
<td>7.SP.6</td>
<td>Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.</td>
</tr>
</tbody>
</table>

Sample Activities

2013-14

Activity 1: Sampling Ourselves (CCSS: 7.SP.1)

Materials List: paper, pencil

In this activity, small groups of students are selected as samples to represent the population of the class. The class collects information about a given characteristic from a sample group of students, describes the distribution, and then decides how well the information from the sample describes the class. They evaluate whether they are a
representative sample of the specified population. Students will use DL-TA (view literacy strategy descriptions), directed learning-thinking activity, to process the concept of sampling. DL-TA is an instructional approach that invites students to make predictions, and then check their predictions during and after the learning.

Begin the activity by asking students what it means to take a sample of something. Brainstorm a list of samples with students. Try to include items that do not involve opinion surveys, such as samples chosen for medical research, or samples that are selected and measured or weighed and counted, and not determined just by asking questions. Once a class list is generated, ask students what all the items have in common. Allow time for them to reflect on the similarities and to generalize: What’s a sample for? Emphasize the idea of using a sample as a way to gather information about a larger group, which is called a population. Be sure students see that samples are chosen to help someone make decisions about the “whole” population.

The first question that the class will try to answer through sampling is, “What part of our class is right-handed?” Select four students to be the sample group. Have these four students sit or stand together. Ask for a report about their dominant hand, and make a tally on the board. What’s the general picture that emerges? For example, most/all/few/half/less than half of the students in the sample are right-handed. Based on this sample of the class, ask students to predict what portion of the whole class to be right-handed. Allow a little time for students to discuss their inferences and their methods. Then collect the “real” data from the class. Be sure to count the four who were in the sample—they are, after all, members of the population from which they were drawn. Display class results on the board alongside the data from the sample (a simple table to organize the data will work). Ask students to compare the “real” outcome with their predictions.

Ask the following questions:
- What can you say in general about the class data?
- How does it compare with the results from the sample?
- How well did the sample provide representative information about the class?

A sample of four is quite small. It may well be too small to be representative. Select a different sample of eight students and ask the same question again. Does this result get closer to the “real” profile of the class? Ask, “What do you think would make a sample representative of a population? What factors should be looked for?”

Select a different sample of four students and ask a new question, “Do you walk to school?” Have students analyze the results of the sample survey. What can they say about the sample results? What would they expect of the whole class? Before asking the whole class for their data, send the sample of four back to the whole group and choose a new sample of eight students to respond to the question. Again, ask the class to respond to the results, and to extrapolate to the whole class. Are the results similar each time? Are they reasonable?
Collect the whole class data—how many walk to school?—and ask them to compare the actual data with their predictions. Did the samples adequately predict the whole class results? Does a larger sample seem more representative?

As a final sample and a way of wrapping up the DL-TA process, show the students that samples can be less than representative by asking the question, “What color eyes are typical of our class?” Stack the deck by selecting the sample of all brown-eyed or all blue-eyed students, or a mixed sample whose proportion is not typical of the class.

Collect the data and ask students to infer from the sample. Then collect data from the entire class. When the whole-class results are quite different, ask students why they think that happened. This would be a good place to introduce the idea of the representativeness of a sample. A sample should be selected so that it is representative of the population.

2013-14
Activity 2: Samples and Representation (CCSS: 7.SP.1)

Materials list: Samples and Representation BLM

In this activity, students will take samples of the class and look at the way different samples can lead to different conclusions. Students will work in groups of four as a sample of the population of the class. Distribute Samples and Representation BLM and have students research the four questions and make predictions about the population (class) based on the sample information (questions). While monitoring each group, help students understand that they are to make predictions about the class from their sample alone. Each sample group will answer the questions and compile their small-group results, and then make predictions or inferences in the space provided on the BLM about the class based on their sample results.

Discuss samples and predictions with the whole class by assigning a spokesperson from each group to respond. Record each small group’s data on the board for each question and see whether the samples were generally accurate. If a sample was not a good predictor, ask students why they think that might have happened. These samples are small ones, and they may not reflect the whole class very well.

Proceed through each question, looking at the similarities and differences among the small-group results and the whole-class data. Students may notice that the sample is more similar to the class’s data when the data are more uniform—if no one has a baby sister, or if everyone is allowed to stay up after 11:00 p.m. on weeknights. When the small-group and whole-class data have been compared for all four questions, review by asking students again what factors they believe affect the representativeness of their samples.

Note: The overall goal of students should be to get experience in sampling, rather than to find out in every case “whether we were right.” Class discussions will be most fruitful if the focus is kept on the relationship between the sample and the population—the size of the sample and how well the sample represents the population.
**2013-14**

**Activity 3: Selecting a Sample (CCSS: 7.SP.2)**

Materials List: pencil, paper, brown paper bags (1 per group), color tiles (10 in each bag: 5 of one color, 3 of another color, 1 of a third color, and 1 red)

To help students see how a random sample is selected, provide them with a brown lunch bag containing 10 cubes or color tiles of 4 colors (5 of one color, 3 of another color, 1 of a third color, and 1 red) and have them shake the bag. Then have one student in each group remove a cube or tile and note its color (do not allow the students to look in the bag prior to their data collection). To simulate a large population, replace the cube drawn, shake the bag and draw another cube, note its color, then replace it. Have each student determine the number of times that the process should be repeated to allow them to make a good guess as to what the colors of the tile in the bag are and how many of each color are in the bag, if there are 10 total cubes or tiles in the bag. Have students make predictions as to what the next randomly selected sample color will be from their collection. Discuss how certain they are about their prediction, and then have them collect the sample. Discuss why their predictions were accurate or not. Instruct the students to use their data and make a prediction of how many tiles or cubes of each color are in the bag if there are a total of 10 tiles or cubes in the bag. They should do this without looking inside the bags. Have students write these predictions and then open the bag and count the number of each color of tile in the bag. Ask how closely each student’s sample of 10 matched the population - this is a good time to discuss the importance of sample size. Combine all the results in the class and then determine how closely the aggregated data match the actual color proportions in the population. A website with an interactive “Let’s Make A Deal” probability page is available at [http://matti.usu.edu/nlvm/nav/frames_asid_117_g_4_t_2.html](http://matti.usu.edu/nlvm/nav/frames_asid_117_g_4_t_2.html)

**2013-14**

**Activity 4: Describe the Data (CCSS: 7.SP.4)**

*This activity has not changed because it already incorporates this CCSS.*

Materials List: pencil, paper, Describe the Data BLM

Conduct a class survey by asking, “What is your shoe size?” Record the data on the board, but do not organize it. In groups of 2 or 3, instruct students to organize the collected data and to write 3 to 4 sentences to describe/analyze the data. Encourage the students to organize the data graphically to assist them in doing this.

After 10 minutes, have groups present their organization and analysis to the class. Allow the class to check for accuracy and discuss the pros and cons of each representation and analysis of the data. Be sure to include a discussion about patterns, clusters, gaps, and outliers in the data. To help students make connections to the vocabulary, describe examples where they use the same vocabulary every day. They might see a cluster of...
grapes in the grocery store or a cluster of flowers in their yards. Ask students to describe what a cluster of grapes or a cluster of flowers looks like. Use this knowledge to describe the data in a similar way.

Examples: A cluster of grapes is a bunch of grapes all connected to one big stem. A cluster of data is similar in that there are several pieces of data surrounding a central spot or piece of data. To demonstrate gaps, have students sit in a row using every chair, then skip several chairs, and then have a student sit in the next chair on the same row. This person is an outlier, and the empty seats represent a gap.

In previous years, students have described data by identifying the patterns they see in the data or the distribution of the data. Be sure they are able to identify the same patterns using the vocabulary: cluster, gaps, and outliers.

After groups have presented, address the types of graphic organizers not used. Ask why each was not used. Could the information collected be organized using every kind of graphic? Is one better to use than another? Discuss the cons and pros of a line plot, frequency table, bar graph, chart, and circle graph.

Distribute to each student a copy of the Describe the Data BLM. Students will describe/analyze the distribution of data in terms of patterns, clusters, gaps, and outliers. After students have completed the Describe the Data BLM, allow them to pair with another student to share their analysis, paying close attention to accuracy.

Have students reflect on the different data displays from the Describe the Data BLM using SPAWN writing (view literacy strategy descriptions). SPAWN is an acronym that stands for five categories of writing prompts (Special Powers, Problem Solving, Alternative Viewpoints, What If?, and Next). Students will respond to the following prompt:

- **What if** the data describing the music were displayed in one of the other graphic organizers on the BLM? Would this be an effective way to describe this data?

Students will extend the “What If?” prompt to include the data describing favorite soft drink and ticket sales. Students can record answers to the SPAWN prompt in their learning logs (view literacy strategy descriptions). After students have completed the SPAWN writing, students can share their responses with the class to stimulate a discussion of the use of appropriate data displays.

**2013-14**

**Activity 5: Fast Food Data (CCSS: 7.SP.4)**

Materials List: menu with nutritional information from a fast food restaurant (may be found on the Internet), computer with Internet access (optional), grid paper, pencils, dry spaghetti
2012-13 and 2013-14 Transitional Comprehensive Curriculum

This activity has not changed because it already incorporates this CCSS. Students will create a scatter plot of data to explore the relationship between grams of fat and grams of protein in menu items at various fast food restaurants. Allow students to access the Internet to collect their own data or print information to give to students. If Internet access is not available, most local fast food restaurants have nutritional brochures that can be copied for student use.


Let students pick at least 5 items they want to chart on a scatter plot. As a class, create a scatter plot showing the grams of fat compared to the grams of protein in individual servings of lunch and dinner items sold at various fast food restaurants. Have the students work with a partner to interpret the data.

Lead a discussion about the patterns in the data. Are there any outliers or gaps? Where does the data seem to cluster? Is there a trend line? Place a piece of uncooked spaghetti on the graph to develop the concept of trend line.

Have students work in pairs to choose 5 additional items from a menu, collect their own data, create a scatter plot, and prepare to discuss any patterns observed in the data. After 15-20 minutes, have student groups take turns presenting their graphs assuming the role of professor-know-it-all (view literacy strategy descriptions). Remind them to write down 2-3 questions they think they will be asked by their peers and 2-3 questions to ask their peers. They also will have to justify their thinking and be prepared to answer questions from the class. Students should hold the professors accountable for the accuracy of their answers to the questions by asking clarify questions.

2013-14
Activity 6: What’s Variable? (CCSS: 7.SP.3)

Materials List: cm grid paper, Math Scores BLM

In this activity students will relate previously learned vocabulary (mean, median, mode) to new vocabulary (variation/variability, mean absolute deviation or MAD). Variability is reflected in everything that is measured. For example, not all statistics test scores are the same, not all newborn babies weigh the same at birth, not all cars sell for the same price,
not all basketball players score the same number of points in a game, not all fingers are
the same length, and not all people have the same color of skin. In short, variability is
inevitable in life.

Begin the activity by having students review key vocabulary (measures of center as in
mean, median, mode) before having them interact with new vocabulary (variability,
measures of variability as in mean deviation). On the board or overhead, display the
following data sets/situations and ask students to work with a partner to determine which
measure of center is most appropriate to use. Then have each pair work with another pair
to share their ideas and justify why they think the situation most appropriately describes
mean, median, or mode. Listen for groups that have insightful explanations/justifications
and have these groups share during whole class discussion.

1. 

<table>
<thead>
<tr>
<th>Maximum Speed Limits</th>
<th>Number of States</th>
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<tbody>
<tr>
<td>55 mph</td>
<td>7</td>
</tr>
<tr>
<td>60 mph</td>
<td>2</td>
</tr>
<tr>
<td>65 mph</td>
<td>21</td>
</tr>
<tr>
<td>70 mph</td>
<td>13</td>
</tr>
<tr>
<td>75 mph</td>
<td>8</td>
</tr>
</tbody>
</table>

The data shows that far more states have a 65 mph speed limit for trucks than any other speed limit. So you could say the typical maximum speed limit for trucks is 65 mph because more states have this limit—mode.

2. In five basketball games, you score 17, 20, 14, 18 and 16 points.
For data sets with no very high or low numbers, mean is usually most appropriate. In this situation, the person scored an average of 17 points per game. Ask students if mean would still be an appropriate average if the person scored 50 points in the next game? No—just one unusually high-scoring game brought the average points-per-game up by more than 5 points! It made the mean bigger than any of the other data points. So the mean no longer describes a typical number in the set of numbers.

3. During one 7-hour shift, “The Burger Barn” kept track of the number of customers served each hour:

<table>
<thead>
<tr>
<th>Hours</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>11-12</th>
<th>12-1</th>
<th>1-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Customers Served</td>
<td>7</td>
<td>16</td>
<td>13</td>
<td>11</td>
<td>14</td>
<td>50</td>
<td>17</td>
</tr>
</tbody>
</table>

The median for the set of data is 14. So the typical number of customers in an hour is 14. The mean for this set of data is about 18.29. The slow first hour and the very hectic lunch hour are not typical of the rest of the shift and they do not affect the median as much as they affect the mean, so this is why the median is most appropriate to use in this situation.
To help students to think of the term variability in context, pose the following situation:

John and Mary received the following scores on their last five math tests:
John: 80, 80, 85, 75, 95
Mary: 65, 75, 90, 90, 95

Ask students if the mean would be an appropriate measure of center to determine which student is more consistent in demonstrating their knowledge in math. Both John and Mary have the same average test score of 83, yet they have not exhibited identical test results, so this example demonstrates that the measure of center of two data sets does not always give us the best picture or interpretation of the data.

NOTE TO TEACHER: To provide a more meaningful interpretation, one also needs to know the score’s spread, or scatter. The spread or scatter of scores is referred to as variability. By knowing the measures of variability, you can determine the amount that the scores spread, or deviate, from the measures of center. The measure of variability used in the subsequent activities involves mean deviation. When mean is used to talk about what’s typical about a set of data, it helps to be able to say by how much most of the numbers in the set tend to differ from the mean. Mean deviation or average deviation can give you this information. A low mean deviation indicates that the data tends to cluster around the mean—there is not much difference between most numbers in the set and the mean. A higher mean deviation indicates that the data are more spread out.

To help students conceptualize variability, refer to the previous situation. Ask students to predict whether John or Mary shows more variation in their test scores. Answers may vary at this point. Distribute grid paper and have students draw line plots to test their answer. In addition, have students write at least three observations they can make from looking at the line plots. Some observations that may come out: John’s data looks clustered together, while Mary’s data is spread apart; John’s scores cluster around the mean of 83 while Mary’s does not; Mary has a very low score of 65 but three very high scores of 90, 90, and 95 which levels off to help her get the same score as John; John shows more consistency in his understanding of math because his scores cluster around the average: Mary’s distribution of data suggests that her scores show more variability.

JOHN:

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<td>X</td>
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<td>X</td>
<td>X</td>
<td>X</td>
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<td>X</td>
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<tr>
<td>65 70</td>
<td>75 80</td>
<td>85 90</td>
<td>95</td>
<td></td>
</tr>
</tbody>
</table>

MARY:

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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>65 70</td>
<td>75 80</td>
<td>85 90</td>
<td>95</td>
<td></td>
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</table>
Next, ask students to describe what it means to deviate. Answers will vary but focus on descriptions of differing, changing, varying, etc. Tell students that there is a method called mean deviation to prove how much the numbers in the data sets differ from the mean. Distribute Math Scores BLM. Refer to the second column, Deviation from Mean, and explain that deviations are like distance—always positive. Students will find the positive difference, or mean absolute deviation of each value from the mean. Model this process using John’s scores, then have students complete Mary’s scores independently and check with a partner. When complete, ask students to discuss with a partner how the mean absolute deviation relates to their original prediction of which student, John or Mary, shows greater variability in their math scores. There is more variability in Mary’s scores. Students should make the connection that a low deviation (John) shows that his scores are closer to the mean (also seen in the line plot) and that a high deviation (Mary) shows that her scores are spread apart from the mean (also seen in the line plot). Mary’s scores differ more from the mean than John’s.

2013-14
Activity 7: Heart Throbs (CCSS: 7.SP.3)

Materials List: cm grid paper (2 sheets per student), clocks with second hands or some other time keeping device (watch, phone, timer, …), Pulse Rates BLM

Data can be used for making a comparison between two populations in a wide variety of ways. One is by comparing a measure of center (mean, median, and mode) of each group. When measures of center are coupled with a measure of variability such as the mean absolute deviation, which measures how far on average the data points vary from the mean, a better analysis of the data is possible. A dot plot provides a visual display of the data.

In this activity, students consider the question, “How much faster is your exercise pulse than your resting pulse?” They gather data about their own pulse rates, describe the shape of the data, and determine the typical resting pulse for the class. Then they exercise and immediately take their post-exercise pulse rates, organize and display these data, and compare the results with the resting pulse rates.

Discuss what students have noticed about the effects of exercise on their heart rate. Ask them to speculate about how much faster their heart beats during exercise. A little faster? Twice as fast? Ten times faster? Tell students that in order to determine how much faster the heart beats when exercising, first they have to know what their resting heart rate is (how fast the heart beats when sitting still). Ask students what their estimates are.

Post the following questions where students can refer to them throughout the activity. Then give an overview of the activity.

- How much greater will the average exercise pulse be than the average resting pulse?
- Will there be greater variability in pulse rates when at rest or exercising?
- What factors account for this variability?
Next, ask for a student assistant to help demonstrate the pulse taking technique. The best place to take a pulse is on the large artery (carotid) on either side of the neck. Use two fingers to press gently on the artery until you find the pulse. Decide how long to count. Based on this decision, how will you get a rate per minute? Discuss students’ ideas about how long to count and what to use as the multiplier for getting the per minute rate. In general, an interval of 10-30 seconds gives students time to get an accurate count. For example, if you choose an interval of 10 seconds, then multiply that amount by 6 to get the per minute rate; for an interval of 20 seconds, multiply by 3; for an interval of 30 seconds, multiply by 2.

Students now pair up and begin taking pulses. Decide what timing device will be used (watch, phone, timer, ...). One student serves as the timer while the other takes his or her own pulse. Each student should take several readings and write down the results of each one.

Ask for a few volunteers to list their results on the board. Were each person’s results the same? If not, how do you decide which number to use? In this discussion, support students in finding a clear set of guidelines that can be applied consistently to each person in identifying resting pulse rates and have each student choose their results that best represents this. For example, a student may have results of 60, 56, 64, and 36. Ask the student which result should be used for the class data and why. The student may say that he didn’t start counting on time when he got 36, so that number shouldn’t be used. Ask if using the middle number of the good tries would be a good strategy. In this case, the middle number of 56, 60, and 64 would be 60, the result used for the class data. Once students have decided which result they will use to represent their resting pulse rate, compile the results on the board. Distribute grid paper and have students create a line plot of the Resting Pulse Rates. Ask students what the typical resting heart rate of the class is. The actual mean will depend on class results.

To prepare for the next collection of data, arrange students in groups of four and ask students to brainstorm ideas about simple exercise they could do to increase their heart rates (running, doing jumping jacks, jogging in place). Make a list of ideas on the board. Each group will decide which exercise to do. As a class, decide how long to do it. Between 1 and 3 minutes is usually sufficient. Students will pair up within the group of four to take pulse rates (alternate being the “time keeper” and “exerciser”). The procedure for finding an exercise pulse rate is slightly different. Pulse rate goes down very quickly after exercising, especially if you are in good shape. Students will need to take their pulse immediately after they stop and only for ten seconds so the heart doesn’t have a chance to slow down. Remind students to exercise at a pace they can keep up with for the whole time period. Ideally, students should have one practice turn at this, followed by a second chance to collect their data. Give the signal to start, and one member of each pair will start exercising. Give a warning when it’s almost time to stop, at which point the time keepers get ready to time and the exercisers get ready to count. Give the signal “Find pulse!” and exercisers will immediately stop and locate their pulse. Tell the time keepers “Start!” and the exercisers count until the time keepers signal “Stop” (after 10
seconds). Time keepers should record the results. Switch roles and complete the process again.

Record results of each student on the board. Ask students what the typical exercise pulse rate of the class is. The actual mean will depend on class results. Distribute grid paper and have students create a line plot of the Exercise Pulse Rates. Distribute Pulse Rates BLM to each student. Students will record class data in each table of the BLM (resting pulse rate and exercise pulse rate) and calculate the mean deviation of each. Next, they will work with a partner to see if they can answer the question posed at the beginning of the activity, “How much faster is your exercise pulse than your resting pulse?” by comparing the line plots and mean deviation. In their comparison of the data, have students consider the following:

- Was there a greater variability in pulse rates when resting or exercising?
- What factors account for this variability?

When monitoring group discussions, ask guiding questions that enable students to make the connection that a high mean deviation indicates greater variability. Where is this evident in the line plots? Is there an overlap between the two sets of data? Listen for groups that have insightful explanations/justifications and have these groups share during whole class discussion. A greater variability should be seen with exercising pulse rates, and possible factors that may account for this variability are that every student is at a different fitness level.

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**2013-14**

**Activity 8: Sports Heights (CCSS: 7.SP.3)**

Materials List: grid paper (2 sheets per student), Sports Heights BLM

Pose the following situation to students:

Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn’t know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of the soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists:

Copy the lists on the board or overhead:

**Basketball Team**—Height of Players in inches for 2010 Season
75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84, 84, 80, 84

**Soccer Team**—Height of Players in inches for 2010
73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

Distribute two sheets of grid paper to each student and have them create two line plots on the same scale. Ask students how they will determine the scale of the line plots. The
range will be from 65 inches to 84 inches because the shortest player is 65 inches and the tallest players are 84 inches.

The line plots should look like this:

<table>
<thead>
<tr>
<th>Height of Soccer Players (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X X X X X X X X X X X X X X</td>
</tr>
<tr>
<td>65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Height of Basketball Players (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X X X X X X X X X X X X X X X</td>
</tr>
<tr>
<td>65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84</td>
</tr>
</tbody>
</table>

Have students independently compare the two line plots and write three observations about the distribution of the data. Then ask students to share and compare their observations with a partner. When monitoring partner discussions, ask probing questions that will enable students to see that there is some overlap between the two data sets. Some players on both teams are between 73 and 78 inches tall.

Distribute Sports Height BLM and ask students to use the mean and mean deviation to compare the data sets. The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches. Students may use round values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The mean deviation is 2.14 inches for the basketball players and 2.53 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams (7.68) is approximately 3 times the variability of the data sets (7.68 ÷ 2.53 = 3.04; 7.68 ÷ 2.14 = 3.59).
2013-14
Activity 9: Is It Likely? (CCSS: 7.SP.5)

Materials List: Colored tiles or colored squares of paper for each group (14 yellow, 4 red, 2 blue), envelopes or small paper bags (1 per group)

In this activity, students represent the likelihood of possible outcomes for chance situations. Students order events of likelihood and use a diagram or appropriate language to compare the chance of each event’s occurrence (impossible, unlikely, equal, likely, certain). Students relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).

Have students brainstorm times in their lives that they used probability. Ask students: When might you hear things about chance? Weather, making the team, winning a prize
When you hear the work “impossible,” what do you think? Something that can’t happen
If you are told that something will probably happen, what do you think? More likely than not, it will happen

Have students create a list of events that are “impossible,” “unlikely” “certain,” “likely” or “equally likely” to happen. Events may include being assigned homework tonight in math, it will rain today, the school bell will ring at 3:00 p.m., someone will knock on our classroom door during math today, etc.

Separate students into groups of four. Distribute colored tiles or colored squares of paper to each group. Begin by explaining to the class that there are 20 colored tiles in the envelope/bag. Tell them that some are red, some are blue and some are yellow. Ask students to predict which color they think will come up most. Some students just choose their favorite colors and some may say there is no way of guessing since they don’t have any way of knowing. Ask the class how they could determine the number of tiles of each color in the bag without looking or dumping the tiles out of the envelope/bag. Record ideas on the board and determine a reasonable method.

Ask students what might be the best way to keep track of the color of the tiles chosen. One possibility is to make a frequency table or chart on the board with the three colors and a column for tally marks. Draw the table below on the board and have one member from each group copy it.

<table>
<thead>
<tr>
<th>Color of Tile</th>
<th>Frequency Picked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td></td>
</tr>
</tbody>
</table>

Allow each student from the group to pick a tile, place the tile back in the envelope/bag and then place a tally on the frequency table next to the color selected. Make sure that students return the tile to the envelope/bag, so the probability remains the same. Observe the tally table to make sure students record the information accurately.
After five tiles have been drawn, have the students make predictions in their learning logs (view literacy strategy descriptions) about the number of tiles of each color in the envelope/bag. Have students discuss predictions with partners and write a reason for their prediction in the learning log. Students may have different predictions. Ask what can be done to improve the predictions. Draw additional tiles. Have students continue to draw tiles, stopping periodically to have students make new predictions and recording them. Next, have students respond to the following question in their learning log: How many of each color do you think was in the envelope/bag? Explain your reasoning.

Have each group pour the tiles out of the envelope/bag and compare to their predictions. Allow students to discuss with their groups these questions: Which color should have come up most often? Yellow Which color was “unlikely” to be picked? Blue Can you name a color that would be “impossible” to pick? Orange What was the probability of choosing a yellow tile? \( \frac{14}{20} \) or \( \frac{7}{10} \) Red tile? \( \frac{4}{20} \) or \( \frac{2}{10} \) Blue tile? \( \frac{2}{20} \) or \( \frac{1}{10} \)

Draw the following likelihood line on the board as shown below.

```
0 __________________________ 1
Impossible          unlikely         equal        likely       certain
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Have students discuss with their groups where each of the three colors belongs on the likelihood line. Have members from each group write the colors above the appropriate word or use colored sticky notes where they think they belong on the line. Discuss conclusions.

Activity 10: Probability Using Spinners (GLE: 38)

Materials List: Spinner BLM, pencil, paper clips, paper

The interactive website below will enable students to understand the difference between theoretical and experimental probability. This may be used to introduce the activity or could be used as closure to the activity.
http://www.glencoe.com/sites/common_assets/mathematics/mc3/cim/interactive_labs/M3_10/M3_10_dev_100.html

Make a fair spinner (spinner with all sections exactly the same size) with numbers 1 through 9 and duplicate one Spinner BLM for each pair of students on card stock. Have students cut out the spinner. Show them how to spin a paper clip around a pencil point to make the spinning device. (If available, a graphing calculator can be used to make a fair spinner with the number 1 through 9.)

Ask students to study the numbers on the spinner and have them predict the probability of spinning a multiple of 3. Have students discuss in their groups how they would collect...
data to determine the accuracy of their predictions. Have students record the steps in data collection that their group will follow to collect the data.

Ask students to conduct the experiment and record the data. Have each group find the probability for spinning a multiple of 3 from the data they collected.

Use discussion (view literacy strategy descriptions) in the form of Inside-Outside Circles to discuss experimental (data collected and probability figured from collected data) and theoretical (the possibilities of each event happening in theory) probability. Class discussion can be used to promote deeper processing of content and rehearsal of newly learned content. Inside-Outside Circles has the advantage of allowing for a variety of inputs through simply rotating the circles of students. Be sure to make enough space in the room for this discussion activity, and move about the circle to listen in on students’ brainstorming. Ask students to stand and face each other in two concentric circles. The inside circle faces out and the outside circle faces in.

- Ask students to discuss theoretical probability in the context of the spinner activity with the person standing most directly in front of them. Listen for students describing that theoretical probability is what SHOULD happen as an outcome. For example, since the spinner included numbers 1-9 and contained three possible multiples of 3, theoretically, the chances of landing on a 3, 6 or 9 is $\frac{3}{9}$ or $\frac{1}{3}$.

- Next, ask the outer circle to rotate clockwise until they hear “stop.” Ask students to discuss experimental probability in the context of the spinner activity with the NEW person standing most directly in front of them. Listen for students describing that experimental probability is what REALLY happened in the experiment. Since every student’s outcome will be different, listen for statements such as, “I should have landed on a multiple of 3 one-third of the time, but I really landed on a multiple of 3 only 1 time, so experimentally, my chances were $\frac{1}{9}$ rather than $\frac{1}{3}$.

- Then ask the outer circle to rotate clockwise once more until they hear “stop.” Ask students to discuss why the probability might be different when data is collected through an experiment with the NEW person standing most directly in front of them. Answers are likely to vary here. Ask guiding questions to help students understand that in this situation, the theoretical probability will always have the same outcome (same spinner 1-9 and same multiples of 3)—outcome will always be $\frac{1}{3}$. The data collected through the experiment will be different every time because of the randomness of where the spinner will land.

After these rotations, randomly ask individual students to share their own ideas or those of the person(s) with whom they have been discussing with the whole class.
Activity 11: Probability Using Markers (GLE: 38)

Materials List: brown lunch bags, 10 marbles, markers, or plastic chips for each pair of students (5 blue, 3 red, and 2 green), paper, pencil

Make bags of 10 marbles, markers, or plastic chips (i.e., 5 blue, 3 red, and 2 green). In this activity, students will predict the number of marbles/markers of each color in the bag and compare their prediction with the theoretical probability of drawing each color.

Have students work in pairs. Instruct Student 1 to draw 1 marble/marker from the bag (without looking), record the color, and then replace it in the bag. Repeat this process 10 times. Have Student 2 complete the same process. Ask, “Using your data, which color marble is most prevalent? Discuss the students’ predictions.” Have the students predict all the colors in the bag based on their data. Discuss their predictions. Ask, “Do you think it is possible to have a color in the bag and never draw that color?” Have students open the bag, then count and record the number of each color. Ask students to compute the experimental probability of drawing each color in the bag based on their data and then to compute the theoretical probability. Ask, “How do these compare?”

Activity 12: Who Did It? (CCSS: 7.SP.6)

Materials List: Who Did It? BLM, brown lunch bags (4 for each group), 10 color tiles of four different colors (in each of 4 bags for each group), pencil, paper

Begin class with a discussion about sampling. Tell the students that today they will collect results without replacement. Discuss what this means. Distribute the Who Did It? BLM to each student, and give four brown lunch bags filled with the following (unknown to them) tile or cube combinations to each group of four students. Bags should be labeled A – D and should each contain a total of 10 tiles or cubes of four different colors. Let the students know that there are 10 tiles in each bag and four different colors. Bag A and one of the other bags should be identical (have the same number of each color of cubes).

Tell the students that the sample in Bag A was found at the scene of a crime. CSI investigators explored the contents of Bag A. When the investigators bagged the contents of Bag A, they duplicated the items from bag A and made a second bag. A new CSI trainee was also on the scene, and he took these bags back to the lab without labeling the second one. These bags were placed in a box with bags from another crime scene. When the CSI went to get the bags, there were four bags and only Bag A was labeled. The CSI knew that there were two identical bags, and these bags are very important to their case. They want to try to determine which are the identical bags and not touch the items any more than necessary.

Discuss as a class that when samples are examined without replacement, the sample size is constantly changing. Suppose this is what happens when the first tile is selected from
the bags: Bag A – red; Bag B – red; Bag C – green; Bag D – red. Now there are only nine tiles in each bag to select from. Challenge the students to devise a plan to sample contents of the bags without replacement so that they can make the best prediction based on experimental probability without looking at the contents of the bags. Have students record their results and make a prediction after the 6th selection from each bag, justifying why this is their selection. Lead a discussion about whether the selections give enough information to make the prediction. Ask if all four bags have to be completely empty to make a valid prediction. Have students explain their thinking and their results.

*Teacher Note:* Student results will be different, and they will have to use some logical reasoning as they compare the results they gather.

**Activity 13: Determine Probability from Data (CCSS: 7.SP.6)**

This activity has not changed because it already incorporates this CCSS.

Materials List: computer with Internet access, basketballs and basketball net or wastepaper baskets and foam balls, paper, pencil

Introduce the lesson with basketball statistics about national players (http://www.basketball.com/ or http://www.nba.com/statistics/index.html) or players from the local high school. Guide the discussion of the school’s team using local newspaper reports or other sources to point out vital statistics (e.g., player with the most 3-point shots completed or most rebounds). Ask students what it means when someone says that a certain basketball player is a 75% free throw shooter. Pick two or three players from the list of statistics, and ask students to determine the probability that the player will make his next free throw. Ask students to explain how to calculate a player’s free throw percentage (or his/her probability of making the next free throw). Make sure that students understand that these values are calculated by dividing the number of free throws made by the number of free throws attempted. Ask students to give fractional equivalents of the percentages (i.e., a free throw percentage of 65% means the player makes 13 out 20 free throws or about 2 out of every 3 free throws).

Have students work in groups of 4. Let students predict the number of free throws they think they can make out of 20 tries. Have students go to the basketball court and let each student shoot 20 times from the free throw line. (Substitute a wastepaper basket or milk crate and a foam ball to create an indoor version of the activity.) Have students take turns recording the data as team members shoot. Ask each group to work cooperatively to create a chart, table, or graph to organize their scores. Have the students prepare a presentation for the class, making sure they give the prediction, actual number of shots made out of 20, and each student’s probability of making the next shot.
Activity 14: It’s Theoretical! (GLE: 38)

Materials List: coins, graphing calculator (optional), paper, pencil, dice, cups, spinners

Introduce the activity by asking students, “What is the probability of getting a head when we toss a single coin? After giving time for students to provide an answer, ask how they determined the answer. Determine that students understand that the probability of getting a head is $\frac{1}{2}$, which is called the theoretical probability. Theoretical probability is determined mathematically by comparing the number of possible favorable outcomes (what you want to happen) to the total number of possible outcomes for a particular event. In this instance, there are two possible outcomes because a coin has two sides. Only one side has a head, so the number of possible favorable outcomes is only one.

Have each student toss a coin 10 times, keeping track of the results. Ask students if they got the same number of heads as tails. Did each result (heads and/or tails) occur the same number of times? What is the probability of getting a head according to the data? Remind students this is called experimental probability. Combine all the data from the class members and then recalculate the number of heads versus the number of tails. Have the students compare their individual results of tossing the coin 10 times with the class results. Discuss as a class. (If students have a graphing calculator available, the coin toss could be done 10 times, then 100 times to make a comparison of the results.) Lead students to understand that the more times the experiment is carried out, the closer the results of the experimental probability get to the theoretical probability.

Create activity centers for students to work in groups.

- Cups with dice (e.g., 1 cup with 1 die; 1 cup with 2 dice, and so on)
- Spinning arrows on cardboard (e.g., 1 with only 2 marked off sections; 1 with 4 different sections, and so on)

Have students determine the theoretical probability of certain situations, and then compare with experimental probability based on results of their experiments. For example, have students determine the theoretical probability of rolling a 6 when rolling one die and the probability of getting a sum of 4 when rolling a pair of dice. Then have students perform the experiments and compare the theoretical and experimental probabilities.

Activity 15: How Do the Chips Fall? (CCSS: 7.SP.6)

This activity has not changed because it already incorporates this CCSS.

Materials List: marbles or chips (blue, red, and green), How Do the Chips Fall? BLM, pencils, paper bags

Before class, prepare a paper bag with 5 blue marbles/chips, 3 red marbles/chips, and 2 green marbles/chips for each pair of students.
Have students work in pairs for this experiment. Give each pair one of the prepared bags and one copy of How Do the Chips Fall? BLM. Have the students look in their bag, find the theoretical probability of each color of marble/chip, and record it on the chart found on How Do the Chips Fall? BLM.

Each group will now conduct an experiment and record the results on the How Do the Chips Fall? BLM. Student 1 draws a marble from the bag, Student 2 tallies the color, Student 1 returns the marble to the bag. Have students repeat the process for a total of 25 draws.

Using the results from the experiment, have students complete the How Do the Chips Fall? BLM, computing the frequency and experimental probability, then compare the experimental and theoretical probabilities. As a class, discuss their comparisons. Give each pair a second copy of How Do the Chips Fall? BLM, and instruct students to switch roles and repeat the experiment with Student 2 drawing and returning the marbles 25 times and Student 1 acting as the recorder.

Combine results for Student 1 and Student 2. How do the experimental and theoretical probabilities compare now? How do the individual experimental probability and the combined experimental probability compare? Discuss. Find the probabilities using the results obtained by the entire class and compare them to the theoretical probabilities.

Have students use a modification of GISTing (view literacy strategy descriptions) to write an accurate 4-5 sentence summary of experimental and theoretical probabilities. GISTing is an excellent strategy for helping students paraphrase and summarize essential information. This is a modification of GISTing because there is no text for students to paraphrase. Students may refer to the information they collected from How Do the Chips Fall? BLM. Students should be able to summarize their findings very succinctly.

Begin the GIST by asking students to look at the data collected for the first experiment. Students will work with a partner to summarize the information in 20 words or less for the theoretical probability of picking a blue chip. Since each pair generated a different data set, each GIST will contain different data but the general GIST should be the same. Elicit various first-sentence GISTs from several pairs of students and display it with their data set using a document camera or a transparency for each pair so that students can see how the GIST matches the data.
Example:

<table>
<thead>
<tr>
<th>Chip Color</th>
<th>Theoretical Probability</th>
<th>Tally</th>
<th>Frequency</th>
<th>Experimental Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>$\frac{5}{10}$</td>
<td>/////</td>
<td>13</td>
<td>$\frac{13}{25}$</td>
</tr>
<tr>
<td>red</td>
<td>$\frac{3}{10}$</td>
<td>//</td>
<td>7</td>
<td>$\frac{7}{25}$</td>
</tr>
<tr>
<td>green</td>
<td>$\frac{2}{10}$</td>
<td>///</td>
<td>5</td>
<td>$\frac{5}{25}$</td>
</tr>
</tbody>
</table>

A bag contained 5 blue, 3 red and 2 green so the theoretical probability of picking blue is 50%.

The next part of the GIST will involve students looking at their data for the experimental probability of picking a blue chip. Again, they will summarize the information in 20 words or less combining information from the first sentence with information from the second.

Although the theoretical probability of picking blue is 5 out of 10, the experimental probability is 13 out of 25.

After the GIST is written, students will exchange with a partner to see how their GISTs compare. In the sample above, students should see that even though they drew blue 50% of the time theoretically, the experiment showed that drawing blue actually occurred 52% of the time. As the number of trials keeps increasing, the experimental probability should be closer to the theoretical probability.

A final GIST might look like this: A bag contained 5 blue, 3 red and 2 green so the theoretical probability of picking blue is 50%. Although the theoretical probability of picking blue is 5 out of 10, the experimental probability is 13 out of 25, or 52%.

Activity 16: Birthdays (CCSS: 7.SP.6)

This activity has not changed because it already incorporates the CCSS

Materials List: pencil, paper

Have the students collect data from the class to find the month each person was born. Have students organize the data in a line plot. Instruct students to analyze the data identifying any clusters, gaps, outliers, or patterns.

Ask the students to find different probabilities from the data such as, “What is the probability that a student was born in July? After April?” Add your month to the data, and have the students describe how the probabilities of the questions changed. Ask what would happen to the probabilities if data from other classes were added to the line plot.
**Activity 17: Probability with *Jumanji* (CCSS: 7.SP.6)**

Materials List: *Jumanji*, paper, pencil, dice, Jumanji BLM

Read the book *Jumanji* to the students. While reading the book, stop at different points in the book to ask mathematical questions. For example, on page 6 (where the two children are sitting at the table with the game board), say, “To play the game of *Jumanji*, the children rolled two dice and found the sum. If there are 48 spaces on the game board, what is the least number of plays it would take one person to win the game? Explain.” Continue reading the book and asking questions periodically.

After reading the entire book, give each pair of students a pair of dice and the Jumanji BLM. Instruct the students to create a list of the different ways the dice could land. How many ways are there? (36) From their lists, have students find the theoretical probability of rolling the sums 2 through 12. Students should roll the pair of dice 12 times and record the sum of the roll each time, then find the experimental probability of getting each sum. Have the students compare experimental and theoretical probability from the roll of their dice.

**Activity 18: Sums Game (GLE: 38)**

Materials List: Sums Game BLM, pencil, brown lunch sacks, 8 same color markers or plastic chips for each pair of students, math learning log

Make sacks containing 8 same color markers or plastic chips for each pair of students. The markers or plastic chips should be marked A-1, B-1, C-2, D-2, E-3, F-3, G-4, H-4. Distribute sacks to each pair of students. *Tell them not to look inside the sack*. Tell them they will play a game involving random draws from the sack, replacing the markers after each draw. Have a whole class discussion about ways to ensure that draws are random. Write these ideas on the board or on chart paper.

Go over the directions with the students. Directions: Player 1 randomly draws 2 markers from the sack, computes the sum of the marker numbers, and writes the letters that are on the markers below the sum of the markers on the score card. Replace the markers in the sack and shake the sack. Player 2 repeats this procedure, recording on a separate score card. Players continue alternating turns. The winner of this game is the *first person* to obtain each different sum at least once or to obtain any single sum 6 times.

Example: Player 1 pulls out C-2 and H-4. The sum of 2 and 4 is 6.
Example of score card:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Combinations:</strong></td>
<td></td>
<td></td>
<td></td>
<td>C, H</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students play the game and record their sums. Have the pairs of students post their data on the wall. Give students 5 minutes to walk around the room and make observations from the score cards posted.

Have the students determine a method of determining the *theoretical probabilities*. Theoretically, on any draw from the sacks, what sum is most likely to occur? Least likely? Make an organized chart showing all the possible sums and how each can occur. Then make one or more graphs showing all the possible sums. Given below are two types of charts and a bar graph showing the possible sums. Students should come up with something similar to these.

<table>
<thead>
<tr>
<th>Sums:</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Combinations:</strong></td>
<td>A,B</td>
<td>A,C</td>
<td>A,E</td>
<td>A,G</td>
<td>C,G</td>
<td>E,G</td>
<td>G,H</td>
</tr>
<tr>
<td>A,D</td>
<td>A,F</td>
<td>A,H</td>
<td>C,H</td>
<td>E,H</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B,C</td>
<td>B,E</td>
<td>B,G</td>
<td>D,G</td>
<td>F,G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B,D</td>
<td>B,F</td>
<td>B,H</td>
<td>D,H</td>
<td>F,H</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,D</td>
<td>C,E</td>
<td>E,F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,F</td>
<td>D,E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D,F</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

The above chart can easily be seen as a bar graph which is shown below.

Grade 7 Mathematics ◊ Unit 4 ◊ Statistics and Probability
Have students respond to the following prompt in their math *learning logs* (view literacy strategy descriptions).

Suppose you make one more draw from the sack. Which sum do you think will be the result? Justify your thinking with math and examples from your experiment. Allow students to share their entries with a partner as they listen for accuracy and logic.

### Sample Assessments

#### General Assessments

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage the student to create his/her own questions.

#### Activity-Specific Assessments

- **Activity 4:** The student will correctly work the following problem:
  
  The students in Mrs. Chance’s 7th grade math class recorded the following scores on their last math test. Organize the data and describe any patterns, clustered data, gaps, or outliers in the data. Test scores:

  77, 76, 97, 90, 95, 89, 76, 100, 95, 77, 90, 77, 96, 77, 88, 97, 88, 97, 33, 96, 100, 97, 100

<table>
<thead>
<tr>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>76</td>
<td>77</td>
<td>78</td>
<td>79</td>
<td>80</td>
<td>81</td>
<td>82</td>
<td>83</td>
<td>84</td>
<td>85</td>
</tr>
<tr>
<td>86</td>
<td>87</td>
<td>88</td>
<td>89</td>
<td>90</td>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
<td>95</td>
</tr>
<tr>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Grade 7 Mathematics ◇ Unit 4 ◇ Statistics and Probability
Solutions may vary. The above is an example of way to organize the data. The data clusters from 76-77, 88-90, and 95-97. There are gaps in the data, 33 could be considered an outlier.

- **Activity 6:** Gregory and his sister shared a computer at home. In order to be fair, they kept track of how much time they spent socializing with friends each day for the last two weeks. Their usage in minutes follows:

  Gregory: 49  48  51  52  68  40  73  68  61  60  69  55  51  59  
  His sister: 49  45  37  63  56  57  62  50  42  48  55  64  40  42

Find the mean deviation for each person’s computer usage. *Gregory: mean is 804 ÷ 14 = 57.4 or 57; mean deviation is 112 ÷ 14 = 8. Sister: mean is 710 ÷ 14 = 50.7 or 51; mean deviation is 106 ÷ 14 = 7.57. The mean deviation is 8 for Gregory and 7.57 for his sister.*

- **Activity 10:** The student will examine the spinner to the right and represent the likelihood of events by labeling them as impossible (0), unlikely, equal, likely, certain (1).
  
  a. The likelihood of landing on the letter Z *impossible*  
  b. The likelihood of landing on either a number or a letter *equal*  
  c. The likelihood of landing on the letter A *unlikely*  
  d. The likelihood of landing on a 1, 2, 3, A, B, or C *likely*  
  e. The likelihood of landing on 1, 2, 3, A, B, C, D, or on a line *certain*

- **Activity 18:** The student will write the theoretical probability of each player’s winning a game and discuss the fairness of the game given the information below:

  Imagine that the following 12 squares are cut apart and placed in a container, and Player 1 and Player 2 play a game by selecting squares. Each player in turn takes a square, records its color, and returns it to the container. Player 1 wins by selecting a red and Player 2 wins by selecting something other than red.
### Solutions

5 out of 12 red squares possible for Player 1 to win (about 42% if they give the percent): 7 out of 12 for Player 2 to win (about 58% if they give percent). It is not a fair game because each player does not have an equal chance of winning.
Grade 7 Mathematics
Unit 5: Angles and Circles

Time Frame: Approximately four weeks

Unit Description

This unit provides practice in solving real-life and mathematical problems involving angles and circles. The relationships among radius, diameter, circumference, and area of a circle are examined. An understanding of angle relationships in triangles is developed.

Student Understandings

Students can use facts about supplementary, complementary, vertical and adjacent angles to write and use them to solve simple equations for an unknown angle in a figure. They apply the formulas for circumference and area of a circle and can give an informal derivation of the relationship between the circumference and area of a circle.

Guiding Questions

1. Can students identify supplementary, complementary, vertical and adjacent angles in a figure?
2. Can students write an algebraic equation and solve for an unknown angle in a figure?
3. Can students illustrate the relationships between a circle’s circumference and area and the measures of its diameter and radius?
4. Can students identify and apply the angle-sum relationship for a triangle in problem-solving situations?

Unit 5 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>Identify and draw angles (using protractors), circles, diameters, radii, altitudes and 2-dimensional figures with given specifications (G-2-M)</td>
</tr>
<tr>
<td>24.</td>
<td>Determine the radius, diameter, circumference, and area of a circle and apply these measures in real-life problems (G-5-M) (G-7-M) (M-6-M)</td>
</tr>
</tbody>
</table>

Grade 7 Mathematics ◇Unit 5◇ Angles and Circles
### CCSS for Mathematical Content

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td><strong>7.G.4</strong></td>
<td>Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td>
</tr>
<tr>
<td><strong>7.G.5</strong></td>
<td>Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.</td>
</tr>
</tbody>
</table>

### Sample Activities

**Activity 1: Complementary, Supplementary, or Vertical? (CCSS: 7.G.5)**

Materials list: paper, pencil, straightedge, protractor, patty paper or tracing paper

In this activity, students will investigate the properties of supplementary, complementary, and vertical angles and write an equation to solve for a missing angle. Before beginning the activity, students will assess their understanding of key terms used in this activity through a vocabulary self-awareness chart (view literacy strategy descriptions). Because students bring a range of word understandings to the learning of new concepts, it is important to assess students’ vocabulary knowledge before interacting with the content. This awareness is valuable for students because it highlights their understanding of what they know, as well as what they still need to learn in order to fully comprehend the content. Have students copy the chart below in their notebook or learning log (view literacy strategy descriptions).

*Teacher note: make sure students leave plenty of room for examples and definitions since they will be adding to or modifying them throughout the activity.*

<table>
<thead>
<tr>
<th>Word</th>
<th>+</th>
<th>✓</th>
<th>-</th>
<th>Example or Illustration</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjacent angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complementary angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supplementary angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask students to complete the chart before the activity begins by rating each vocabulary word according to their level of familiarity and understanding. A plus sign (+) indicates a high degree of comfort and knowledge, a check mark (✓) indicates uncertainty, and a minus sign (-) indicates the word is brand new to them. Also, ask students to try to supply a definition and/or example for each word. For words with check marks or minus signs, students may have to make guesses about definitions and examples. Over the course of
the activity, allow time for students to revisit their self-awareness charts to add new information and update their growing knowledge about key vocabulary. Students may want to add new knowledge with a different color or with ink to indicate how their understanding is changing. Make sure students keep the chart handy and remind them to add to their original entry throughout the activity.

Draw the following pair of linear angles on the board and ask students to use a straightedge to draw a similar sketch labeling the left angle, “1” and the right angle, “2.” Depending on the formative assessment resulting from the vocabulary self-awareness chart, a discussion of angles and adjacent angles may be appropriate, at this point.

\[ m \angle 1 \approx \underline{\text{_____}} \]
\[ m \angle 2 \approx \underline{\text{_____}} \]

Students will use a protractor to measure each of the angles formed and write the angle measures to the side. Use this time to review how to use this tool appropriately, if needed. Next, students will draw a different pair of linear angles. Have them switch with two other people so that one student will measure \( \angle 1 \) and record the measure and a different student will measure \( \angle 2 \) and record the measure. When students have finished, make sure the original students get their sketch back. As a class, compile a list of students’ results. The list may look like the one below:

<table>
<thead>
<tr>
<th>Angle Measured</th>
<th>Angle Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m \angle 1 \approx 58^\circ )</td>
<td>( m \angle 2 \approx 122^\circ )</td>
</tr>
<tr>
<td>( m \angle 1 \approx 30^\circ )</td>
<td>( m \angle 2 \approx 155^\circ )</td>
</tr>
<tr>
<td>( m \angle 1 \approx 140^\circ )</td>
<td>( m \angle 2 \approx 40^\circ )</td>
</tr>
<tr>
<td>( m \angle 1 \approx 37^\circ )</td>
<td>( m \angle 2 \approx 145^\circ )</td>
</tr>
</tbody>
</table>

The teacher’s list should be longer than this so that students have enough data to make a conjecture.

After the class has compiled a list big enough, ask students to make a conjecture about the measures of the two angles. The students should notice that the angle pairs sum to 180º. Note: not all angle pairs will sum to exactly 180º, however, the sums should all be close enough to 180º for the students to make a proper conjecture. Ask students why they think some of the angle pairs do not add up to the same number. Allow students to discuss some of the reasons they came up with. Reasons should include, but not be limited to, human error, thickness of the lines, and improper use of the protractor. Discuss the importance of accuracy and precision when measuring.

Ask students what pairs of angles that add up to 180º are called. Discuss what it means for two angles to be supplementary. Include examples of nonadjacent supplementary angles. Next, ask students if the same relationship would be true for pairs of angles that add up to 90º. Discuss what it means for two angles to be complementary and include examples of nonadjacent complementary angles. Note: This may be a good time to have a discussion about parallel, perpendicular and intersecting lines. It should be noted that adjacent complementary angles are formed by perpendicular lines.

Next, have students use a straightedge to draw two intersecting lines and label angles formed 1, 2, 3, 4 in either a clockwise or counterclockwise rotation.
Ask students to make a conjecture describing which angles are congruent. *Students should see that angles 1 and 3 are congruent and that angles 2 and 4 are congruent.* To verify this conjecture, students will trace angle 1 with patty paper and see if they can find other angles congruent to it. Repeat this procedure for all the angles. For example, students can see that the measure of angle 1 has to be the same as the measure of angle 3.

![Diagram of intersecting lines with angles labeled 1, 2, 3, 4]

Ask students if they know the name of the non-adjacent angles formed when two lines intersect. Using *discussion* (*view literacy strategy descriptions*), specifically *Think-Pair-Square-Share*, students will describe how pairs of vertical angles are related in terms of their locations and what conjecture can be made about their measures. *Discussion* can improve learning and remembering when students participate in the dialog about concepts. Class discussion can be used to promote deeper processing of content and rehearsal of newly learned content. Ask students to think alone about how to define pairs of vertical angles in terms of their locations and what conjectures can be made about the measures of vertical angles. After students have thought about this for a short period of time, have them pair up with someone to share their thoughts. Then have pairs of students pair with other pairs, forming, in effect, small groups of four students. Monitor the brief discussions and elicit responses afterward. Be sure to encourage student pairs not to automatically adopt the ideas and solutions of their partners. These short-term discussion strategies work best when a diversity of perspectives are expressed. Allow students time after the discussion to record new learning on the *vocabulary self-awareness* chart. Remind students that they are allowed to use the chart as a reference when solving problems related to these terms.

**Activity 2: Find that Angle! (CCSS: 7.G.5)**

Materials list: pencil, paper, Find that Angle! BLM

In this activity, students will use facts about supplementary, complementary, vertical and adjacent angles to write and solve simple equations for an unknown angle. Sketch the following diagram on the board:

![Diagram of intersecting lines with angles labeled ABD, ABE, DBC, ACE]

Students will use *SQPL* (*view literacy strategy descriptions*) to write questions that would need to be answered before they can find the missing measure of one of the angles. *SQPL* promotes purposeful learning by prompting students to ask and answer their own questions about content. Pair up students and based on the sketch, have them generate 2-3 questions that would need to be answered before finding the missing measure of $\angle ABD$. *Questions may include*: What is the relationship between $\angle ABD$ and $\angle ABE$? What is relationship between $\angle DBC$ and $\angle ABE$? Can I use what I know about vertical angles to
find the missing measure? Can I use what I know about supplementary angles to find the missing measure? How do I find the missing measure of angle if they are supplementary? What are the relationships between all of the angles shown?

When all student pairs have thought of their questions, ask someone from each team to share questions with the whole class. As students ask their questions aloud, write them on the board. Similar questions will be asked by more than one pair. These should be starred or highlighted in some way. Once all questions have been shared, look over the student-generated list and decide whether teacher-generated questions need to be added. This may be necessary when students have failed to ask about important information they need to be sure to learn.

Next, have students work with their partner to write a simple equation and then solve to find the measure of \( \angle ABD \). \((x + 145^\circ = 180^\circ; x = 35^\circ)\). Tell them as they work through the problem to pay attention to information that helps answer the questions from the board. They should be especially focused on material related to the questions that were starred or highlighted. These might be considered class consensus questions.

Have a volunteer come to the overhead to explain how they found the missing angle measure. Ask students how they know the answer makes sense (because if they add 145 and 35, they will get 180. If not, then there is an error in finding the missing measure). Then have another volunteer describe which questions were answered by the students’ solution path.

Distribute Find that Angle BLM. In the example problem, students will use a process guide (view literacy strategy descriptions) to enable them to progress independently through the steps required to apply previous learning about angles to a new situation. Process guides scaffold students’ comprehension and are designed to stimulate thinking during or after their involvement in content instruction. Begin by explaining the guide’s features, intent, and benefit to students. Students will be able to use this guide as a reference with the remaining problems.

Pair students with a partner to complete the process guide found at the top of the Find that Angle BLM. Monitor students’ discussion and completion providing assistance where needed. Engage the whole class in discussion based on their responses to the guide, and use this feedback to provide additional explanation and to make any necessary modifications to the guide.

Students will work independently to complete the remaining problems on the BLM using the process guide as needed. Then students will exchange their papers with a partner to compare the responses and steps used to solve the problems. Discuss the responses to each problem as a whole class by inviting volunteers to come to the overhead to explain. If a document camera is available, students can display their work and describe the steps they used to solve the problem.
Activity 3: What’s Your Angle? (GLE: 24)

Materials List: What’s Your Angle BLM, protractor, pencil

Give each student a copy of the What’s Your Angle BLM. Individually, have students place three points on each circle to act as the vertices of a triangle, and then use the straight edge of a protractor to construct a triangle. Ask students to measure each angle of the triangle using the protractor, and record the measurements.

Have students share by comparing and discussing within their group the types of triangles drawn and the sum of angles of each of their triangles. Have students recognize that the sum of the angles in a triangle appears to be 180 degrees. Because of variances in measurements, this may not be obvious. This concept is revisited in the next activity.

Discuss the term altitude. Have the students draw an altitude in each of the triangles they drew inside the circles. Be sure to emphasize that the two perpendicular sides in right triangles are also altitudes.

For practice, have students draw triangles with specific side lengths and/or specific size angles. Give students the opportunity to draw 2-dimensional figures with specified angular measures.

Activity 4: Interior Angles of a Triangle (GLE: 24)

Materials List: construction paper, protractors, scissors, pencils, tape, math learning log

In this activity, the understanding that the sum of the interior angles of a triangle is 180 degrees is reinforced by using a method in which none of the individual angle measures is known. Since this process works for any triangle, it is considered a proof of the triangle sum theorem.

Divide students into teams of three students each. Give each student a sheet of construction paper and a protractor. Each group should also have at least one pair of scissors. Ask each student in the group to draw a different type triangle (acute, obtuse, right).

Have students cut out their triangles, label each angle with a letter (put the letter in the interior of the triangle near the vertex), and then tear off the angles of the triangle. Instruct students to place the angles adjacent to one another with vertices touching. Ask students how this proves that the sum of the three angles is 180 degrees.
Next, have each student draw a triangle on construction paper and label two of the angles with measures they find by using the protractor. The third angle should be marked with a question mark. On the back of the triangle, place the numerical value of the missing angle. Have teams exchange their triangles with another team, and then find the missing angle using the triangle sum theorem (rather than measuring the angle). They can quickly check each other’s work by comparing their answers to the one written on the back of the triangle. Students should discuss if there are differences of opinion regarding the answers.

In their math learning log (view literacy strategy descriptions), have students describe how to find the sum of the interior angles of a triangle as well as any new understandings they now have about the angles in triangles. Afterward, they can share their log entry with a partner to check for accuracy and logic.

Activity 5: π! (CCSS: 7.G.4)

Materials List: 3-5 different sized lids per group (place a mark on the edge of each lid with a permanent marker to indicate a starting point), 2 sheets of cm grid paper per group, tape, rulers, pencils, Around the Lid BLM, calculator, Circumference and the Dragon of Pi (optional)

The purpose of this activity is to have students investigate or discover π. Have students work in pairs for this activity. Distribute three to five different-sized lids for each group, two sheets of cm grid paper, and rulers. Have students tape the two sheets of grid paper together. (8 1/2 in edge to 8 1/2 inch edge to make the paper 22” long). Draw an x-axis along the bottom and a y-axis along the left edge to represent quadrant I of a coordinate plane.

Instruct students to place each lid on the x-axis so that the diameter of the lid rests along the x-axis, and the outside edge of the lid rests on the origin. Ask students to mark the endpoint of the diameter and then draw a line through that point which is perpendicular to the x-axis. (Note: Depending on the length of the diameter, this may or may not be one of the vertical lines on the grid.) Have students trace around the lid showing the semi-circle along the x-axis.

Next, instruct students to place the point marked as the starting point on the x-axis at the point where the end of the diameter of the lid is marked. Have the students roll the circle along the perpendicular line that they drew earlier by first placing the starting point at the intersection of the x-axis and the perpendicular line. When the starting point rotates back to the vertical line, have students mark this point.

Discuss the fact that this one rotation is called the circumference of the circle. Have the students find the lengths of diameter (marked on the x-axis) and circumference (marked on the perpendicular line) by reading the units on the grid and record their answers on Around the Lid BLM. Repeat the process with all lids.
Allow students to use a calculator to compute the ratio of the circumference to the diameter as a decimal correct to five places and place their answers in a table. Once students have completed the table on the Around the Lid BLM, ask them to record three observations for the data in the chart.

Lead a discussion concerning the students’ findings and the relationship between the circumference and diameter of a circle. Have students see that the pattern is multiplicative and that the decimal ratios are close to the value of $\pi$. Make sure that students understand that the relationship between the circumference and diameter indicates that the circumference is a little more than three times the length of the diameter. After the discussion, have the students calculate the average ratio for all five.

For a closure to this activity, read the book *Circumference and the Dragon of Pi* to the class to reinforce the concepts of pi, circumference, diameter, and radius. An alternative method is to begin the activity, read up through page 13, do the activity, and then finish reading the book.

**Activity 6: Pricing Pizza (CCSS: 7.G.4)**

Materials List: Pricing Pizza BLM, centimeter grid paper, compass, string, 1-inch grid paper (optional for below level students) and pencil

This activity asks students to think about how a pizza is priced relative to its diameter, radius, circumference, and area. Students are asked to find these measures for three different circular pizzas and to decide which measures are most closely related to price. The purpose of the activity is to encourage students to think about measuring circles, not to introduce the formulas. They should use counting and estimating strategies to find area and circumference. As students work through the activity, they should be looking for connections between a circle’s diameter, radius, area and circumference. Students should also search for clues that tell when each of these measurements gives useful information about a circular object in a given situation.

Prior to starting the activity, make sure students understand circumference, radius, and diameter. Students will complete vocabulary cards to demonstrate their understanding of these terms and will continue to use the card to add new knowledge and examples.

Begin the activity by discussing with students that many pizza restaurants sell small, medium, and large pizzas—usually measured by the diameter of a circular pie. Of course, the prices are different for the three sizes. Ask students if they think a large pizza is usually the best buy and why. Distribute Pricing Pizza BLM and centimeter grid paper.
Read the problem with the class so that they understand the context in which they will be investigating. Have students use centimeter grid paper and a compass to construct models of the three pizzas in the problem. Use the scale 1 in = 1 cm. Note: Below level students that have trouble with this scaled model can use large sheets of inch grid paper or tape together several sheets to make circles the actual size of the pizzas.

Have students work in pairs or small groups to find the radius, circumference, and area of each circle. Make sure students are recording their findings on the BLM and can explain how they arrived at their answers.

On the board or overhead projector, record the measurements that students found for the radius, circumference, and area of each circle. Discuss how they found the measurements and whether any seem unreasonable. Because students are counting and estimating, circumferences and areas will vary but should be within a reasonable range. Ask students if they see any patterns in the measurements that might help to predict circumference and area. Do not give the formulas now unless students see the relationships among diameter, radius, circumference and area and can explain why they make sense. Discuss the answers to part B. Many students will say that the diameter is most closely related to price because as it changes 3 inches, the price changes by $3.00.

**Activity 7: Covering a Circle (CCSS: 7.G.4)**

**Materials List:** Graphic Organizer BLM, Covering a Circle BLM

In this activity, students will find the area of a given circle. The purpose is not for them to find or use a formula but to see the need for a shortcut—a formula—for finding the area of a circle.

A review of how one finds the area of other shapes might help students begin to think about strategies for finding the area of a circle. Students will use a **graphic organizer (view literacy strategy descriptions)**, to record the name of the shape and the rules for finding its perimeter and area. **Graphic organizers** are effective in enabling students to assimilate new information by organizing it in visual and logical ways. Organizing previously learned information about rectangles, squares, parallelograms, and triangles will enable students to assimilate new information about circles. It is not important for students to present the rules as they are written here. What is important is that they have a correct formula that makes sense to them. The goal is not only to have efficient methods of finding area and perimeter, but to be flexible enough to realize that their method and another’s method may sound different but accomplish the same thing.

Next, students will explore how to find the area of a circle using what they know about finding the area of other polygons. Distribute Covering a Circle BLM and have them work with a partner to experiment in finding an easier way to find the area of a circle. Encourage them to think about different methods and strategies. Check that they are
prepared to share their answers and strategies. If students are struggling, ask questions that will help them cut down on the work by using the symmetry of the circle.

Have students share their measures and their methods. Some strategies students may report include:

- Find the area of the largest rectangle made from whole squares that could fit inside the circle. Then count and add the whole squares and parts of squares that were not inside the rectangle.
- Find the area of half of the circle and double it.
- Find the area of a quarter of the circle and multiply by 4.
- Surround the circle with a large square and find the area of it. Then find the area of the region outside the circle but inside the large square by counting the grid squares and parts of grid squares. Subtract this area from the area of the large square to get the area of the circle.

Students will revisit their graphic organizer and add a new “branch” to the back that includes circles. They should include and/or modify their previous explanation of the perimeter and area of a circle.

Activity 8: “Squaring” a Circle (CCSS: 7.G.4)

Materials List: Circles and Radius Squares BLM, Grid Paper BLM, lids from Activity 5

In this activity, students make squares with sides the same length as the radius of a circle and then determine how many of these “radius squares” are needed to cover the circle. They will easily see that four is too many and three are too few. The goal of the activity is to help students to discover the formula for finding the area of a circle and to understand why it makes sense.

Distribute Circles and Radius Squares BLM and Grid Paper BLM to students. For each circle, have students cut out several copies of the radius squares it takes to cover the circle. Then students will find out how many radius squares it takes to cover the circle.
cutting the radius squares into parts if needed. Students will record their data in a table with these column headings:

<table>
<thead>
<tr>
<th>Circle</th>
<th>Radius of circle</th>
<th>Area of radius square</th>
<th>Area of circle</th>
<th>Number of radius squares needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 units</td>
<td>36 square units</td>
<td>About 113 square units</td>
<td>A bit more than 3</td>
</tr>
<tr>
<td>2</td>
<td>4 units</td>
<td>16 square units</td>
<td>About 50.3 square units</td>
<td>A bit more than 3</td>
</tr>
<tr>
<td>3</td>
<td>3 units</td>
<td>9 square units</td>
<td>About 28.3 square units</td>
<td>A bit more than 3</td>
</tr>
</tbody>
</table>

Next, students will draw a couple of their own circles on centimeter grid paper. They can use the circles from objects in Activity 5. Students will make radius squares for each circle, and find out how many radius squares it takes to cover each circle. Students will add this data to their table. Then have students compare answers with a partner. Students should see that for each of the three circles, four radius squares are too many to cover the circle and three radius squares are too few.

<table>
<thead>
<tr>
<th>Circle</th>
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<th>Area of radius square</th>
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</tr>
</tbody>
</table>

Have students share their strategies for finding the number of radius squares needed to cover a circle. Then ask students to look at the table and identify patterns they notice. Students should be able to determine that it takes just over three radius squares to cover a circle. The answer, of course, is that it takes \( \pi \) squares to cover a circle. Help them to formulate a way to describe finding the area of a circle:

1. The area of a circle is a little more than 3 times the area of a square that has the circle’s radius as its side length.
2. The area of the radius square is found by multiplying the length by its width, which is radius times the radius, \( r \times r \) or \( r^2 \).
3. The area of a circle is thus (a little bit more than 3) \( r \times r \) or (a little bit more than 3) \( r^2 \)


This activity did not change because it already addresses the CCSS.

Materials List: paper plates (not Styrofoam), ruler or straight edge, scissors, pencil, paper, tape

Have the students cut a paper plate into eight sectors and position the sectors of the circle as shown to the right.
Lead students to understand that the shape formed is beginning to look like a parallelogram. You may want to have the students cut each of the eight pieces in half and use the resulting sixteen pieces to reform the parallelogram shape. The smaller the pieces, the more the shape will look like a parallelogram. Ask students leading questions so that they understand 1) the base of the rectangular shape is $\frac{1}{2}$ of the circumference and 2) the height of the parallelogram is the radius of the circle. Develop the formula for the area of a circle by starting with the formula for the area of a parallelogram, $A = bh$, and substituting $\frac{1}{2}C$ for $b$ and $r$ for the $h$. This results in the following:

$$A = bh = \frac{1}{2}C(r) = \frac{1}{2}\pi d(r) = \frac{1}{2}\pi(2r)(r) = \pi r^2,$$

the formula for finding the area of a circle.

Provide students with practice in using the formula.

**Activity 10: Real Life Measures (GLE: 28)**

Materials List: a round/circular object for each pair, measuring tapes, paper, pencil, Circles in Real Life BLM

Have a class discussion of the need to use correct terminology and how to measure circular objects. Ask, “What is someone referring to when he/she speaks of 22-inch rims on a vehicle? A 45-inch round table? Where on each of these objects would you measure to verify the information stated?” (diameter)

Ask students to work in groups of four for this activity. Give each pair of students a round or circular object and a tape measure. Have the students measure the diameter, radius, and circumference of their object. Record these measurements. Trade objects with the other pair in the group, measure, and record the measurements. Have the groups study and discuss the relationships between the radius and diameter measurements and between the diameter and the circumference measurements. Ask, “Do your measurements have the same relationships that are found in the circles in the previous activity? Why or Why not?”

Have each pair of students use their measurements to find the area of its object, then trade objects with another pair and find the area of the new object. Groups can compare their answers and check for accuracy and logic.

Provide real-life problems in which students find circumference, area, diameter or radius of given objects (e.g., car or truck tires, bicycle tires, gasoline drums, circular pools, silos, tree trunks, water tanks). Additional practice can be found on Circles in Real Life BLM.
Activity 11: Circumference and Area (GLEs: 24, 28)

Materials List: rulers, pencils, compasses, Circumference and Area BLM, computer with internet access (optional)

Break the students into groups of 4. Give each group rulers and three radii measurements. Have the students draw three different circles with the given radii. Give \( \frac{1}{3} \) of the groups the measurements 4 cm, 8 cm, 16 cm, another third the measurements 2 cm, 4 cm, 8 cm, and the last third of the groups the measurements 3 cm, 6 cm, 12 cm.

Have students complete the Circumference and Area BLM chart with given radii, diameter, circumference, and area and answer the questions that follow.

Discuss as a class the three different sets of numbers used and each group’s observations and reasoning.

An example of the chart with some answers is given.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Area</th>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cm</td>
<td>12.56cm²</td>
<td>4 cm</td>
<td>12.56 cm</td>
</tr>
<tr>
<td>4 cm</td>
<td>50.24cm²</td>
<td>8 cm</td>
<td>25.12 cm</td>
</tr>
<tr>
<td>8 cm</td>
<td>200.96cm²</td>
<td>16 cm</td>
<td>50.24 cm</td>
</tr>
</tbody>
</table>

After discussing each group’s observations and reasoning, have students complete this statement: “If the radius of a circle tripled, then …” Be sure to have students justify their reasoning.

An interactive math lesson for calculating the circumference of a circle can be found at http://www.aaamath.com/geo612-circumference-circle.html.

An interactive math lesson to teach the finding the area of a circle can be found at http://www.aaamath.com/geo612-area-circle.html.

Activity 12: Replacing Trees (CCSS: 7.G.4)

Materials List: Replacing Trees BLM

Begin by explaining to students that they will have a chance to use their knowledge of areas of circles to investigate a real-world problem. Present the following words to students: city, tree, law, replacement, diameter, circumference, and area. Students will use these words to write a lesson impression (view literacy strategy descriptions) text that will enable them to make a guess as to what the activity will involve. Lesson impressions create situational interest in the content to be covered by capitalizing on students’ curiosity. By asking students to form a written impression of the topic to be discussed, they become eager to discover how closely their impression text matches the actual content. Have students write a short paragraph using the given words in the context they
think will be used for the activity. When students finish their impression texts, invite volunteers to read what they have written to the class. Anticipation is heightened when several students share their different impressions, leaving students to wonder whose is the closest to the actual content.

**Example of Lesson Impression for a real-world problem**

**Impression Words:** city, tree, law, replacement, diameter, circumference, and area

**Impression Text:** My dad was telling me this morning about a new law he read about in the newspaper. The city is planning on building a new park with a walking track around the circumference of the park. There are a lot of trees in the park and the law says the city will have to pay a replacement cost for every tree that is cut down. The cost is figured using the area of the park and the diameter of each tree that is cut down. I'm glad that the city cares about the trees and is trying to protect them by passing this law.

Distribute Replacing Trees BLM and read the problem with the class. Make sure they understand the idea of a cross-section of a tree trunk and the two replacement rules. Have students work on the problem and follow up in groups of three or four. Remind students they must offer an explanation for their choice and give mathematical reasons why they chose one rule over the other. For struggling students, have them trace the diagram from the BLM onto grid paper and draw a picture of what it would mean to follow each of the rules.

Have groups share their answers and strategies. Challenge them to offer better explanations than simply, “If you want more trees in your neighborhood, the area rule is better.” Have them talk about the fact that, in the given example, the area rule will provide four times as many new trees as the diameter rule. Some students may talk about the cost of the trees and say that it is not fair to make someone come up with the money necessary to buy all the trees needed to satisfy the area model.

Ask students to go back to their impression text and compare their text with the actual information presented. Have students describe the similarities and differences in their learning logs ([view literacy strategy descriptions](#)), and exchange with a partner to see whose impression text came closest to the actual context of the problem.

**Sample Assessments**

**General Assessments**

- Determine student understanding as the student engages in the various activities.
• Create extensions to an activity by increasing the difficulty or by asking “what if” questions.
• Encourage the student to create his/her own questions to evaluate his/her understanding of circle concepts.
• Have the student create a portfolio containing samples to demonstrate understanding of one of the concepts in the unit. The following is an example: Anne called The Pizza Man to order a 12-inch pizza for her and her best friend. When the pizza delivery boy came to the door with her pizza, he had two boxes. He told Anne that they had run out of twelve-inch pizza boxes and the owner had told him to deliver two, six-inch pizzas instead. He told Anne that it was the same amount of pizza and each of them would have her own pizza. Anne disagreed with the pizza boy and convinced him that she was correct. Explain how you think Anne might have proven this to the pizza boy. Use diagrams and/or mathematics in your proof.
• Have the student trace, measure and record the diameter of three circular objects and then find the circumference and area of each.
• Have the student complete journal writings using such topics as:
  o Explain the difference between circumference and perimeter.
  o Form a conjecture about the relationship between the shape of rectangles and their areas and perimeters.
• Have the student use the given measurements of a circle to find the other measurements. Example: A dinner plate has a diameter of about 9 inches. Find its circumference and area.

Activity-Specific Assessments

• Activity 3: Jordan rides his skateboard over a ramp shown below. What is the measure of the angle that the ramp makes with the ground?

![Diagram of a ramp]

Solution: The angle measures 18°

• Activity 4: The student will find the sum of the measures of the angles of each shaded region and explain his/her reasoning.

a) ![Diagram of a shaded region]

b) ![Diagram of a shaded region]

Solutions: a) 360° b) 540°
• Activity 6: The student can demonstrate an understanding of radius, circumference and diameter.

Which measurement of a circular pizza—diameter, radius, circumference or area—best indicates its size?

Solutions: Pizza sizes are usually given by diameter, so the logical measure that best indicates its size is area. However, the radius and circumference are also defensible answers because as any of the four measures of a circle increase or decrease, so do the other three. Therefore, since all four measures are related, any of them are technically plausible as indicators of the size of the pizza.

• Activity 10: The student can describe the connection of area and circumference to real-life.

Some everyday circular objects are commonly described by giving their radius or diameter. In the examples below, explain what useful information (if any) you would get from calculating the area or circumference of the circle.

  o A 3.5-inch diameter computer disk
    Solution: the area of a computer disk tells you something about the storage space on the disk.
  o A 21-inch diameter bicycle wheel
    Solution: the circumference of a bicycle wheel tells you how far the bike travels in one revolution.
  o A 12-inch water pipe
    Solution: the diameter and related cross-sectional area of a water pipe will tell you how much water can flow through the pipe.
  o A lawn sprinkler that sprays a 15-meter radius section of lawn
    Solution: the area of a lawn sprinkler’s spray would let you estimate how much of your lawn will get watered at each location the device is used and will allow you to estimate how long it will take to water your lawn.

• Activity 11: The student can correctly find radius, circumference and diameter.

A large burner on a standard electric stove is about 8 inches in diameter.

a) What are the radius, circumference, and area of the burner? Solution: radius = 4 in., circumference = about 25 in., area = about 50 sq. in.

b) How would the area and circumference of a smaller 4-inch diameter burner compare to the area and circumference of the 8-inch burner?

Solution: the circumference will be about half—about 12.5 inches. The area, however, will be only one fourth as much—about 12.5 sq. in.

• Activity 12: The student will work the following problem correctly:

Randy’s dad cuts circular targets for archery practice. He asked Randy to help him by cutting one circle with a diameter of 12 inches out of red plastic and a second circle with a diameter of 18 inches out of blue plastic. He wanted a white center bull’s eye circle cut with a diameter of 6 inches. Randy was to glue these circles together into a target that looked like the
picture. Determine the area of each color on the target, and explain your method of solving the problem.

Solution: The white bull’s eye has an area of 28.26 sq in
The red area that is showing has an area of 84.78 sq in
(Whole red circle area of 113.04 – white circle area of 28.26)
The blue area that is showing has an area of 141.3 sq in
(Whole blue area circle of 254.34 – whole red circle of 113.04)
Grade 7
Mathematics
Unit 6: Measurement

Time Frame: Approximately five weeks

Unit Description

This unit extends the work with measurement conversion and the application of perimeter and area concepts to irregular and regular polygons. Building on work with surface area and volume in previous grades, students solve mathematical and real-world problems related to two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Scaling and drawing to scale is revisited in the context of reproducing a scale drawing at a different scale. The concept of slicing is introduced as students describe the two-dimensional figures that result from slicing three-dimensional figures.

Student Understandings

Students will convert, within the same system, between units and compare measurements between the systems using common reference points. Students will calculate area, perimeter, surface area and volume mathematically and in the context of real-world situations. Students will build conceptual understanding of scale factor in different situations. Being able to identify polyhedrons and use this knowledge to calculate surface area and volume as well as to describe the resulting two-dimensional figure from slicing a three-dimensional figure will be developed.

Guiding Questions

1. Can students convert between measures of area within the same system of measurement?
2. Can students work with the changes in scale?
3. Can students identify situations in which area, surface area or volume are used and be able to calculate with accuracy?
4. Can students identify polyhedrons?
5. Can students apply these understandings in problem-solving situations?

Unit 6 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>GLE #</th>
<th>GLE Text and Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)</td>
</tr>
</tbody>
</table>
### Sample Activities

**Activity 1: Measuring Scavenger Hunt (CCSS: 7.EE.3)**

*This activity has not changed because it already incorporates the CCSS.*

**Materials List:** a list of measurements describing various objects (several for each pair of students), yard sticks and/or measuring tapes, meter sticks, protractors, pencil, paper, Measuring Scavenger Hunt BLM

Make a list of measurements describing various objects found in the classroom or in a specified area outside on the school grounds. Provide measures in the U.S. and metric systems with angle measurements included. Be sure to include descriptions that are given in units of area. Measurements may be added to the Measuring Scavenger Hunt BLM and given to students to find the objects. Give each pair of students a list of objects, a yard stick (or tape measure), a meter stick, and a protractor. Have students hunt for each object described, measuring objects to find the ones on the list and writing the name of the object found on paper. Specify a time limit for completion of the hunt.

(Example of description of objects: This object is 6 inches off the ground, and its dimensions are 12 inches by 4 inches, or this object has an angle that measures 60º and the sides that form the angle each have a length of 15cm, or the top of this object has an area of 10 square inches and sits on the ground.)
When the students return to the classroom, have them convert specified measurements within the same system (e.g., 6 inches = _____ foot, 12 in = _____ ft, 2 square feet = _____ square inches, 36 square feet = _____ square yards).

As an extension, allow each pair of students to find and write a measurement description for an object, then swap descriptions with another pair to find the object.

**Activity 2: Metric Madness (CCSS: 7.EE.3)**

*This activity has not changed because it already incorporates the CCSS.*

Materials List: metric rulers, meter sticks, paper, pencil, place value chart (can write on board)

Pass out metric rulers and meter sticks to groups of students. Have the students study the rulers and meter sticks, and write down observations about the relationships between units. Lead a class discussion to help students develop an understanding of equivalencies. Discuss how the metric system is based on 10, the same as the place-value system. Help the students connect the metric prefixes with the decimal place value system. Place metric prefixes with decimal names on a place value chart to help students remember their values (i.e., thousands–kilo, hundreds–hecto, tens–deca, ones–meter, tenths–deci, hundredths–centi, thousandths–milli). Discuss this chart emphasizing the most needed units–kilo, centi and milli. Using the meter stick, have the students convert millimeters to centimeters and kilometers and vice versa. Record each conversion on the board, study the conversions, and discuss how converting from a smaller unit to a larger unit requires division and converting from a larger unit to a smaller unit requires multiplication. Make sure that students are aware that the prefixes work the same with grams and liters.

Extend the activity with explanation, examples, and problems involving converting between measures of area in the metric system.

Example: Joshua is planning to cover a coffee table with material that is sold in square meters. His table measures 60 cm by 32 cm. How many square meters of material should he purchase?

Present additional real-life problems in which students convert between units of area.

**Activity 3: Building a Cube (CCSS: 7.EE.3)**

*This activity has not changed because it already incorporates the CCSS.*

Materials List: ruler, pencil, scissors, and a brown grocery bag of piece of newsprint for each student, tape, string or yarn, cube-shaped box, solid wooden or plastic blocks

This activity lays a conceptual foundation for understanding squares and cubes and conversions within the same system. These two concepts are very difficult for students to grasp.

Distribute a ruler and brown paper bag (available at grocery stores) to each student.

Have each student draw a 12 inch by 12 inch grid on the paper bag, and then cut out the grid in order to make a square foot. Be sure to identify the area of the square as
12 in by 12 in = (12 in)² = 144 in².

Discuss the conversion. Make sure that students understand that they are changing two of the dimensions, not just one when converting to square inches to square feet and that 144 sq inches = 1 square foot.

Discuss how students could make a square yard from the square foot grids. A square yard can be made by taping 9 student square foot grids together (3 square foot grids by three square foot grids). Discuss the conversion. Make sure students understand that they are changing two of the dimensions, not just one when converting square feet to square yards. 9 square feet = 1 square yard.

Tape 6 of the students’ square grids together that are 1 foot by 1 foot by 1 foot to form a cubic foot. Explain that the six faces represent only the surface area of the cubic foot; a cubic foot is a solid. To help students visualize this concept, have them fill a cube-shaped box with solid wooden or plastic blocks to allow students to find the volume of the cube they create.

12 in x 12 in x 12 in = (12 in)³ = 1728 in³

Discuss with students how to build a cubic yard. How many cubic feet will be needed to make a cubic yard? Have students tape 6 of the 3 foot by 3 foot (1 square yard) sections to form a cubic yard. Again, remind students the six faces only represent the surface area of the cubic yard. Discuss the conversion. Make sure students understand that they are changing three of the dimensions, not just one or two when converting cubic measurements.

If possible, hang the square foot and cubic foot (square yard and cubic yard also) from the ceiling for reference during the year.

**Activity 4: Break it Down (CCSS: 7.G.6)**

**Materials List:** Break it Down BLM, 2 sheets of cm grid paper per student

Begin by distributing copies of the Break it Down BLM and one sheet of grid paper to each student. Allow students to work independently to determine the area of the two given figures. The activity requires the students to build on their earlier experiences of counting whole and partial squares on a grid to find the areas of figures. Move around the room to get a sense of the
strategies they are using and encourage students to try different strategies from what they may have already shown. The students should see that each figure is a composite of rectangles, squares, and triangles, and then will use different approaches to determine the area. Once the students have completed their measurements and calculations, discuss their strategies as a class.

As the students discuss the different ways in which they found the areas for Shapes A and B, the following strategies for finding areas should come up: counting squares and partial squares within the figure; decomposing the original figure and applying formulas to the various subfigures; or “enclosing” each figure in a rectangle, calculating the area of the rectangle, and subtracting areas of the “added” pieces. Be sure to discuss all methods and any other strategies the students used so that the entire class will come away with multiple ways to think about finding areas.

Allow students to apply this skill to a real-world situation. Copy the following sketch of a playroom on the board and distribute the second sheet of grid paper to students. Ask students to use a scale of 1 unit = 2 feet to draw the sketch on their grid paper. Next, students will calculate the amount of carpeting needed to cover this room. Next, students will calculate the cost of carpeting this room if the price is $15 per square yard. Students will need to convert square feet to square yards to determine the cost.

Students may subdivide the figure as follows:

To calculate the cost of carpeting the room, students must know there are 9 sq ft in 1 sq yd:

\[
\frac{9\text{sq ft}}{1\text{sq yd}} = \frac{108\text{sq ft}}{x} \quad \text{so} \quad 9x = 108 \quad \text{and} \quad \frac{108}{9} \quad \text{is 12 square yards.} \quad (108 \text{ sq ft} = 12 \text{ sq yd})
\]

12 sq yd x $15 per yard = $180 to carpet the playroom.

Activity 5: Calculate Perimeter and Area of a Plane Figure (CCSS: 7.G.6)
This activity has not changed because it already incorporates the CCSS.

Materials List: video of home improvement show and TV or projector (optional), House Plan BLM, newsprint, color markers, rulers, catalogs or sale papers from home improvement stores, scissors, glue and/or tape, paper
Allow students to view a clip from a home improvement show for motivation. Tell students that when beginning various home improvement projects, homeowners often must know the dimensions of each room. Using the House Plan BLM or another simple house plan that includes several rooms composed of composite plane figures with a scale of $\frac{1}{2}$-inch equals 5 feet (this can be changed to suit the project), have a class discussion on how to measure and find the dimensions of the floors in a room on the house plan. Review how to find the area of the walls of the rooms with the assumption that the walls are 8 feet tall. Have a discussion about how to convert between units of area. Tell students that the area of a kitchen is 75 square yards, and is being covered by tiles that measure 1 square foot. How many square foot tiles are needed to cover the floor? 675 tiles or 675 square feet

Working in groups of four, have each group pick a room that they want to redo. Have them draw a scale model of the room on newsprint and provide a key for the scale used. Then have students find the perimeter and area of the walls, floor and ceiling of the room. Instruct students to make clear explanations of the processes used at each phase of the problem. Using catalogs from home improvement stores, have students select molding, flooring, paint, wallpaper, and other improvement items. Have students cut out pictures and prices of materials, glue them to a sheet of paper, and then calculate the cost for each. To facilitate evaluation, have students show their work on the paper next to the pictures. Allow students to decorate by cutting out pictures to show how they want each room to look.

This project will take several days depending on the number and type of improvements incorporated. Include area unit conversions within the same system, U.S. or metric, in the activity, having students show any conversions on their papers. For example, if carpeting is sold by the square yard and the living room has an area of 200 square feet, how many square yards of carpet are needed? Students should include the cost per improvement and a total cost for the room.

After each group has finished, have them assume the role of professor know-it-all (view literacy strategy descriptions) and present their room remodeling to the class, explaining how they figured the cost of each improvement. Remind other students to formulate questions to ask the professors and to hold them accountable for the accuracy of their answers. After all presentations have been made, place all the rooms together to build the house. Display on a classroom wall.

Activity 6: Pool and Hot Tub Addition (CCSS: 7.G.6)
This activity has not changed because it already incorporates the CCSS.

Materials List: Pool & Hot Tub BLM, pencil, math learning log

Pose this situation as an addition to Activity 1 and have students work in groups. The Pool and Hot Tub BLM may be used.

The swimming pool that is to be put in the back yard has an irregular shape as shown below. A pool cover is needed to keep the leaves out this winter. Find the area of the pool. All corners are
90°. Pool covering material costs $4.95 per square yard; how many square yards are needed, and how much will the pool cover cost? Explain how you arrived at finding the area of the pool and the cost of the pool cover. We also need to know the perimeter of the pool, so that we can buy bricks to go around the edge of the pool. Find the perimeter. Bricks are 6 inches long. How many bricks are needed to buy for one row of bricks end to end around the pool? Bricks cost 60¢ each. How much will be spent on bricks? Explain and show how you determined the perimeter of the pool and the cost of the bricks.

A hot tub in the shape of a trapezoid with the dimensions shown will be built along the right side of the pool and adjacent to the bricks. A top view of the hot tub is shown. Find the cost of making a cover for the hot tub. Since the hot tub will be placed next to the swimming pool, the side with a length of 4 ft. will not be bricked. Find the cost of bricking the remaining three sides. Show all work for determining the cost of the cover and the bricks.

Ask students to respond to the following prompt in their math learning logs (view literacy strategy descriptions). Depending on the students, there may need to be a similar discussion prior to the students completing the math learning log.

Are the answers you found during the Pool and Hot Tub activity realistic? Give at least three reasons to support your answer or describe how the answers should be adjusted to fit the real-life situation.

Example answers: If the pool covering material is sold by the yard, it will have to be sewn together and extra material should be bought. For the bricks around the pool, the end bricks will need to extend the width of the brick to make an even corner. Also, the lengths are not all multiples of 6, and the bricks will need to be broken. Will there be mortar between the bricks? The cover of the hot tub is a trapezoid, so extra material will be needed for the slanted side. These things encourage higher order thinking.

Activity 7: Designing a Park (CCSS: 7.G.1, 7.G.6)

Materials List: Designing a Park BLM, centimeter grid paper, ½ cm grid paper, grid paper on large rolls (optional), rulers, compass, calculator

Begin by telling students they will be designing a city park with various specifications in the form of a scale drawing. Facilitate a whole class discussion for students to consider the total
space appropriate for a park. Be sure to include terms such as feet, yards, square feet, square yards, and dimensions. What do students consider to be an appropriate size for a city park? Ask students to state their ideas in units of square feet or square yards. To help students visualize some of their ideas, divide students into groups of 4 and take them outside to walk off various areas. First, ask groups to form an area of land that is 4 square yards. 4 students should form a square that is 2 yards by 2 yards or 6 feet by 6 feet. While listening to the discussion of each group, be sure students are using appropriate units of measurement. Ask groups to give ideas about park features that could take up this amount of space (water fountain area, snowcone stand . . .) Next, ask groups to form an area of land that is 9 square yards. Students should form a square that is 3 yards by 3 yards or 9 feet by 9 feet. Ask groups to give ideas about park features that could take up this amount of space (small picnic table spot, bike rack, ...) Ask groups if their original idea about an appropriate park size is reasonable now that they have had an opportunity to “walk out” smaller areas. Were their estimates too small? Too large? In order to answer these questions, the students may need to “step off” the size of the park as a class.

Continue the discussion in the classroom by asking students about the kind of shapes that could be made if the city park involved 2500 square yards of land (rectangle, square, circle?). Distribute centimeter grid paper (for students that need to “see” the dimensions) and calculators. Ask the following questions:

- What would be the dimensions of a park that had an area of 2500 square yards? Let students come up with several variations that include rectangles and a square.
- Would it be better to build the park on a piece of land that was 5 yards by 500 yards?
- Why or why not? What about a piece of land that was 10 yards by 250 yards? How functional would these shapes be? What would a park with these dimensions look like? Small group exploration may be best for these questions.
- What would a circular park look like? The closest circular park with an area of 2500 square yards would have a radius of about 28 yards to give an approximate area of 2,463 square yards.
- What would be the advantages and/or disadvantages of designing the park as a square? As a rectangle? Students may need to consider which figure gives a larger perimeter.

The next part of the discussion centers on drawing to scale using cm grid paper. Have students consider the advantages and disadvantages of having each square represent ½ square yard through 10 square yards. For example, could they draw a small object in the park if each square equaled 10 square yards? If they made each square represent only ½ square yard, how large would their park blueprints become?

Distribute Designing a Park BLM and discuss park specifications. This activity could be assigned as an individual, partner or small-group project. Students have the choice of sketching their scale drawing on cm grid paper, ½ cm grid paper or a sheet from a large roll of grid paper. Encourage students to visit local parks or school playgrounds and make measurements of things they might put in their designs.
Activity 8: What’s the Difference? (CCSS: 7.G.1)

Materials List: ½ cm grid paper (2 sheets per student), Similarity and Scaling BLM (1 copy per group)

Students will use grid paper to see the effects of changing the scale of a figure. Pose this question to students: How does changing the scale change the drawing? *Students will give varying responses but should see that it depends on the scale.* Ask students to predict the effects on a scale drawing if the scale is changed from 1:1 to 1:2. Will the drawing be bigger or smaller? How will the new scale affect the area of the figure on the drawing? *Students will have the opportunity to explore this idea.*

Distribute one sheet of grid paper and ask students to sketch a rectangle with sides of 6 cm and 8 cm using a scale of 1 unit = 1 cm. Ask students what this scale means. *This means that 1 unit represents 1 cm on the drawing.*

Next, ask students to sketch the same rectangle, but using a scale of 1 unit = 2 cm instead. Ask students what this scale means. *This means that 1 unit represents 2 cm on the drawing.* Be careful here. Students will want to draw 2 cm for 1 unit, which is not the same thing. If this comes up, ask students why this is not the same. When students have drawn the rectangle using the new scale, ask: What was the effect of changing the scale on this drawing? *Doubling the scale shortens the length of each side by half: the sides are now 3 units by 4 units. The area of the figure with the original scale was 48 square units and now the area is 12 square units. The area of the scaled down rectangle is ¼ of the area of the original one. Students should see that both dimensions were decreased by ½, which affects the area by ¼ (½ x ½).*

Ask students the next series of questions to help them see the difference between scales that may cause confusion. The “big idea” that needs to surface is that the scale of a drawing is the ratio of the size of the drawing to the size of the actual object or scale = \[
\frac{\text{size of drawing}}{\text{size of actual object}}
\]

- What does a scale of 1:2 mean? *The ratio of the size of the drawing is ½ the size of the actual object or \( \frac{1}{2} = \frac{\text{drawing}}{\text{actual}} \). Another way to say this is if the scale factor is less than 1, it is a reduction.* What real-life situation could be represented by this scale? *A map or model is a reduction of the actual object.*

- What does a scale of 2:1 mean? *The ratio of the size of the drawing is twice the size of the actual object or \( \frac{2}{1} = \frac{\text{drawing}}{\text{actual}} \). Another way to say this is if the scale factor is greater than 1, it is an enlargement.* What real-life situation could be represented by this scale? *A diagram of a plant cell is an enlargement of the actual object.*
• How is a scale of 2:1 different from a scale of 1:2? A 2:1 scale means that the actual object is smaller than the drawing, and a 1:2 scale means that the actual object is bigger than the drawing.

To help students see the relationship between the side length and area of a square, draw a 1 x 1 and a 2 x 2 on the board. Ask students to use grid paper to sketch these two squares and then to sketch the next three squares in the sequence (3 x 3, 4 x 4, and 5 x 5). Ask students to describe their observations about the relationships they notice in this pattern. Then, ask guiding questions until students realize that the area is the square of the sides or $s^2$.

Arrange students in groups of four. Distribute the second sheet of grid paper to each student and one copy of Scaling and Similarity BLM to each group. Ask students to sketch Square A and Square B on the grid paper labeling the area, side length, and perimeter of each square. Invite volunteers to share their sketches with the class, describing the reasoning their group used to determine Square B. It may be helpful to sketch edge pieces when comparing lengths. Repeat for Squares C through I.

Possible student reasoning used to determine each square follows. For example, students may determine Square C by sketching edge pieces along one side of Square B and then doubling that length to determine the length of Square C. Students may notice that the ratio of the areas of 2 squares is the square of the ratio of the side lengths: $\frac{B}{A} = \frac{9}{1} = \left(\frac{3}{1}\right)^2$ and the ratio of the perimeters is equal to the ratio of the side lengths: $\frac{12}{4} = \frac{3}{1}$. Encourage students to discuss their ideas about why this is so.

To reinforce vocabulary, ask students the following:
• What is the scale factor of Square B to Square A? scale factor of 3, it is an enlargement of Square A
• What is the scale factor of Square A to Square B? scale factor of $\frac{1}{3}$, it is a reduction of Square B.

Continue this line of questioning by choosing pairs of squares from A through I and ask the students to identify scale factors of the sides that relate the squares to each other. Make sure the following ideas and related math language comes out:
• When the lengths of the sides of a figure are each changed by multiplying by the same number and the angles of the figure are not changed, that number is called a \textit{scale factor} and the figures are \textit{similar}.
• When the scale factor is greater than 1, the result is an enlarged figure.
• When the scale factor is between 0 and 1, the result is a reduced figure.

Ask students to discuss examples of enlargements and reductions they have seen or used. \textit{Students will likely have many examples, including photographs, maps, and models of different sorts.}

\textbf{Activity 9: Scaling Shapes (CCSS: 7.G.1)}

Materials List: Scaling Shapes BLM, calculators, a set of two similar two-dimensional shapes with a linear scale factor of 2:1 for the teacher

The purpose of this activity is not to learn how to find perimeter and area, but to understand the relationship between the scale factors of similar figures and the resulting scale factors of the perimeters and areas. Begin by holding up the two similar two-dimensional shapes, such as two triangles, two rectangles, two parallelograms or two other quadrilaterals with a scale factor of 2:1. Ask the students how the two shapes are related. \textit{Same shape but one is larger than the other.} Ask the students how they think the \textbf{linear measurements} of the two shapes compare. Select a few pairs of students to demonstrate with the two shapes how they think the lengths compare using as much math language as possible in their explanation. \textit{Students should see that the length of the larger shape is twice the length of the smaller shape.} Next, ask students how they think the \textbf{areas} of the two shapes compare. Select pairs of students to demonstrate how they think the areas compare justifying their reasoning with appropriate math language. \textit{Students should see that the area of the larger shape is four times the area of the smaller shape.}

Sketch the following two figures on the board or overhead:

![Diagram of two triangles](image)

Ask students to write a ratio that compares the large triangle to the small triangle. \( \frac{B}{A} \)
Next, ask students to write a ratio of the base of Triangle B to Triangle A. \( \frac{6}{2} \) Then ask students to write a ratio of the height of Triangle B to Triangle A. \( \frac{12}{4} \)
Ask students if there is a relationship between the ratio of bases with the ratio of heights and what they think this relationship means. \textit{Students should see that both ratios reduce to \( \frac{3}{1} \) which means that the lengths of the larger triangle is 3 times the size of the smaller one (scale factor).}
Continue with the same line of questioning to compare the small triangle to the large one. 
*Students should see that the scale factor is now \( \frac{1}{3} \) which means that the lengths of the smaller triangle is one-third the size of the larger one.* It is also important to mention here that scale factor can also be written as 3:1 (three times larger) or 1:3 (three times smaller or \( \frac{1}{3} \) the size of the larger one). An *enlargement* has a scale factor greater than 1, and a *reduction* has a scale factor less than 1.

Distribute Scaling Shapes BLM and calculators to students and have them work in small groups to complete the BLM. Discuss their findings. Be sure that the students realize that although the rectangles on the activity sheet are all similar, different scale factors can exist among the pairs. The students should also see that pairs of shapes must have the same scale factor to be considered similar.

For an extension, see the Illuminations link:
http://www.nctm.org/standards/content.aspx?id=26770  Side Length and Area of Similar Figures (applet): The user can manipulate the side lengths of one of two similar rectangles and the scale factor to learn about how the side lengths, perimeters, and areas of the two rectangles are related.

**Activity 10: Scaling in the Real World (CCSS: 7.G.1)**

Materials List:  Group cards BLM cut into 6 problems (1 card per group), chart paper, markers, sticky notes, Scaling in the Real World BLM for each student

Write the following situation on the board and ask students to solve independently first, then pair with a partner to discuss.

The scale of a Louisiana map is 1 in = 50 mi. Find the actual distance from Zachary to Franklin if the distance on the map is 2.75 in. The actual distance is 137.5 miles.

When most have completed the problem and discussed with a partner, call on students who used different methods to solve.

**Possible Method 1:**

\[
\text{scale length} = \frac{\text{map length}}{\text{actual distance}} \quad \text{or} \quad \frac{1\text{in}}{50\text{mi}} = \frac{2.75\text{in}}{d}
\]

**Possible Method 2:**

<table>
<thead>
<tr>
<th>1 inch</th>
<th>1 inch</th>
<th>⅔ inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 miles</td>
<td>50 miles</td>
<td>37 ½ miles</td>
</tr>
</tbody>
</table>

Divide students into groups of four. Distribute Group Cards BLM (1 card per group) assigning each group a different problem. Allow students to work the assigned problem independently.
before the group discusses the processes used to solve it. Once each group reaches consensus, distribute chart paper and markers for students to show how they worked the problem. Post groups’ solutions around the room and allow students to circulate with their group to each problem using sticky notes to write clarifying questions on the problem. Once all groups have circulated, distribute group problems back to the original group to discuss clarifying questions as a group before presenting solutions to the class.

Distribute Scaling in the Real World BLM to each student to take notes about each problem presented by the groups.

Activity 11: Any Way You Slice It! (CCSS: 7.G.3)

Materials List: Styrofoam cone (teacher demo), Styrofoam blocks cut into rectangular prisms and cubes (1 per group), wax-coated dental floss, Classifying Solids BLM, What Slice is It BLM

Teacher Note: Prior to this activity, make two cuts on the cone. One cut should be made horizontally (parallel to the base). It is helpful to insert a small dowel down through the cone from the center of the base up to vertex of the cone. Do not go through the vertex, but tape the end of the dowel to the interior of the cone at the vertex. The dowel will hold the cone together, allowing the teacher to hold the cone up without its falling apart.

Put the cone back together and make a vertical cut (perpendicular to the base). Optional: use two cones and have one representing the horizontal cross section and the other cone representing the vertical cross section. When discussing the cross sections formed with students, open the cone to reveal the inside by holding the appropriate pieces of the cone. Use a large sheet of styrofoam and cut cubes and rectangular prisms from it (one per group). Depending on the depth of the foam, the base of the prism should be no larger than a 4 in x 3 in piece.

Begin the activity by reviewing vocabulary related to 3-dimensional figures. Ask students to describe a polyhedron. A polyhedron is a three-dimensional figure in which all faces are polygons. Ask students to describe the difference between a prism and a pyramid in words or sketches. Call on volunteers to sketch at the overhead or whiteboard what they think a prism and/or a pyramid looks like. A prism is a 3-dimensional figure that has two congruent and parallel faces that are polygons with the remaining faces being parallelograms. A pyramid is a three-dimensional figure whose base is a polygon and whose other faces are triangles that share a common vertex.

Next, distribute Classifying Solids BLM and have students complete the word grid. A word grid (view literacy strategy descriptions) is an effective visual technique for helping students learn important related terms and concepts. Before students are able to understand “cross sections,” they must first be able to classify the three-dimensional solids from which the two-dimensional cross sections come from. First, ask students to mark X’s in each row that describe each shape. Then ask students the following questions about the first solid:
What property helped you to determine whether this figure was a polyhedron or non-polyhedron? All faces are polygons.

What property(ies) helped you to determine what kind of polyhedron it was? No parallel faces, the base is a polygon and the other faces are triangles that share a common vertex.

How can these properties help you to name the solid? Because there were no parallel faces, it had to be a pyramid and since the base is a rectangle, the name is “rectangular pyramid.”

Ask students if they need to go back and correct any of the columns they checked off previously after discussing the solids’ properties.

Have students record in the last column the name of the solid and the properties that helped them to classify it based on the previous discussion. Students will work with a partner to complete the last column for all solids. While monitoring pair discussions, listen for the justifications for which students classify each solid. When recording the properties that helped them to classify the solid, students may change their minds on how they checked off the descriptions of each at the beginning of this activity. Look for a correct match in descriptions and properties. Ask student pairs to find another pair to share out.

As a whole class, ask students if they noticed any similarities or differences in which the solids were classified. Answers will vary but students should see that the only non-polyhedrons contained a circle as one of the faces (cylinder and cone). Students may also have classified the triangular prism as a pyramid. If this comes up, ask guiding questions to ensure understanding that a pyramid will only have one face that is a polygon while the remaining faces are triangles. The triangular prism on the BLM contains 2 triangular faces (parallel)and the remaining faces are parallelograms (rectangles). Refer to the initial discussion of prism vs. pyramid. Students will refer to the word grid when completing the next part of the activity and may add additional solids to the grid as they come up during the activity.

To introduce “cross sections,” find pictures of several real-world examples to show students (cross section of a leaf, a planet, a hurricane, a tree…) Next, ask students the following opening question: If you place a cone on the table and cut a slice that is parallel to the table (parallel to the base), what will that face look like? Ask students to sketch their prediction. After students sketch the resulting cross section, show them the pre-sliced cone to see if they were correct. Next, ask them to predict the resulting cross section if the cone were cut perpendicular to the base and sketch their prediction. The cross section will be an isosceles triangle. After sketching their prediction, show them the pre-sliced cone to see if they were correct.

As a class, discuss how you can predict what a particular cross section will look like. Distribute Styrofoam prisms and cubes (one per group) and a 6 inch piece of dental floss for students to explore this concept. Waxed dental floss can be used to slice the Styrofoam using a back-and-forth sawing motion. Have students explain their method and why it would work. To experiment with cross sections that are created from a slice other than a horizontal or vertical cut, the following applet may be useful:
The big idea that students should grasp is that a slice made parallel to the base will result in a two-dimensional face similar to the base, and a slice made perpendicular to the base will result in a two-dimensional face similar to the lateral (side) faces.

Distribute What Slice is It BLM for students to complete with a partner. Students may need to refer to the word grid at the beginning of the activity for clarification on the names of solids. As students are completing the What Slice is It BLM, have them add other solids to the word grid to classify and name. Call on volunteers to come to the overhead or white board to demonstrate or explain how they determined what the resulting cross sections of each solid would be. A document camera is especially helpful for this activity, if available. Finally, ask students which solids were added to the original word grid. Remind them that the grid can be used as a reference sheet when exploring subsequent concepts such as surface area and volume.

Activity 12: Let’s Build It (CCSS: 7.G.6)

Materials List: centimeter cubes (30 per group), cm grid paper (2 sheets per student), scissors, calculator, Build It BLM

In this activity, students will explore the concepts of volume and surface area by using cubes to build solids and cutting nets from grid paper to cover the solids. Before beginning the activity, students will assess their understanding of key terms used in this activity through a vocabulary self-awareness chart (view literacy strategy descriptions). Because students bring a range of word understandings to the learning of new concepts, it is important to assess students’ vocabulary knowledge before interacting with the content. This awareness is valuable for students because it highlights their understanding of what they know, as well as what they still need to learn in order to fully comprehend the content. Have students copy the chart below in their notebook or learning log (view literacy strategy descriptions). Teacher note: make sure students leave plenty of room for examples and definitions since they will be adding to or modifying them throughout the activity.

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<th>Word</th>
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<th>✓</th>
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</tbody>
</table>

Ask students to complete the chart before the activity begins by rating each vocabulary word according to their level of familiarity and understanding. A plus sign (+) indicates a high degree of comfort and knowledge, a check mark (✓) indicates uncertainty, and a minus sign (-) indicates the word is brand new to them. Also, ask students to try to supply a definition and/or example for each word. For words with check marks or minus signs, students may have to make guesses about definitions and examples. Over the course of the activity, allow time for students to revisit their self-awareness charts to add new information and update their growing knowledge about
key vocabulary. *Students may want to add new knowledge with a different color or with ink to indicate how their understanding is changing. Make sure students keep the chart handy and remind them to add to their original entry throughout the activity.*

Below the chart, ask students to explain how a unit of volume, a unit of surface area, and a unit of length are related and how they are different. *Encourage the use of sketches to aid in their explanation.*

**Note to teacher:** The *vocabulary self-awareness* chart is a great formative assessment tool used to create flexible grouping arrangements as understanding of the concepts increases.

Arrange the students in groups and give each group 30 cubes. Give each student 2 sheets of grid paper and scissors. Ask students to suppose that 2 cubes have been glued together to form a rectangular solid as shown to the right. Have them cut out several nets for this solid. Collect and display a variety of the nets produced.

Here are two possible nets for the given 2-cube solid:

![Possible nets for 2-cube solid](image)

Discuss the students’ ideas about what is meant by the volume, surface area, and dimensions of a rectangular solid. *The volume of a rectangular solid is the number of cubes needed to exactly “fill” a jacket; its surface area is the number of unit squares in a jacket which covers each face of the solid exactly once; and its dimensions are the numbers of linear units in its length, width, and height. The volume of the 2-cube solid is 2 cubic units; the surface area of the 2-cube solid is 10 square units, and the dimensions of the 2-cube solid are 2 x 1 x 1. Clarify as needed, using cubes and cut out squares to illustrate differences and relationships. This may be a good time for students to revisit their vocabulary self-awareness chart to make changes.*

![Volume, area, and linear unit](image)

Next, have students build a rectangular solid with dimension 3 x 1 x 1. Ask students to determine how many squares are in the net for this solid. Ask for volunteers to explain how they arrived at
their answer. Some students need to build the net while others are comfortable visualizing the net or counting the exposed faces from the model. Volume of this solid is 3 cubic units, and the surface area is 14 square units.

Note that all rectangular solids have 6 faces. This solid has 4 faces with each having an area of 3 square units (4 x 3) and 2 faces with each having an area of 1 square unit (2 x 1) to give a total surface area of 14 square units. Students may also need to sketch the net and label each face with the dimensions to see the area of each face.

Ask students to think about what a rectangular solid formed by a single row of 10 cubes would look like. Ask each student to independently determine the surface area of this 10-cube, then ask volunteers to describe their methods. A 10-cube rectangular solid arranged in a single row would have a surface area of 42 square units. Encourage discussion of ways of recording the students’ verbalizations using numbers and arithmetic symbols. This could lead to discussions of notation, commutative property, and order of operations. Students may show the notation in the following ways: 4(10x1) + 2(1x1) or 4(10) + 2(1). Students should also make the generalization that when cubes are built in a single as these examples, each of the four long faces of this type of rectangular solid is covered by as many squares as there are cubes and each of the 2 ends is covered by 1 square, so the surface area is equal to four times the number of cubes plus 2 or 4n +2.

Pair students and distribute Build It BLM for student pairs to work together. Then have student pairs join another student pair to form a group of 4 to share what their solutions are. Note: The figures are not all rectangular solids. The intent is that students build on the understanding that surface area is the sum of exposed faces and that volume is the number of cubes used to build the figure.

Close the activity by having the students add to the vocabulary self-awareness chart. Remind students to keep this chart handy as volume and surface area concepts are explored in greater depth during subsequent activities.


Materials List: different sized boxes (one per group of 4) that are cubes and rectangular prisms (pizza, shoe, juice, cereal, etc.), newsprint, rulers, calculators, sticky notes (1 set per group)

In this activity, students will measure real rectangular prisms and cubes, construct the net to scale, and determine the surface area and volume. Prior to beginning the activity, set out the boxes for the activity in the front of the room. Have students predict independently on a sheet of paper the order of the boxes from greatest to least as related to surface area then volume. Determine how students will refer to the boxes ahead of time—either label each box as A, B, C, D, etc., or students can simply refer to the boxes as “pizza” box, “tissue” box, etc. Make sure students justify why their choice makes sense.
Divide students into groups of four and distribute one box to each group, rulers, newsprint and calculators. Students will measure the dimensions of the box and calculate the surface area and volume of the box. *Mark through any volume measurements that may be shown on any of the boxes. Make sure that all groups are using the same unit of measurement (inches, cm or mm) so that the surface area and volume can be compared at the end of the activity.* Next, distribute newsprint so that students can construct a net of the box to scale (depending on the size of the box, they may need to draw each face on several pieces of newsprint and then tape together). On the net, students should label all dimensions and then the area dimensions of each face. All calculations should be shown on a separate sheet of paper. *While monitoring each group, make sure that the labels of the linear dimensions match the area dimension of each face and the calculations for surface area and volume are correct.*

Next, each group will rotate to each box and check the measurements, nets, and calculations. If any corrections or clarifications need to be made, groups will record questions and corrections on sticky notes and place the note on the component they have an issue with. *For example, if the box measurement is incorrect, the sticky note should be placed on the box; if labeling on the net is incorrect or if the net constructed doesn’t match the net of the box, the sticky note should be placed on the net; and if the calculations are incorrect or missing any steps, the sticky note should be on the calculation page.* The surface area and volume of each box should be recorded on a single group sheet of paper before rotating to the next box so that the students can check their original predictions regarding the order of surface area and volume from greatest to least at the end of the activity. Each group will rotate until all boxes are examined.

Students will return to their original box and be ready to justify or explain any comments or questions left by other groups on sticky notes. Allow any groups that need to clarify their solutions to do so for the whole class.

Have students go back to their predictions at the beginning of the activity and compare them to the actual surface area and volume of each of the boxes. *Make a chart with the dimensions for each box, surface area and volume and display on overhead or whiteboard for ease in comparing.* Discuss the comparisons. Were the students accurate in their prediction? Now that they know what the accurate measurements are, how does this make sense? Make sure any misconceptions regarding what surface area and volume “looks” like are addressed.

**Activity 14: Real-World Practice with Rectangular Solids (CCSS: 7.G.6)**

Materials List: Cover It, Fill It BLM, calculators

The intent of this activity is for students to apply surface area and volume skills to real-world situations. To assess understanding of volume, write the following statement on the board: *It would take more than 10,000 one-inch cubes to fill a cube that is 8 feet on each edge.*

Students will use *SQPL (view literacy strategy descriptions)* to write questions that would need to be answered in order to determine whether this statement is true or false. *SQPL promotes*
purposeful learning by prompting students to ask and answer their own questions about content. Pair up students and based on the statement, have them generate 2-3 questions that would need to be answered before determining whether this statement is true or false. Questions might include:

- How many inches are in 1 ft?
- How many inches are in 8 ft?
- Should we use 8 ft or 96 in.?
- How many cubes would be in the bottom layer? How many layers would be needed?

When all student pairs have thought of their questions, ask someone from each team to share questions with the whole class. As students ask their questions aloud, write them on the board. Similar questions will be asked by more than one pair. These should be starred or highlighted in some way. Once all questions have been shared, look over the student-generated list and decide whether additional questions need to be added. This may be necessary when students have failed to ask about important information they need to be sure to learn.

Have students solve the problem with a partner. While monitoring students’ solution strategies, record strategies that will be particularly interesting to discuss as a whole class. Ask the student pair if they would be willing to share their solution strategy during whole class discussion.

Call on pre-determined student pairs to share their solution strategy with the class. When all pre-determined pairs have shared, ask if any other student pairs would like to share how their strategy was similar or different from the strategies presented. Return students’ attention to the list of SQPL questions and ask them to identify which questions were answered. The statement is true. It would take 884,736 one-inch cubes to fill the large cube. Solution strategies may include the following: find the volume of an 8 ft x 8 ft x 8 ft cube, then convert to square inches; convert 8 ft to 96 inches, then find the volume of a 96 in x 96 in x 96 in cube; find the number of cubes in the bottom layer (either ft or in), then multiply by the number of layers (either 8 if using ft or 96 if using in). Refer students to the paper bag models created in Activity 3 to facilitate a discussion about measurement conversion, if needed.

Distribute Cover It, Fill It BLM and calculators to students. Students may work independently or with a partner. Monitor student work by asking guiding questions. To discuss, call student pairs randomly to go to the front of the room and explain/demonstrate the method used to find the solution.

**Activity 15: Practice with Other Prisms (CCSS: 7.G.6)**

Materials List: Prism Practice BLM, calculators, plastic cm cubes, snap cubes or wooden cubes, chart paper, markers

Students have had practice with finding the surface area and volume of rectangular solids and cubes. In this activity, students will find the surface area and volume of figures involving triangles and other quadrilaterals.
Use the sketch shown to generate discussion with students on how to find the surface area and volume of a triangular prism. Ask students to determine the surface area using any method that makes sense to them. *Some students may sketch a net, then label the area dimensions, and other students may simply list the shape of the faces with dimensions.*

1 rectangle face: \(10 \times 14 = 140\) sq. in.
2 rectangle faces: \(13 \times 14 = 2(182) = 364\) sq. in.
2 triangle faces: \(\frac{1}{2} (10)(12) = \frac{1}{2} (120) = 60(2) = 120\) sq. in.

Surface area = \(140 + 364 + 120 = 624\) sq. in.

Next, ask students how they might find the volume of the triangular prism. *Responses will vary according to level of understanding.* To help students see volume in a different way, distribute cubes and ask them to build a rectangular prism with the dimensions of \(2 \times 2 \times 3\). Ask students to determine the volume of this prism. *12 cubic units* Ask students if they could use what they know about the area of one of the faces to help them determine surface area. *Some students may see the volume as 3 “stacks” (height) of \(2\times2\)s or \(3(2\times2)\). Try this with a couple more until students make the generalization that if they know the area of the base of the figure and the height (“stacks”), then they can calculate the area of any prism in which they know how to calculate the area of the base.* Challenge students to use this idea to determine the volume of the triangular prism. *The area of the triangular base \(\frac{1}{2} (10)(12) = 60\) sq in. Since the height of the prism is 14 in, then this means there are 14 stacks of 60, which is 840, so the volume of the triangular prism is 840 cubic inches.* This may be a good time to show that volume can be shown as \(V = Bh\) where \(B\) represents the area of the base. A discussion may need to take place about how to determine which face is the base (\(B\))—on a prism, the parallel faces are considered to be the bases.

Next, students will use a *graphic organizer (view literacy strategy descriptions)* to compare ideas needed in finding the surface area and volume of a rectangular prism with that of a triangular prism. Graphic organizers are visual displays teachers use to organize information in a manner that makes the information easier to understand and learn. Since students have had numerous experiences with finding surface area and volume of *rectangular* prisms, they will process what they know about this concept to transfer it to the new context of finding surface area and volume of a *triangular* prism. Graphic organizers are effective in enabling students to assimilate new information by organizing it in visual and logical ways. Have students sketch a Venn diagram similar to the one below:

---

**Rectangular prism**

**Triangular prism**
Ask students to write the ideas needed to find the surface area and volume of rectangular prisms and triangular prisms that are similar in the space that overlaps both and to write the ideas needed to find the surface area and volume of rectangular prisms and triangular prisms that are different in the corresponding space of either the rectangular prism or the triangular prism. Encourage students to think about the methods used so far and to include graphical ideas in addition to ideas written verbally. An example of a completed Venn diagram is shown below:

Form students into groups of four and distribute 1 sheet of chart paper and a set of markers to each group. Using Round Robin discussion (view literacy strategy descriptions), students will share the content of their individual Venn diagrams one at a time. Students have the opportunity to “pass” on a response, but eventually every student must respond. After initial clockwise sharing, students will determine which responses should be recorded on the group Venn diagram. Using chart paper and markers, students will create a group Venn diagram based on responses shared during Round Robin discussion. This allows all opinions and ideas of the groups to be brought to the teacher’s and the rest of the classmates’ attention. Ask each group to present their group Venn diagram to the whole class. After sharing, ask students if they can generalize how to find the surface area and volume of a trapezoidal prism. Students should be able to generalize if they know how to find the area of any base, if they know that the volume can be found by multiplying the area of the base by the height of the prism, and that the surface area can be found by finding the sum of the areas of all the faces. After this whole class discussion, allow students to add to or modify their individual Venn diagram for reference when solving problems related to the surface area and volume of prisms.

Distribute Prism Practice BLM to give students the opportunity to find the surface area and volume of other prisms besides rectangular solids and cubes. Call on volunteers to explain/demonstrate the method used to find the solution.
Sample Assessments

General Assessments

- Create and use checklists to determine the students’ understanding of measurement concepts.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage the student to create his/her own questions to evaluate his/her understanding of measurement concepts.
- Have the student accurately measure different objects using a variety of measurement tools.
- Have the student draw a scale model of his/her bedroom and create a makeover of the room. The makeover will include the purchase of flooring (carpet, tiles, and so on), paint for the walls and relocation of furniture based on scaled drawings of the pieces of furniture. The student will show all mathematical steps for the work. Work will be placed in the student’s portfolio.
- Have the student complete journal entries using such topics as:
  1. Explain the difference between measurements reported in ft and ft²
  2. Explain what may have happened if two people had different results when measuring the same item. For example, one person measures a board as 18 feet, and another measures it to be 6 feet.

Activity-Specific Assessments

- **Activity 1**: Use the example rubric provided in the blackline masters to evaluate the student’s understanding of the remodeling activity.

- **Activity 2**: The student will solve the problem below correctly:
  Your backyard is a rectangular shape that is 100 feet by 40 feet. The patio in the backyard is 18 feet by 20 feet. How much of the backyard is not covered by the patio? 

  3,640 sq ft is not covered by the patio

- **Activity 8**: The student will find the scale factor correctly of the similar figures below:

  The ratios of corresponding sides are 6/3, 8/4, 10/5. These all reduce to 2/1 so the scale factor of these two similar triangles is 2:1.

- **Activity 10**: The student will solve the problems below correctly:

  The local school district has made a scale model of the campus at West Brook Middle School including a proposed new building. The scale of the model is 1 inch = 4 feet.
a) An existing gymnasium is 8 inches tall in the model. How tall is the actual gymnasium? *32 feet*

b) The new building is 22.5 inches from the gymnasium in the model. What will be the actual distance from the gymnasium to the new building if it is built? *90 feet*

- **Activity 14:** The student can calculate volume correctly in the problem below:

  A tent used for camping is shown below. Find the volume of the tent. *120 cu ft*
Grade 7 Mathematics
Unit 7: Rational Number Fluency

Time Frame: Approximately 3 weeks

Unit Description

Adding, subtracting, multiplying and dividing rational numbers is the culmination of numerical work with the four basic operations. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in Grade 7 depends on rational number arithmetic, fluency with rational number arithmetic should be the goal in Grade 7. In this unit, students will apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers and solve real-life and mathematical problems using numerical equations. The intent of the activities contained in this unit is to enhance understanding of rational number operations and to extend ideas explored in previous units related to operations with fractions and integers.

Student Understandings

Students demonstrate their understanding of rational number operations by solving real-world and mathematical problems involving the four operations with rational numbers.

Guiding Questions

1. Can students demonstrate understanding of rational number operations fluently?
2. Can students apply properties of operations to add and subtract rational numbers?
3. Can students apply properties of operations to multiply and divide rational numbers?

Unit 7 Common Core State Standards (CCSS)

<table>
<thead>
<tr>
<th>CCSS#</th>
<th>CCSS Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.NS.1</td>
<td>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</td>
</tr>
<tr>
<td>7.NS.2</td>
<td>Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</td>
</tr>
<tr>
<td>7.NS.3</td>
<td>Solve real-world and mathematical problems involving the four operations</td>
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Expressions and Equations

| 7.EE.3 | Solve real-life and mathematical problems using numerical and algebraic expressions and equations. |

Sample Activities

Activity 1: What’s the Key? (CCSS: 7.EE.3)

Materials List: 3 x 5 lined index cards (4 per student)

Sketch the graphic organizer [(view literacy strategy descriptions)] below and have students copy. Make sure students leave plenty of room in each block. Graphic organizers are effective in enabling students to assimilate rational number operations by organizing the words that imply each operation in a visual and logical way.

Ask students to think of key words or phrases that mean *add, subtract, multiply* and *divide* and place the words in the appropriate boxes. It may help if they think about situation problems they have solved in prior units. For example, *key words that mean addition may include add, sum, increase, altogether...*

Next, have students share the words in their graphic organizer with a partner allowing them to modify their own list. Call on volunteers to add words to the class graphic organizer on the board until complete. Some words may overlap categories. An example may be the word “altogether” which could mean both addition and multiplication. An example of key words for each operation may include:

- **Addition:** *add, sum, increased by, altogether, more than, combined, total of*
- **Subtraction:** *subtract, difference, minus, decreased by, how many more, less than*
- **Multiplication:** *multiply, product, groups of, increased by a factor of*
- **Division:** *divide, quotient, separated, per, out of, percent (divide by 100)*

Students will use this graphic organizer to identify key words for rational number operations to help them to select the appropriate operation when solving real-life problems. Explain to students that they can modify their graphic organizer throughout this unit as they encounter situation problems with keywords excluded from their initial graphic organizer.

Write each of the following situations on the board in any order. Students will write the expression used to solve the problem, then solve and write the key words used to
determine which operation to use. Ask for volunteers to share their solutions with the class.

1. Erin has \(\frac{1}{2}\) bag of candy and Mary has \(\frac{1}{4}\) bag of candy. How much candy will they have if they combine both bags?
   
   \[
   \text{Numerical expression(s): } \frac{1}{2} + \frac{1}{4} \text{ or } \frac{1}{4} + \frac{1}{2} \\
   \text{Solution: } \text{Erin and Mary will have } \frac{3}{4} \text{ bag when they combine each of theirs.}
   \]
   
   Key word(s): and, combine

2. Larry jumped 10 feet and 6 inches in the long jump, which is 18 inches farther than Will jumped. How far did Will jump?
   
   \[
   \text{Numerical expression: } 126 - 18 \\
   \text{Solution: } \text{Will jumped } 9 \text{ feet.}
   \]
   
   Key word: further (difference)

3. The 7th grade class planted a flower garden at the entrance to the school. How much space can be used to plant flowers if the dimensions of the garden are 8 feet by 3 feet?
   
   \[
   \text{Numerical expression(s): } 8 \times 3 \text{ or } 3 \times 8 \\
   \text{Solution: The space that can be used to plant flowers is } 24 \text{ square feet.}
   \]
   
   Key word: dimensions \((l \times w)\)

4. Jamie and Alex baked 50 cookies for the bake sale and packaged them 6 cookies per bag. How many packages did they make?
   
   \[
   \text{Numerical expression: } 50 \div 6 \\
   \text{Solution: They were able to make } 8 \text{ packages of cookies with } \frac{1}{3} \text{ of a package remaining. Since 2 cookies make up } \frac{1}{3} \text{ of a package, that is not enough to make a package of 6.}
   \]
   
   Key word: per

Allow students to modify their graphic organizer at this time to include any key words not previously recorded.

Distribute index cards (four per student). On the lined side, have students write a situation problem on each card (one for each operation). According to student readiness, assign some students whole numbers while others use fractions, percents or integers. The difficulty level can also be increased by requiring students to write problems using more than one operation to solve. If students are having difficulty in writing problems that make sense, have them refer to the graphic organizer. Next, students will switch their cards with a partner. On the blank side, students will write the numerical expression used to solve the problem, solve, then circle the key words that the student used to help them determine which operation to use. Cards will be returned to the author of the cards to be checked for accuracy. While monitoring each pair, choose cards with interesting or challenging situations to have the class solve. Close the activity by asking students to add to their graphic organizer. Remind them to use it during the next several activities when they need assistance in determining the correct operation to use in solving real-life problems.
Activity 2: Basic Operations—Addition and Subtraction (CCSS: 7.NS.1, 7.EE.3)

Materials List: colored tiles (15 per student), ½ cm grid paper (2 sheets per student), Add or Subtract BLM

In this activity, students will use tiles to express their perceptions of addition and subtraction and draw diagrams using grid paper to illustrate the meanings of the operations. When students develop the conceptual understanding of rational number operations, they are able to choose the appropriate operation when solving real-life problems. This activity is particularly useful to students still struggling with fraction and/or integer operations. Additionally, students will write situation problems modeled by their diagrams. NOTE: Using the number line to model addition and subtraction was introduced in an earlier unit, but it is definitely worth the review for students still struggling with the concept.

Distribute tiles and one sheet of grid paper to students. If available, a document camera will be particularly useful for discussion purposes.

Write “9 + 5” on the board and ask students to use their tiles to represent the meaning of this expression and share with a partner. Most students will count out 9 tiles and 5 tiles combining the groups to show addition as the process of “putting together” sets. Ask for volunteers to come to the board to explain their thinking. Students will use grid paper to sketch the tile arrangement. A review of the commutative property may be useful here.

Ask students if addition meaning the “putting together” of sets applies to fractions and integers and to give examples. Addition of all rational numbers is simply combining two sets. For example, -2 + 1 means that you are combining a -2 with a positive 1 to get a -1; and ½ + ¾ means that you are combining ½ with ¾ to get 1 ¼ . For another model of integer addition, review with students how to use a number line to add integers.

Write “9 – 5” on the board and ask students to use their tiles to represent the meaning of this expression and share with a partner. Many students will form a group of 9 and take 5 tiles from this group. Other students may determine how many tiles must be added to a group of 5 tiles to match the group of 9 tiles (counting up). Still others may compare the two groups of tiles and state that 9 – 5 is the difference between the two groups. Students will sketch their tile arrangements on grid paper.

Sketch the following diagrams on the board and ask students to work with a partner to create word problems whose solution could be found using each method:

9 – 5 = 4
In the first diagram, carrying out $9 - 5$ by taking 5 tiles from 9 tiles is an example of the \textit{take-away} method of subtraction. This method is appropriate in responding to the question: “If a student has 9 pencils and gives 5 of them away, how many pencils does the student have left?” The second diagram is an example of determining $9 - 5$ by finding how many tiles must be added to 5 tiles to match 9 tiles and is called the \textit{difference method} of subtraction. This method is appropriate for this question: “If one student has 9 pencils and another has 5, how many more pencils does the first student have than the second?” Ask students for the key words/phrases that mean subtraction and insert in the \textit{graphic organizer} (left, \textit{how many more}). Continue extending the idea of subtraction as taking away or difference using a number line as a review.

Next, ask students to use their tiles to show ways they think the operations of addition and subtraction are related. Have students share with a partner before bringing the whole class together to share.

\textbf{If it hasn’t come up, the next two ideas of how addition and subtraction are related are helpful when thinking of writing algebraic equations:}

- \textit{8 + 5: In the process of addition, both sets of tiles are known and the total number of tiles are unknown.}

- \textit{13 – 8: In the process of subtraction, the number of tiles in one set is known and the total number of tiles is known, but the number of tiles in the other group is unknown.}

Separate students into groups of four and distribute the second sheet of grid paper to each student. Place a transparency of Add or Subtract BLM on the overhead, revealing only Situation A. Ask the groups to:

1. Use tiles to build a model of the situation (or use grid paper to sketch).
2. Make one or more observations about the mathematical relationships they notice in their model and write those observations under the model on grid paper.
3. Write a mathematical equation to describe the mathematical relationships they noticed in #2.

Since students are asked only to model and observe the relationships in this situation and there are no questions to answer, there are many possible observations. For example, in Situation A, students may sketch a set of 12 tiles and a set of 15 tiles and observe that the total number is $12 + 15 = 27$ or they may sketch a set of 12 tiles beneath a set of 15 tiles and compare to show the difference between the numbers is $15 - 12 = 3$.

Have volunteers share their models, observations and equations at the overhead.

Activity 3: Basic Operations—Multiplication and Division (CCSS: 7.NS.2, 7.EE.3)

Materials List: colored tiles (15 per student), ½ cm grid paper (2 sheets per student), Multiply or Divide BLM

Students will use tiles to express their perceptions of multiplication and division and draw diagrams using grid paper to illustrate the meanings of the operations. Distribute tiles and grid paper.

Write “3 x 4” on the board and ask each student to use tiles or grid paper to show what this expression means to them. Have students explain to a partner how their model represents “3 x 4.” Students may think of 3 x 4 as 3 groups of 4. Others may see it as 4 groups of 3. These are “repeated addition” models of multiplication. Have students sketch their model on grid paper. Discuss the commutative property of multiplication here.

If it doesn’t come up, form a rectangle with the dimension of 3 by 4 and ask students how this model represents multiplication. This model is the “area” representation of multiplication.

Ask students to think about how the model of multiplication can be used with fractions and integers. Pair students and assign fractions to one student and integers to the other student. Each student will use tiles or grid paper to prove how the multiplication models work for their assigned rational number. Using discussion (view literacy strategy descriptions), specifically Think-Pair-Square-Share, students will share thoughts with their partner of how they know the multiplication model works with their
rational number form. Discussion enables students to improve learning and remembering when they participate in the dialogue about rational number operations. After pairs have had a chance to share with each other, have pairs of students share with other pairs, forming, in effect, small groups of four students. Monitor the brief discussions and elicit responses afterward. Challenge students to find an alternative perspective to the ones given by others. Call on volunteers to come to the overhead to demonstrate how the multiplication model works for their assigned rational number. This short-term discussion works best when a diversity of perspectives is expressed. Listen for the following key ideas when monitoring the discussion:

**Fractions:** 3 x \( \frac{1}{2} \) means “3 groups of \( \frac{1}{2} \)” or “\( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \)” which is the repeated addition model of multiplication. So 3 x \( \frac{1}{2} \) = 1 \( \frac{1}{2} \)

An area model will also work here.

**Integers:** 2 x -3 means “2 groups of -3” or “-3 + -3 + -3” which is the repeated addition model of multiplication. So -3 + -3 = -6 and -6 + -3 = -9

Write “15 ÷ 3” on the board and ask students to use tiles or grid paper to show what this expression means. Have students explain to a partner how their model represents “15 ÷ 3.” Then have students write a word problem that matches this expression to determine if the model makes sense.

**Method 1:** Representing 15 ÷ 3 as 15 divided into groups of 3 is an example of the “grouping” method of division, as in the question: “If each student is to receive 3 pencils, how many students will 15 pencils supply?”

**Method 2:** Representing 15 ÷ 3 as 15 divided into 3 groups is an example of the “sharing” method of division, as in the question: “If 15 pencils are divided equally among 3 students, how many will each student get?”

Note that the quotient “five” is represented differently in each model and will look differently in the context of a real-world situation problem.

Have students take turns reading their word problems and asking the class to identify the division method that is associated with that problem. If students have difficulty in determining which method works, ask them to sketch or use tiles to model what the problem represents. This will help them identify the correct division method.
Ask students to think about how the model of division can be used with fractions and integers and explain to a partner how they think this model applies to fractions and integers. Use the previous discussion strategy for students to share.

The model makes sense for fractions but not for integers:

Fractions: $\frac{3}{4} \div \frac{1}{2}$ means $\frac{3}{4}$ divided into groups of $\frac{1}{2}$ such as the following problem:

There is $1 \frac{3}{4}$ of a pizza left and we want to see how many people can enjoy $\frac{1}{2}$ a pizza each.

Integers: $15 \div -3$ means 15 divided in groups of -3 (not really an effective model) but can be thought of as the opposite of multiplication as in: $-3x = 15$.

Separate students into groups of four and distribute the second sheet of grid paper to each student. Place a transparency of Multiply or Divide BLM on the overhead, revealing only Situation A. Ask the groups to:

1. Use tiles to build a model of the situation (or use grid paper to sketch).
2. Make one or more observations about the mathematical relationships they notice in their model and write those observations under the model on grid paper.
3. Write a mathematical equation to describe the mathematical relationships they noticed in #2.

Since students are asked only to model and observe the relationships in this situation and there are no questions to answer, there are many possible observations. For example, in Situation A (which students saw in the previous addition and subtraction activity), students should show that there are 1 $\frac{1}{4}$ as many CDs in Mary’s collection as in Erin’s or that Erin has $\frac{4}{5}$ as many CDs as Mary (see diagram which follows).

Have volunteers share their models, observations and equations.
Activity 4: Let’s Party (CCSS: 7.NS.3)

Materials List: Party planning books or catalogs (optional), Recipe BLM, Party Requirements BLM, Budget Spreadsheet BLM, ½ cm grid paper

In this activity, students are in charge of a catering company and bidding on a job to plan a birthday party. They are given a budget and must submit a party proposal that includes a budget spreadsheet, written description of party and events, menu, and map of room. Begin by reviewing operations with fractions in the context of modifying a recipe. Distribute Recipe BLM and have students work with a partner to find the amount of each ingredient when the serving size is decreased to 18 servings and increased to 108 servings. Monitor student work to ensure accuracy. Encourage students who may be struggling to sketch a model using smaller numbers to help them to remember the process of fraction multiplication.

Present the following words to students: party, menu, percentage, budget, scale drawing, expenses, and guests. Students will use these words to write a lesson impression (view literacy strategy descriptions) text that will enable them to make a guess as to what the activity will involve. Lesson impressions create situational interest in the content to be covered by capitalizing on students’ curiosity. By asking students to form a written impression of the topic to be discussed, they become eager to discover how closely their impression text matches the actual content. Have students write a short paragraph using the given words in the context they think will be used for the activity. When students finish their impression texts, invite volunteers to read what they have written to the class. Anticipation is heightened when several students share their different impressions, leaving students to wonder whose is the closest to the actual content.

Example of Lesson Impression for a real-world problem

Impression Words: party, menu, percentage, budget, scale drawing, expenses and guests.

Impression Text: My mom asked me how many guests I want to invite to my birthday party next month because she needed to plan a budget and the menu. She said that only a small percentage would be for decorations because the other expenses included food, entertainment, and invitations. I asked if we could have the party in the garage but she said to give her a scale drawing to see how to arrange everything, and then we could decide if this is the best place.

Explain to students that they own a catering company and their task is to bid on a job to plan a birthday party for a 13-year-old. Given a budget of $250, they will submit a party proposal for 30 guests that includes a budget spreadsheet, written description of party and
events, menu, and map of the room. Distribute Party Requirements BLM and ask students to name their company and to decide on a party theme. Discuss the required budget items and the specific components of the proposal (see BLM). Have students write a rough draft menu for the party (as they begin shopping, they may change their minds and have to modify the menu!)

Students will begin this activity by researching costs to purchase food, decorations, consumables, entertainment, party favors and miscellaneous expenses and record on the Budget Spreadsheet BLM. Provide party sale ads and grocery ads from actual stores or copies of Internet ads. If students have access to computers, the following websites will be helpful for this activity:

- http://www.orientaltrading.com
- http://www.birthdaypartyideas.com
- http://www.birthdaydirect.com
- http://www.birthdayexpress.com
- http://www.celbrateexpress.com
- http://www.netgrocer.com

Based on their budget, have students write a final menu and description of the party and events. Finally, students will create a scale drawing of the birthday party room including where all the food, tables and entertainment will go. Have students assemble all finished products together into a bid package. Optional product: students may want to create an electronic bid package using PowerPoint but must include all components given on the Party Requirements BLM.

At the conclusion of the activity, have students return to their impression texts and discuss how closely they predicted the actual content of the lesson.

**Activity 5: Integer Sums and Differences (CCSS: 7.NS.1, 7.EE.3)**

Materials List: paper and pencil

Pose the following questions to students and have them work with a partner to develop a strategy to find a solution. They should be prepared to justify their reasoning.

- Which integers, when added to -15, give a sum greater than 0? *Any integer greater than 15—guide students to generalize this statement.*
- Which integers, when added to -15, give a sum less than 0? *Any integer less than 15—guide students to generalize this statement.*
- Which integers, when added to -15, give a sum of 0? *The only integer is 15.*

Ask students to describe to a partner what they think the following terms mean in the context of opening a bike shop: profit, start-up costs, and down payment. Part of the discussion should include the idea that most businesses start “in the hole,” a negative value. Tell students that their family has decided to open a bike shop, and to get started,
they will have to make a down payment on the shop, buy bicycles and other supplies to stock the shop, and invest in business equipment and paper to keep track of income and expenses.

Display the following business transactions on the overhead or white board and have students write an addition sentence for each transaction that shows how the new balance is calculated from the old balance:

<table>
<thead>
<tr>
<th>Business Transaction</th>
<th>Cost</th>
<th>Addition sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Down payment of two months’ shop rent</td>
<td>$1800</td>
<td>$0 + (-$1800) = -$1800</td>
</tr>
<tr>
<td>2. Payment for 20 new bicycles:</td>
<td>$2150</td>
<td>-$1800 + (-$2150) = -$3190</td>
</tr>
<tr>
<td>3. Down payment for rent of office equipment:</td>
<td>$675</td>
<td>-$3950 + (-$675) = -$4625</td>
</tr>
<tr>
<td>4. Business insurance for 6 months</td>
<td>$2300</td>
<td>-$4625 + (-$2300) = -$6925</td>
</tr>
<tr>
<td>5. Sale of 3 bicycles</td>
<td>$665</td>
<td>-$6925 + $665 = -$6260</td>
</tr>
<tr>
<td>6. Sale of two helmets and one baby seat</td>
<td>$95</td>
<td>-$6260 + $95 = -$6165</td>
</tr>
<tr>
<td>7. Advertising in the yellow pages</td>
<td>$250</td>
<td>-$6165 + (-$250) = -$6415</td>
</tr>
<tr>
<td>8. Sale of 6 bicycles</td>
<td>$1150</td>
<td>-$6415 + $1150 = -$5265</td>
</tr>
<tr>
<td>9. Refund to an unhappy customer</td>
<td>$225</td>
<td>-$5265 + (-$225) = -$5490</td>
</tr>
<tr>
<td>10. Sale of two bicycles, two helmets and two air pumps</td>
<td>$750</td>
<td>-$5490 + $750 = -$4740</td>
</tr>
<tr>
<td>11. Return of 5 bicycles to manufacturer</td>
<td>$530</td>
<td>-$4740 + $530 = -$4210</td>
</tr>
</tbody>
</table>

Note: This is a refund from return of five bicycles to the manufacturer which is a profit to the bike shop.

Ask students to make observations about the process of addition in these transactions. Answers will vary. Students should see that “adding a negative transaction” will make the business go further “in the hole” and “adding a positive transaction” will help the business “break even.” Have students use a number line to prove this generalization.

Next, ask students to describe how the operations of addition and subtraction of integers are related. Answers will vary. Students should see that addition and subtraction are inverse operations and that all subtraction statements can be written as equivalent addition statements. For example, 8 – 10 is the same expression as 8 + -10. Ask students if any of the above addition sentences can be written as subtractions sentences. The following transactions can be written: down payment of two months shop rent 0 – 1800 = 1800; payment for 20 new bicycles -1800 – 2150 = 3190; down payment for rent of office equipment -3950 – 675 = -4625; business insurance for 6 months -4625 – 2300 = -6925; Advertising in the yellow pages -6165 – 250 = -6415; refund to an unhappy customer -5265 – 225 = -5490.

Then ask students if they can determine whether -8 - -10 is 2. You may need to use the following debt example to explain: Suppose you owe your mother $8 (represented by -8). She agrees to let you work off the debt by doing yard work so that you can subtract $10 of the debt. This is subtracting a negative number, or -8 - -10. The result is that your mother now owes you $2, so you are $2 ahead rather than $2 in debt.

An excellent extension to this activity is found at the link below: http://illuminations.nctm.org/LessonDetail.aspx?ID=L699
Using old batteries and a voltage sensor, students get a real feel of the meaning of negative and positive numbers. Students explore addition of signed numbers by placing batteries end to end (in the same direction or opposite directions) and observe the sum of the batteries’ voltages. The lesson includes an instructional plan, student activity page and assessment options.

**Activity 6: Integer Products and Quotients (CCSS: 7.NS.2, 7.EE.3)**

Materials List: paper and pencil, Integer Practice BLM

In this activity, students explore patterns of multiplication to derive the rule for multiplying and dividing integers.

Present the situation below to students:

The temperature for the past 8 hours has been changing at the rate of -1.5° each hour. The meteorologist predicts that the temperature will continue changing like this for the next 6 hours. The present reading is 0°.

Ask students the following questions. Have them write the expression used to solve.

Students can sketch a thermometer (vertical number line) to help them “see” the solution.

<table>
<thead>
<tr>
<th>Question</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) What was temperature reading 7 hours ago?</td>
<td>-7 × -1.5 = 10.5°</td>
</tr>
<tr>
<td>b) What temperature is predicted for 6 hours from now?</td>
<td>6 × -1.5 = -9°</td>
</tr>
<tr>
<td>c) When was the temperature reading 6°?</td>
<td>6 ÷ 1.5 = 4 hours ago</td>
</tr>
<tr>
<td>d) When is the temperature expected to be -8°?</td>
<td>8 ÷ 1.5 = 5 1/3 hours from now; or between 5 and 6 hours from now</td>
</tr>
</tbody>
</table>

Have students explain to a partner how multiplication can be related to addition in the context of the temperature situation above. Students should see when figuring out what the temperature was 6 hours ago, they can add -1.5° six times. So -1.5 × 6 = -1.5 + -1.5 + -1.5 + -1.5 + -1.5 + -1.5. Ask students why it makes sense that -1.5 × 6 is a negative number. Because the temperature was already below zero and kept dropping, which makes it “more negative.”

Write the following pattern on the board and ask students to describe to a partner any patterns observed in the way the products change as the integers multiplied by 5 get smaller. As the size of the group decreases, the products decrease by 5 at each step. Also ask what multiplication “means” at each step. For example, 5 × 5 means 5 groups of 5, 5 × 4 means 5 groups of 4 and so on.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 × 5</td>
<td>25</td>
</tr>
<tr>
<td>5 × 4</td>
<td>20</td>
</tr>
<tr>
<td>5 × 3</td>
<td>15</td>
</tr>
<tr>
<td>5 × 2</td>
<td>10</td>
</tr>
<tr>
<td>5 × 1</td>
<td>5</td>
</tr>
<tr>
<td>5 × 0</td>
<td>0</td>
</tr>
</tbody>
</table>
Next, ask students to use the patterns observed to predict 5 × -1. Students may need to think about the meaning of 5 × -1 as 5 groups of -1 or a picture of - - - - - to see why the answer is -5. Students could also sketch a number line and count by 5s to see the answer as -5 since the pattern of the product is decreasing by 5 each time.

Have students write the next four equations in the pattern.

- 5 × -2 = -10
- 5 × -3 = -15
- 5 × -4 = -20
- 5 × -5 = -25

What pattern do students notice? Pattern is continuing to decrease by 5. Five groups (positive) of any negative number is a negative product. So, 5 × -100 would be -500.

Explore another pattern to enable students to derive the rule for integer multiplication. Have students complete the following equations and use the pattern they observe to predict the product of -1 × -4. Tell students to be ready to justify their prediction.

5 × -4 = ?
4 × -4 = ?
3 × -4 = ?
2 × -4 = ?
1 × -4 = ?
0 × -4 = ?
-1 × -4 = ?

Ask students to write the next four equations in the pattern and ask them to write a rule that can be used to multiply ANY integers.

- -2 × -4 = 8
- -3 × -4 = 12
- -4 × -4 = 16
- -5 × -4 = 20

Students should be able to generalize from the previous patterns that multiplying integers with the same sign results in a positive product and multiplying integers with different signs results in a negative product. A discussion about the commutative property may be necessary.

Next, ask students to describe how the operations of multiplication and division of integers are related. Answers will vary. Students should see that multiplication and division are inverse operations. Ask students how they can use what they know about multiplication to find the quotient of -25 ÷ 5. Since multiplication is the inverse of division, students could think of the quotient this way: 5 × [ ] = -25. Students know from multiplication rules that multiplying numbers with different signs will result in a negative product, so the answer must be -5. Give students a few more division problems until they are able to generalize that dividing numbers with the same sign results in a positive quotient and dividing numbers with different signs results in a negative quotient.

Distribute Integer Practice BLM and have students complete independently, then share strategies and solutions with a partner. Call on volunteers to share with the whole class at the overhead or board justifying the strategy used.
Sample Assessments

General Assessments:

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage students to create their own questions.
- Ask students to create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- Have students complete math learning log entries (view literacy strategy descriptions) by responding to prompts such as:
  A. Without calculating the sum, how can you decide if the sum of two integers is positive? Negative? Zero? Explain using mathematics, words, and/or a sketch. *Answers will vary*
  B. Without calculating the difference, how can you decide if the difference of two integers is positive? Negative? Zero? Explain using mathematics, words, and/or a sketch. *Answers will vary*
  C. Without actually multiplying, how can you decide whether the product of two integers is positive? Negative? Zero? Explain using mathematics, words, and/or a sketch. *Answers will vary*
  D. Without actually dividing, how can you decide whether the quotient of two integers is positive? Negative? Zero? Explain using mathematics, words, and/or a sketch. *Answers will vary*

Activity-Specific Assessments:

- **Activity 5**: Records at Jefferson Hospital showed the following information about the number of patients received and discharged.
  - Day 1: received 12 patients and discharged 9 patients
  - Day 2: received 14 patients and discharged 21 patients
  - Day 3: received 5 patients and discharged 14 patients
  - Day 4: received 11 patients and discharged 10 patients

The students will write a number statement to determine how the number of patients in the hospital at the end of the four-day period compared with the number of patients at the start of the four-day period.

\[12 - 9 + 14 - 21 + 5 - 14 + 11 - 10 = -12 \text{ patients or } 12 + -9 +14 + -21 + 5 + -14 + 11 + -10 = -12 \text{ patients. There were 12 fewer patients at the end of the four-day period than at the start of the four-day period.}\]

- **Activity 6**: Students will write an equation to represent the situations described below.
A. The temperature at noon was -13°C. For the next six hours, the temperature changed by an average of 1.8° per hour. What was the temperature at 6:00 a.m.? 

-13 + (6 \times 1.8) = -2.2°

B. The average temperatures (in °C) for Fairbanks, Alaska, for each month of the year are -25, -20, -13, -2, 9, 15, 17, 14, 7, -4, -16 and -23. What is the average of these monthly temperatures? 

\((-25 + -20 + -13 + -2 + 9 + 15 + 17 + 14 + 7 + -4 + -16 + -23) ÷ 12 = -3.4°\)